Nonlinear dynamical analysis for coupled fluid-structure systems

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ABSTRACT
The present research concerns the numerical dynamical analysis in elasto-acoustics, taking into account the geometrical nonlinearities induced by the large displacements/deformations of the structure and assuming the internal acoustic fluid occupying an internal cavity coupled to the structure to remain in a linear range of vibration. More particularly, the modeling includes sloshing and capillarity effects on the free surface. A numerical application is presented.

Keywords: Fluid-structure interaction – Sloshing – Capillarity – Reduced-order model – Geometric nonlinearities

INTRODUCTION
The structural-acoustic system under consideration is made up of a tank structure filled with a linear inviscid compressible fluid. Gravity effects and surface-tension effects of the free surface and corresponding coupling terms are taken into account as described in [1]. A linear elastic constitutive equation is considered for the structure. It is also assumed that the structure undergoes sufficiently large deformations and large displacements in order to consider the geometrical nonlinear effects [2], but also sufficiently moderate so that the fluid behavior remains linear [3]. A total Lagrangian formulation around a static equilibrium state taken as a reference configuration is used.

DESCRIPTION OF THE COMPUTATIONAL MODEL
Let $\mathbf{U}(t)$, $\mathbf{P}(t)$, and $\mathbf{H}(t)$ be the $\mathbb{R}^{n_S}$, $\mathbb{R}^{n_F}$, and $\mathbb{R}^{n_H}$-vectors corresponding to the finite element discretization of the structural displacement, fluid pressure, and free-surface elevation fields. The computational model is then written [3] as,

$$[M_S]\,\ddot{\mathbf{U}}(t) + [D_S]\,\dot{\mathbf{U}}(t) + [K_S]\,\mathbf{U}(t) + \mathbf{F}^{\mathrm{NL}}(\mathbf{U}) + [C_{pu}]\,\mathbf{P}(t) + [C_{\eta u}]^T\,\mathbf{H}(t) = \mathbf{F}^S(t)\ , \quad (1)$$

$$-[C_{pu}]^T\,\ddot{\mathbf{U}}(t) + [M]\,\ddot{\mathbf{P}}(t) + [D]\,\dot{\mathbf{P}}(t) + [K]\,\mathbf{P}(t) - [C_{p\eta}]^T\,\ddot{\mathbf{H}}(t) = \mathbf{0}\ , \quad (2)$$

$$[C_{\eta u}]\,\mathbf{U}(t) + [C_{p\eta}]\,\mathbf{P}(t) + [K_{gc}]\,\mathbf{H}(t) = \mathbf{0}\ , \quad (3)$$

in which $[M_S]$, $[D_S]$, $[K_S]$ and $[M]$, $[D]$, $[K]$ are the mass, dissipation and stiffness matrices for the structure and the corresponding one for the acoustic fluid, where $[C_{pu}]$, $[C_{\eta u}]$, and $[C_{p\eta}]$ are coupling matrices, and where $[K_{gc}]$ is the stiffness matrix of the free surface induced by the gravitational and the capillarity effects [4, 1, 3]. The $\mathbb{R}^{n_S}$-vector $\mathbf{F}^{NL}(\mathbf{U})$ is the nonlinear term issued from the large displacements/deformations induced by the geometrical nonlinearities. An adapted numerical nonlinear reduced-order model of order N requiring the numerical computation of the elastic modes of the structure with fluid added mass effect, of the acoustic modes of the fluid, and of the sloshing modes of the free surface [1] is proposed in [3]. Such computation on mid-power computers can be very challenging when large finite element meshes are involved. The original computational strategy [5] that allows for circumventing these difficulties is used in this analysis. The nonlinear reduced-order model is then written as

$$\mathbb{X}(t) = \begin{bmatrix} \mathbf{U}(t) \\ \mathbf{P}(t) \\ \mathbf{H}(t) \end{bmatrix} = [\,\mathbf{\Phi}\,]\,\mathbf{Q} \quad , \quad [\,\mathbf{\Phi}\,] = \begin{bmatrix} [\Phi^S] & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\Phi^F] & [\Phi^{FH}] \\ \mathbf{0} & \mathbf{0} & [\Phi^H] \end{bmatrix} \tag{4}$$

in which $\mathbf{Q}$ is the $\mathbb{R}^N-$vector of the generalized coordinates, solution of the nonlinear dynamical equation

$$[\mathcal{M}_{FSI}]\,\ddot{\mathbf{Q}} + [\mathcal{D}_{FSI}]\,\dot{\mathbf{Q}} + [\mathcal{K}_{FSI}]\,\mathbf{Q} + \boldsymbol{\mathcal{F}}^{\mathrm{NL}}(\mathbf{Q}) \;=\; \boldsymbol{\mathcal{F}}, \tag{5}$$

in which $[\mathcal{M}_{FSI}]$, $[\mathcal{D}_{FSI}]$, and $[\mathcal{K}_{FSI}]$ are the reduced mass, damping, and stiffness matrices of the coupled fluid-structure system, where $\boldsymbol{\mathcal{F}}$ and $\boldsymbol{\mathcal{F}}^{\mathrm{NL}}(\mathbf{Q})$ are the $\mathbb{R}^N-$vectors of the reduced external force and of the reduced nonlinear force that is obtained numerically from the explicit construction of the quadratic and cubic reduced stiffness terms with the finite element method [2].

NUMERICAL APPLICATION

The coupled fluid-structure system under consideration is made up of a cylindrical tank with external radius $R_e = 3.06\times 10^{-2}\,m$, thickness $e = 2.79\times 10^{-4}\,m$, and height $h = 7.97\times 10^{-2}\,m$, partially filled with an acoustic fluid with height $h_f = 3.05\times 10^{-2}\,m$. It is described in a global cartesian coordinate system $(O, \mathbf{e}_1, \mathbf{e}_1, \mathbf{e}_3)$, where O is the center of the cylinder basis, and where the cylinder axis is defined along $\mathbf{e}_3$. The structure is composed of a linear isotropic homogeneous elastic material for which the Young modulus, the Poisson ration, and the mass density are $E = 2.05\times 10^{11}\,N.m^{-2}$, $\nu = 0.29$, and $\rho_S = 7{,}800\,Kg.m^{-3}$. The fluid has mass density $\rho_F = 1{,}000\,Kg.m^{-3}$ and sound velocity $c_F = 1{,}460\,m.s^{-1}$. A fixed boundary condition is applied at the bottom of the cylinder. Furthermore, capillarity effects are added in the numerical model with surface tension coefficient $\sigma_\Gamma = 0.0728$ and contact angle $\alpha = 83°$. The main curvature radii R_1 and R_2 of the free surface and the numerical coefficients that characterize the triple line are computationally obtained in each node of the mesh according [4].

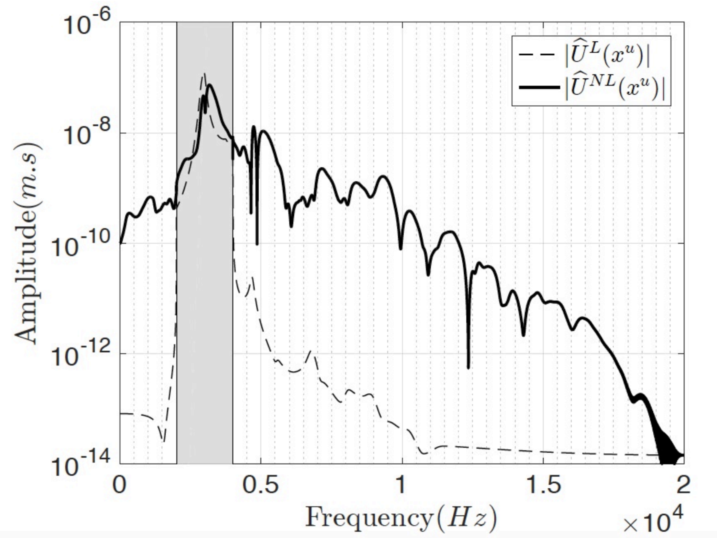

Fig. 1: Graph $\nu \mapsto ||\widehat{u}(2\pi\nu)||$ of the structural displacement for both linear (dashed line) and nonlinear (full line) cases.

The finite element model of the coupled fluid structure system is constructed using $m_S = 111{,}746$ and $m_F = 133{,}719$ three dimensional solid finite elements with 10 nodes for the structure and for the acoustic fluid (tetrahedral finite elements), $m_H =$ 3,392 bi-dimensional finite elements with 6 nodes for the free-surface and $m_\gamma = 144$ one-dimensional finite elements with 3 nodes for the triple line. Thus, there are $n_S = 658{,}209$ dofs, $n_F = 194{,}354$ dofs and $n_H = 6{,}929$ dofs corresponding to the displacement unknowns of the structure, to the pressure unknowns of the fluid, and to the normal displacement unknowns of the free surface. The damping matrices of the acoustic fluid and of the structure are defined as $[D] = \tau_F\,[K]$ and $[D_S] = \tau_S\,[K_S]$ in which $\tau_F = 10^{-5}$ and $\tau_S = 10^{-6}$. A small patch located on one side of the structure is subjected to a transverse load. Such load is of intensity $s_0 = 0.001\,N$ and uniformly excites the frequency band $\mathbb{B}_e = [2000\,,\,4{,}000]\,Hz$. In such excitation frequency band, it should be noted that 11 structural modes and no acoustic modes neither sloshing modes are excited. The computations are carried out with a nonlinear reduced-order model whose optimal order, issued from a convergence analysis is set to $\{N_S, N_F, N_H\} = \{100, 40, 1000\}$. Let ν be the frequency in Hz. The figures **1** and **2** display the graph $\nu \mapsto ||\widehat{u}(2\pi\nu)||$

of the structural displacement and the graph $\nu \mapsto \widehat{h}(2\pi\nu)$ of the normal displacement of the free surface located on the triple line for both linear and nonlinear cases. It can be seen that the presence of geometrical nonlinearities modifies the nonlinear behavior of the fluid-structure system and that non-expected resonances do appear outside $\mathbb{B}_e$.

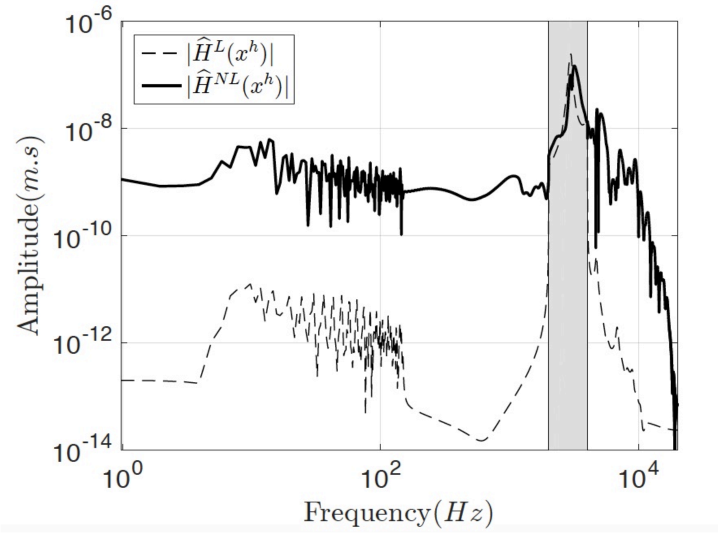

Fig. 2: Graph $\nu \mapsto \widehat{h}(2\pi\nu)$ of the normal elevation for both linear (dashed line) and nonlinear (full line) cases.

CONCLUSIONS

The nonlinear dynamical analysis of a coupled fluid-structure system taking into account both sloshing and capillarity effects has been investigated. Its efficiency is demonstrated through a numerical application.

ACKNOWLEDGEMENTS

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