Multiscale Method with Patches for the Solution of Linear Parabolic Equations with Localized Uncertainties

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Uncertainty quantification in computational engineering is nowadays an essential step to perform the robust design of mechanical systems. Analyzing the propagation of localized uncertainties in computational models allows for robust predictions of the model response with respect to the input uncertainties. Such uncertainties may represent either natural variabilities in the material properties or epistemic variabilities due to a lack of knowledge in the geometry, the boundary or initial conditions. Traditional monoscale approaches based on local refinement or enrichment techniques can be relatively difficult to implement in the existing commercial softwares and computationally demanding for solving high-dimensional stochastic problems. Consequently, concurrent multiscale approaches based on substructuring, domain decomposition or multigrid methods have been proposed to tackle large-scale engineering applications and perform stochastic computations of multiscale problems. A multiscale method has been recently introduced in [1] for solving linear elliptic equations with localized uncertainties and extended to a wider class of semi-linear elliptic equations with localized uncertainties and non-linearities in [2]. It relies on a decomposition of the domain into several subdomains of interest (called patches) containing the sources of uncertainties and possible non-linearities, and a complementary subdomain. A global-local iterative algorithm is then introduced to compute the multiscale solution and calls for the solution of a sequence of linear global problems (with deterministic operators and uncertain right-hand sides) over a deterministic domain, and (non-)linear local problems (with uncertain operators and right-hand sides) over the patches.

In this work, the method is extended to linear parabolic equations involving localized uncertainties. The convergence of the iterative algorithm is analyzed. The proposed multiscale approach allows for considering independent computational models, adapted discretization spaces and solvers for both types of problems. The stochastic local problems are solved using sampling-based approaches along with adaptive sparse approximation methods [3] to efficiently compute sparse representations of high-dimensional stochastic local solutions with arbitrarily-high accuracy. The performances of the multiscale method are illustrated on a transient linear advection-diffusion-reaction stochastic problem with localized random material heterogeneities.

References

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