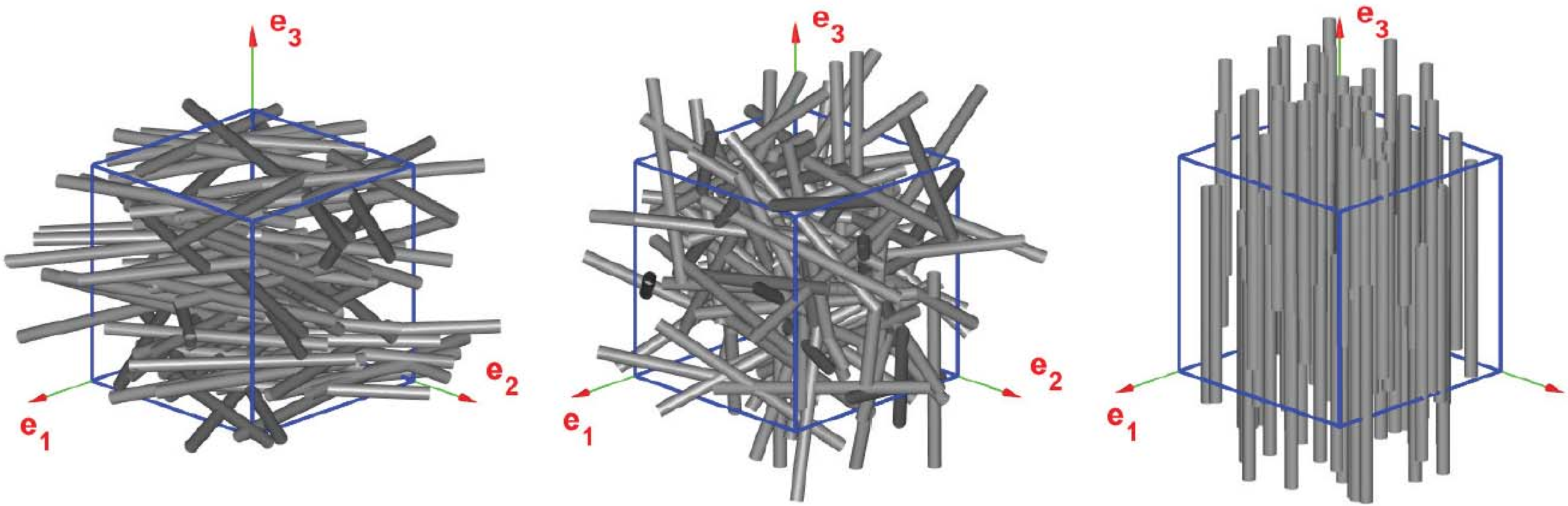
INFLUENCE OF POROSITY, FIBER RADIUS, AND FIBER ORIENTATION ON THE TRANSPORT AND ACOUSTIC PROPERTIES OF RANDOM FIBER STRUCTURES

By Hoang Tuan LUU[1, 2], Camille PERROT[1, a)], and Raymond PANNETON[2]

[1]*Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME UMR 8208 CNRS*

[2]*Groupe d'Acoustique e l'Université de Sherbrooke (GAUS), Mech. Eng. Dpt. , Québec, Canada*

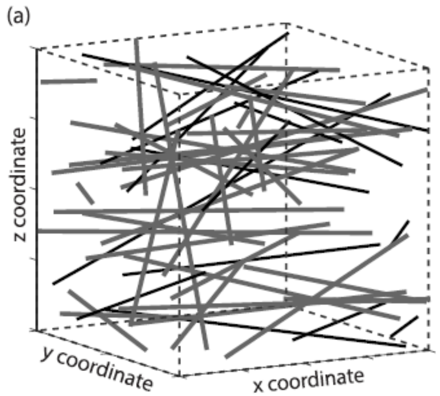
[a)]*Email: camille.perrot@univ-paris-est.fr*

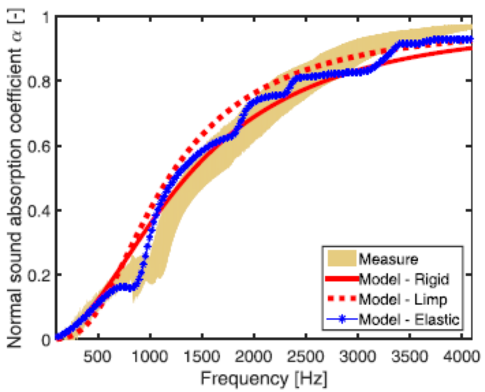


Anisotropic Random Fiber Structures

- **Motivation**. The ability of air-saturated fibrous media to mitigate sound waves in controlled by their transport properties.
- **Objective**. Propose micro-/macro relationships to link microstructural features to macroscopic transport properties (random fiber structures).
- **Methodology**.

1) Generate representative elementary volumes (REV) of random fiber structures (scale separation, rigid frame, overlapping fiber assumptions).
2) Transport equations are numerically solved.
3) Micro-/macro relationships are derived.

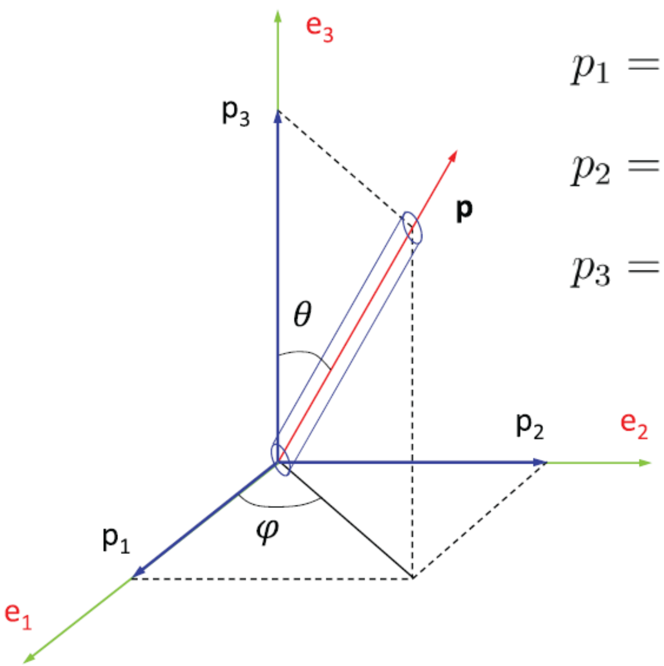




[H. T. Luu, C. Perrot, V. Monchiet, R. Panneton, “Three-dimensional reconstruction of a random fibrous medium: Geometry, transports, and sound absorbing properties” J. Acoust. Soc. Am. 141, 4768 (2017).]

A. Orientation distribution function

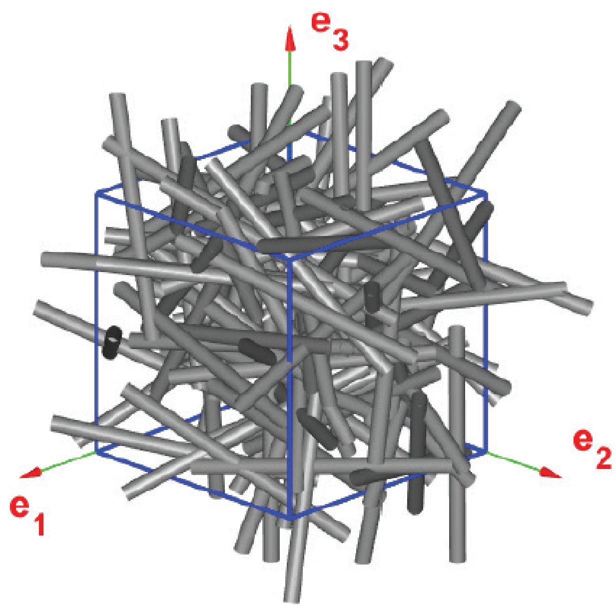
- For the purpose of the present research, the random fiber structures result from the successive generation of rigid uniform cylinders of the same diameter.
- Let 's associate a vector **p** to the fiber.



$$p_1 = sin\theta cos\varphi$$

$$p_2 = sin\theta sin\varphi$$

$$p_3 = cos\theta$$



- **Definition**. A random structure, an arrangement of fibers for which the orientation distribution function ψ(φ, θ) is a function of two variables defining the orientation of a single fiber.

B. Orientation tensor definition

- The use of tensor to describe fiber orientation of composite fibers [Advani and Tucker, J. Rheol. 31, 751 (1987)]
- The second order orientation tensor Ω_{ij} is obtained by forming diadic products of the vector p and then averaging the products with the distribution function ψ over all possible directions:

$$\Omega_{ij} = \int p_i p_j \Psi(\overrightarrow{p}) d\overrightarrow{p}$$

- For a discrete set of fibers:

$$p_1 = sin\theta cos\varphi$$
$$p_2 = sin\theta sin\varphi$$
$$p_3 = cos\theta$$

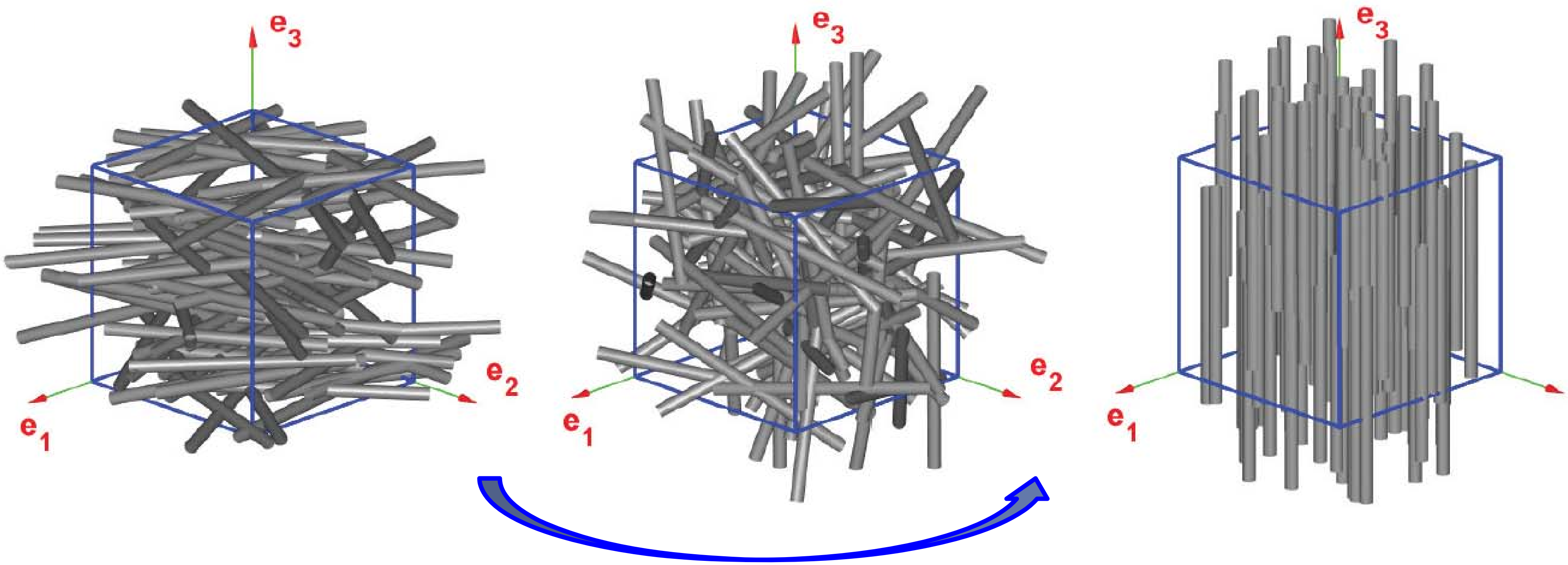
$$[\Omega] = \frac{1}{N_f}\sum_{i=1}^{N_f}\begin{bmatrix} sin^2\theta^{(i)}cos^2\varphi^{(i)} & sin^2\theta^{(i)}cos\varphi^{(i)}sin\varphi^{(i)} & sin\theta^{(i)}cos\theta^{(i)}cos\varphi^{(i)} \\ sin^2\theta^{(i)}cos\varphi^{(i)}sin\varphi^{(i)} & sin^2\theta^{(i)}sin^2\varphi^{(i)} & sin\theta^{(i)}cos\theta^{(i)}sin\varphi^{(i)} \\ sin\theta^{(i)}cos\theta^{(i)}cos\varphi^{(i)} & sin\theta^{(i)}cos\theta^{(i)}sin\varphi^{(i)} & cos^2\theta^{(i)} \end{bmatrix}$$

- **Properties**: [Ω] is symmetric. Since Trace [Ω] = 1 (renormalization condition), for transversely isotropic materials [Ω] completely determined by Ω_{zz} .

Anisotropic Random Fiber Structures

B. Orientation tensor properties

- By varying Ω_{zz} from planar (Ω_{zz} = 0) to aligned (Ω_{zz} = 1) random fibers, one can study the influence of fiber orientation on the transport properties of random fibrous media.



- Done by adjusting (μ_θ, σ_θ) of a normal distribution for θ; with a uniform distribution for φ.

Anisotropic Random Fiber Structures

C. Generation of random fiber structures

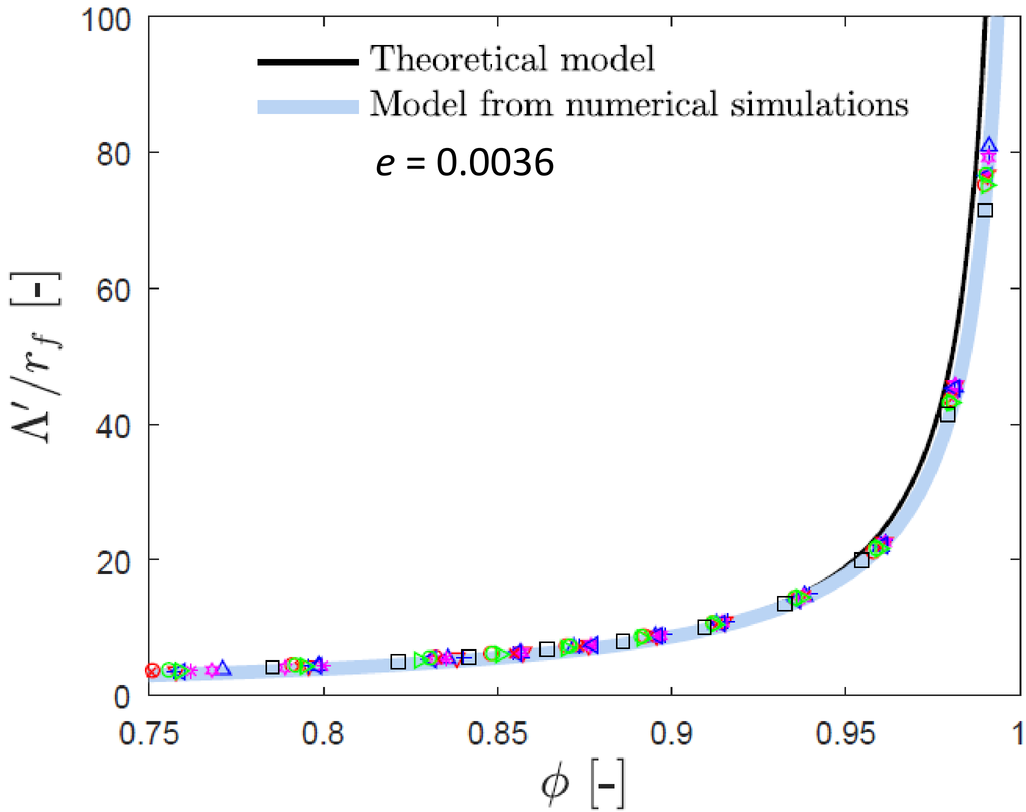
- Algorithm used to generate a representative elementary volume (REV) for a given fiber orientation coefficient Ω_{zz}.
- Fibers allowed to overlap (does not affect transport properties).
- As a general rule, a characteristic fiber radius r_f and target open porosity Φ are imposed (supposed to be known from measurements).
- Main features of the algorithm:

- Iteratively increased the size of the box $L^{(i)}$.
- The solid volume fraction of fibers is known, $V_f^{(i)} = (1-\Phi)L^{(i)3}$.
- Compute the required number of fibers such that $V_f^{(j)} \geq V_f^{(i)}$
with $V_f^{(i)} = \pi r_f^{\,2} \sum_{i=1}^{k} l_f^{(k)}$;
and $l_f(^{k})$ determined analytically from the knowledge of ($\mathbf{r}$, θ, φ).
- Average this process over several realizations (1 000).
- If $|\phi^{(i)} - \phi|/\phi \leq \varepsilon$, then $L = L^{(i)}$.
- In practice, LRVE $\leq$ 500 µm, for $0.75 \leq \phi \leq 0.99$, with $\varepsilon = 0.3$ %.

III. Identification of the geometrical properties

- (ϕ, r_f) measured or imposed.
- The thermal characteristic length Λ' remains to be identified, $\Lambda' = 2V_p/S_w$

where: V_p the fluide phase volume, S_w the wet surface area; are directly computed from the discretized REV



- Indépendant of Ω_{zz}.
- Scales as $\frac{\Phi}{1-\Phi+e}$ where e is identified numerically
- e ~ { r_f/l, number of intersections, intersections shape}

--> Λ' can be deduced from (ϕ, r_f).

VI. Identification of the transport properties

A) Tortuosity and viscous characteristic length

1) Theoretical framework – electric conduction problem (potential flow)

$$\vec{E} = -\vec{\nabla}\pi + \vec{e}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{E} \cdot \vec{n} = 0 \quad \text{at the wall } \partial\Omega$$

π spatially periodic

where

- $\vec{E}$ electric field
- π microscopic potential
- $\vec{e}$ macroscopic electric field
- $\vec{n}$ unit normal vector to $\partial\Omega$

- Tortuosity tensor : $e_i = \alpha_{\infty ij} \langle E_j \rangle$
- Viscous characteristic length Λ:

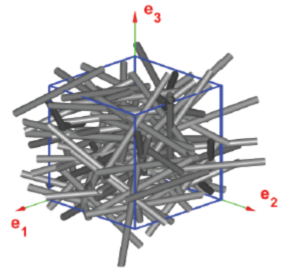
$$\Lambda = 2\frac{\int_\Omega ||\vec{E}||^2 dV}{\int_{\partial\Omega} ||\vec{E}||^2 dS}$$

- ✓ Setting **e** along x and y directions: Λ_{xy}
- ✓ Setting **e** along z direction: Λ_z

[Johnson, Koplik, Dashen, J. Fluid Mech. 176, 379 (1987)]

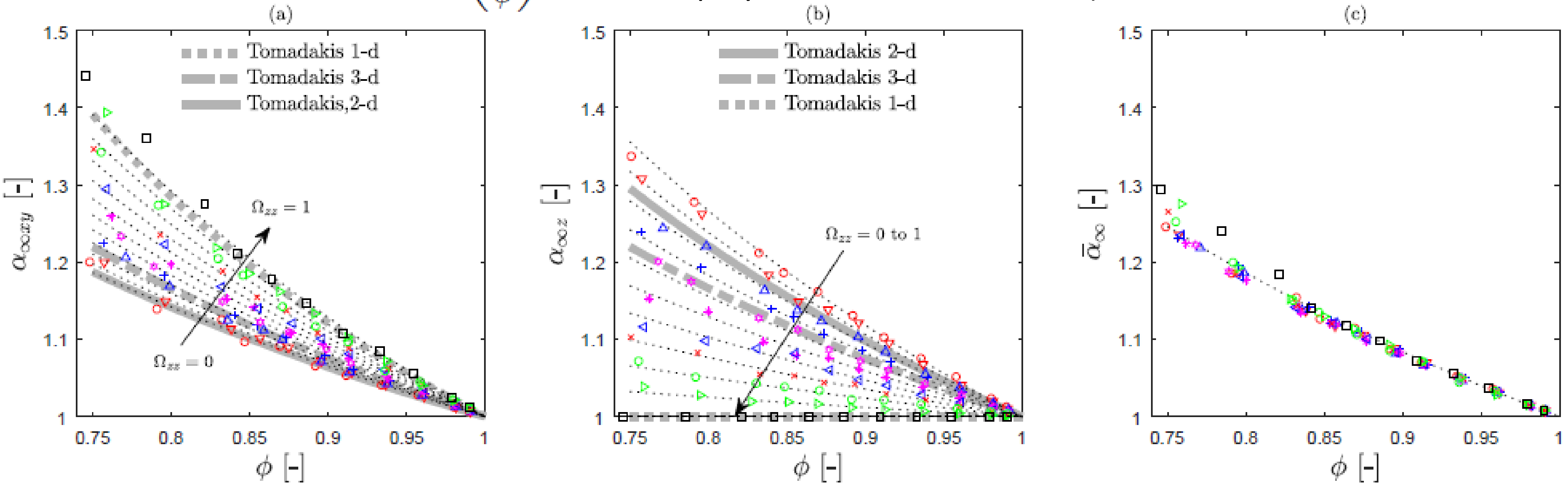
VI. Identification of the transport properties

A) Tortuosity and viscous characteristic length



2a) Numerical results and discussion (tortuosity)

Simulations fit with $\alpha_\infty = \left(\frac{1}{\phi}\right)^{M(\Omega_{zz})}$ (generalized Archie's law, with $M(\Omega_{zz})$ a polynomial of the 2nd order)



- ✓ The transverse tortuosity $\alpha_{\infty xy}$ and the longituinal tortuosity $\alpha_{\infty z}$ are very sensitive to Ω_{zz}
- ✓ Fibers that are orthogonal to the direction of wave propagation yield higher tortuosity values than fibers that are // to the direction of wave propagation.

- ✓ $\bar{\alpha}_\infty$ fewly sensitive to Ω_{zz}.

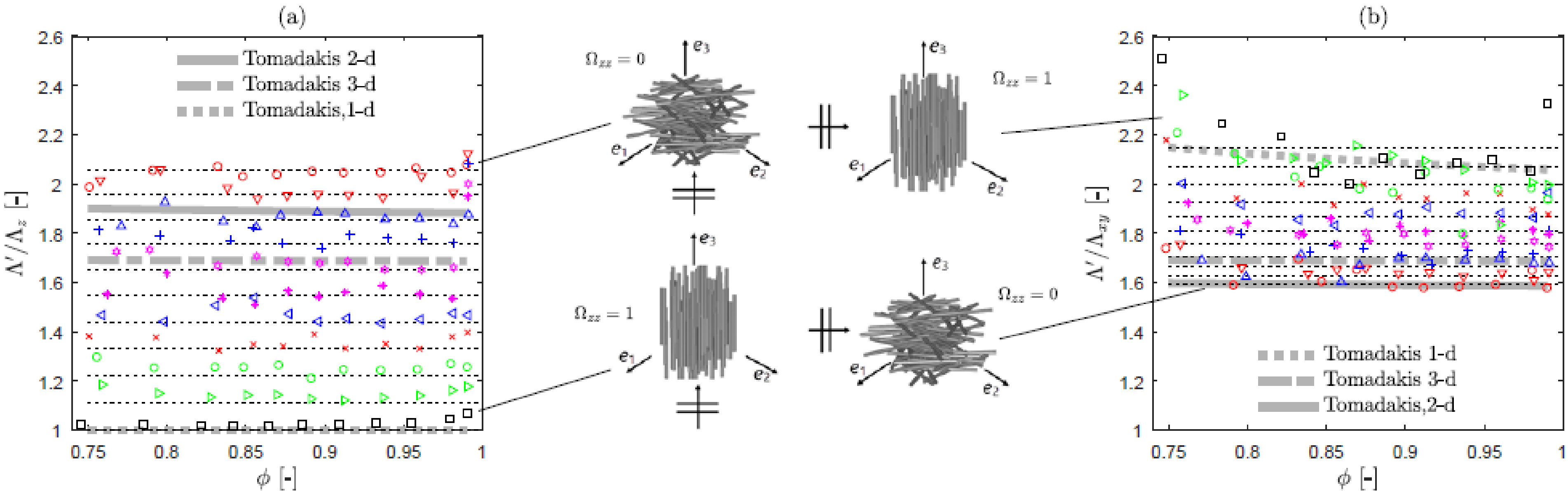
Anisotropic Random Fiber Structures

VI. Identification of the transport properties

A) Tortuosity and viscous characteristic length

2b) Numerical results and discussion (viscous characteristic length)

Simulations (approximately) fit with $\Lambda'/\Lambda = 1 + M(\Omega_{zz})$ (with $M(\Omega_{zz})$ a 2nd order polynomial)



- ✓ Along **e**$_3$, no dependence of Λ'/Λ_z with ϕ.

- ✓ Almost the case along the transverse direction for Λ'/Λ_{xy} ;
  however, as Ω_{zz} ↗, Λ'/Λ_{xy} ↗ when ϕ ↘.
  (Strong effect of the weighting according to the value of **E** in the definition of Λ when ϕ↘)

VI. Identification of the transport properties

B) Static viscous permeability

1) Theoretical framework – Stokes equations

$$\eta \nabla^2 \vec{v} - \vec{\nabla} p = -\vec{G}$$

$$\vec{\nabla} \cdot \vec{v} = 0$$

$$\vec{v} = 0 \quad on \quad \partial\Omega$$

$\vec{v}$ and p spatially periodic

$\vec{v}$ velocity

p pressure

η dynamic viscosity

$\vec{G} = \vec{\nabla} p^m$

macroscopic pressure gradient

- Permeability tensor : $k_{0ij} = \phi \langle k^*_{0ij} \rangle$

  defined from

  $$v_i = -\frac{k^*_{0ij}}{\eta} G_j$$

- Kozeny-Carman equation: $\frac{k_0}{r_f^2} = \zeta \frac{\phi^3}{(1-\phi)^2}$

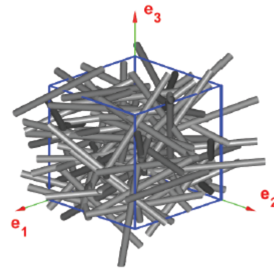
ζ is the Kozeny "constant"

[Johnson, Koplik, Dashen, J. Fluid Mech. 176, 379 (1987)]

VI. Identification of the transport properties

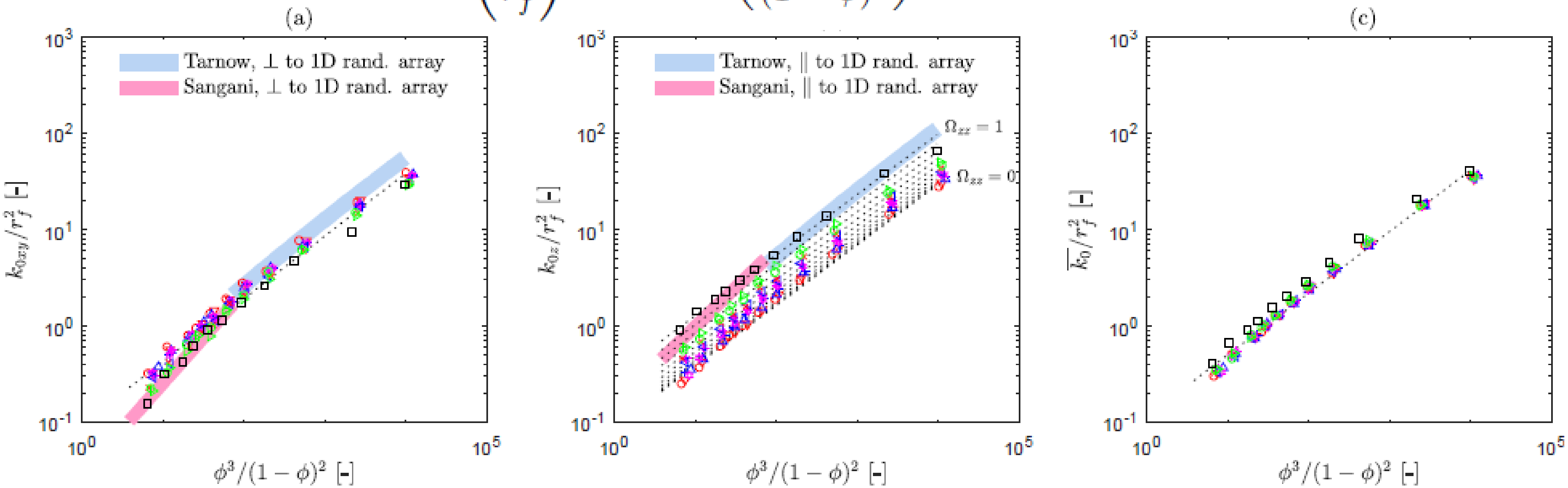
B) Static viscous permeability

2) Numerical results and discussion



Simulations fit with

$$log_{10}\left(\frac{k_0}{r_f^2}\right) = A log_{10}\left(\frac{\phi^3}{(1-\phi)^2}\right) + B\Omega_{zz}^2 + C\Omega_{zz} + D$$



✓ The through plane permeability k_{0z} is more sensitive to fiber orientation Ω_{zz} than the in-plane permeability k_{0xz}:

(a) k_{0xy} varies linearly with $\phi^3/(1-\phi)^2$, which is consistent with the KC Eq.

(b) k_{0z} also depends on Ω_{zz}.

VI. Identification of the transport properties

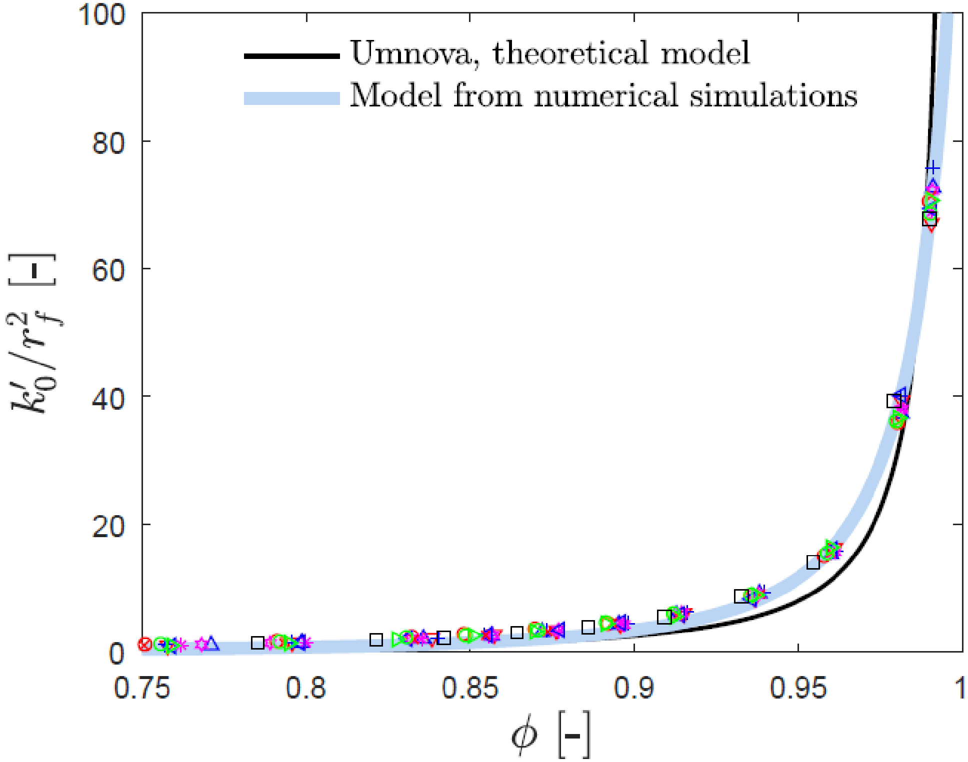
C) Static thermal permeability

1) Theoretical framework – diffusion controlled reactions

$$\nabla^2\tau = -1$$
$$\tau = 0 \quad \text{on} \quad \partial\Omega$$
τ is spatially periodic.

τ excess temperature

- Static thermal permeability $k_0' = \langle\tau\rangle$



2) Numerical results

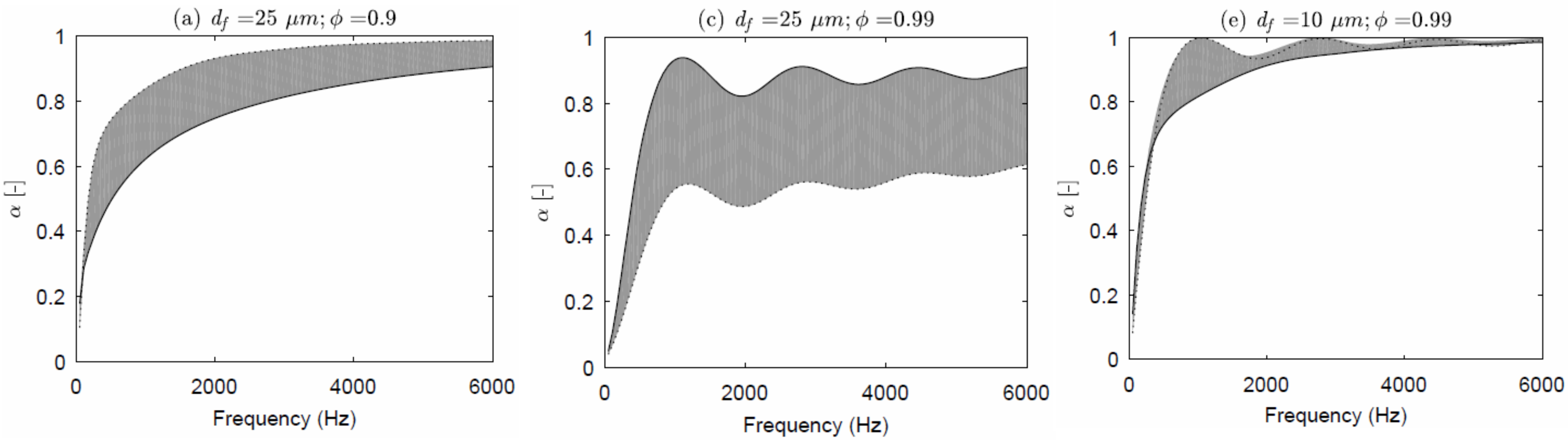
- ✓ k_0' independent of fiber orientation (the diffusion heat does not provide any preferred direction
- ✓ Simulations fit with:

$$\frac{k_0'}{r_f^2} = \frac{1}{M}\frac{\phi^3}{(1-\phi+e)^2}$$

$$M = 14.4718 \text{ and } e = 0.0216$$

V. Concluding remarks

- Knowing the fiber radius r_f, the fiber orientation Ω_{zz}, and the open porosity ϕ; the proposed relations completely define the input macroscopic parameters to be used in an equivalent fluid model.



[H. T. Luu, C. Perrot, R. Panneton, “Infuence of porosity, fiber radius and fiber orientation on the transport and acoustic properties of random fiber structures” *under review* in Acta Acust united Ac., Ms. No. AAA-D-16-00177 (2017)]