Effective fiber diameter for modeling the acoustic properties of polydisperse fiber networks

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Abstract: The purpose of this research is to determine whether the acoustic properties of polydisperse fibrous medium (PDFM) and bidisperse fibrous medium (BDFM) can be modeled by monodisperse fiber media (MDFM) with an effective fiber diameter. Multi-scale numerical simulations on representative elementary volumes of these media are performed to retrieve the transport and geometrical properties governing their acoustic properties. Results show no significant difference between predictions obtained by PDFM or BDFM, and their corresponding effective MDFM.

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1. Introduction

There are a wide variety of fibrous materials used for noise mitigation. Most common ones are glass wools, rock wools, and some natural wools. Their porosity and fiber diameter distribution largely determine their transport and acoustic properties.[1,2] The determination of these properties based on a multi-scale approach was recently investigated.[3–5] The principle of the approach is to numerically solve the governing physical problems at the local scale, based on a reconstructed representative elementary volume (REV) of the fibrous network. From the numerical solutions, the transport and acoustic properties of the fibrous network are deduced by the application of the homogenization technique.[6,7] Such a multi-scale approach may be well suited to design fibrous materials having acoustic properties tailored to given end-applications. However, when dealing with a fiber diameter distribution instead of a unique fiber diameter, the link between the fiber diameter and the transport or acoustic properties is not straightforward. It follows that the tailoring of the acoustic properties may be difficult to achieve. Consequently, the objective of this paper is to check whether the transport properties, and thereby the acoustic properties, of a polydispere fiber network can be retrieved by the multi-scale modeling of a monodisperse fiber network of effective fiber diameter.

To the authors' knowledge, no research work has specifically addressed the aforementioned objective. Nevertheless, some works suggest that the use of an average diameter is sufficient, but they do not present any in-depth investigations. For instance, Peyrega *et al.*[4] determined the acoustic properties of a transversally isotropic fibrous medium based on a multi-scale approach. The diameter of the constitutive fibers follows a gamma distribution; however, in their modelling all the fibers have the same diameter, that is the weighted averaged diameter of the distribution. While their multi-scale results fit with measurements, no indication is given regarding the generality of using an averaged diameter. In a previous work by Luu *et al.*,[5] a multi-scale approach was also used for determining and evaluating the influence of fiber orientation on the transport and acoustic properties of a natural fiber network. When reconstructing the REVs, they used a mean value instead of the real fiber diameter normal distribution. Again, while their simulations fit with measurements, no demonstration is given with regard to the validity of using a monodisperse mixture to acoustically model a polydisperse one. In a similar way, Schladitz *et al.*[3] worked out a model for optimizing the

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acoustic properties of a nonwoven from the reconstruction of the fibrous microstructure. This time, the authors used a full range of fiber diameter distribution. Unfortunately, they have not explored whether an effective diameter could lead to the same results.

In order to assess the validity of modeling a polydisperse fibrous medium (PDFM) by an effective monodisperse one, this paper is structured as follows. First, the multi-scale theory to retrieve the REVs and acoustic properties of polydisperse, bidisperse and monodisperse fiber media is recalled. Second, from the numerical solutions of the local equations on the REVs, results from polydisperse and bidisperse fiber media are compared to their effective monodisperse counterparts. Here, the effective fiber diameter used for each monodisperse medium is the weighted average diameter of its corresponding polydisperse medium. Finally, conclusions are drawn from these comparisons to validate the research question of this paper. Note that for most nonwoven fibrous materials, the fiber mixture is transversely isotropic, where all fibers are randomly stacked in parallel planes. Therefore, the present study is limited to this type of configuration.

2. Theory and methodology

As presented in a previous work,[5] the determination of the transport properties of fiber networks from their microstructural features starts with the reconstruction of corresponding REVs. In the following, two types of polydisperse fiber mixtures will be considered during reconstruction of REVs: (i) the first type corresponds to materials containing fiber mixtures, where the fiber diameter follows a Gamma distribution; (ii) the second type is the combination of two groups of fibers, where each group has a constant fiber diameter and occupies a given proportion of the mixture. While the first type is called PDFM, the second is called bidisperse fibrous medium (BDFM).

When considering PDFM, a literature review shows that a realistic representation of the fiber diameter distribution is given by a Gamma distribution.[2,8] Following the shape-scale parametrization, the probability density function of the Gamma distribution is given by

$$f(d;\kappa,\theta) = \frac{d^{\kappa-1}e^{-d/\theta}}{\theta^{\kappa}\Gamma(\kappa)}, \tag{1}$$

where d is the random fiber diameter, θ is the scale, and $\Gamma(\kappa)$ is the Gamma function evaluated for shape κ. The Gamma distribution ranges between a normal distribution ($\kappa \to \infty$) and a log-normal distribution ($\kappa \to 0$) with a corresponding mean value $\kappa\theta$. Histograms representing the Gamma distribution for various shape factors κ are shown in Fig. 1(a).

When considering BDFM, a discrete uniform distribution is used with the probability mass function given by

$$P(d) = \frac{1}{n}, \tag{2}$$

where $n = 2$ since only two discrete fiber diameters are possible.

For both types of fibrous medium (PDFM and BDFM), the multi-scale approach presented in a previous work[5] is applied. For a given fibrous medium, the open porosity is first fixed together with the corresponding probability function [Eq. (1) or (2)], and the possible discrete values, or continuous range, of fiber diameters: (i) for BDFM, d_1 and d_2 are defined; (ii) for PDFM, mean fiber diameter $\kappa\theta$, normalized by scale θ and shape parameter κ are defined. This information is used as input to the multi-scale algorithm presented by Luu *et al.* (Fig. 3.2 of Ref. 5) to define the size of the corresponding REV and the number of fibers of each diameter it contains. Once the REV is determined, the finite element method is used to discretized the fluid domain between fibers and to solve the three local problems: Stokes flow, potential flow, and heat conduction. It is worth mentioning that for each problem the mesh was selected properly to ensure convergence of the calculated fields. Once the numerical solutions are obtained, the transport properties are deduced from these fields: (1) the Stokes flow problem allows determination of the static transverse viscous permeability k_0,[9] also called the through-plane permeability; (2) the potential flow problem allows determination of the tortuosity α_∞, and viscous characteristic length Λ,[10] and (3) the heat conduction problem allows determination of the static thermal permeability k'_0.[11] The two remain geometrical parameters (open porosity ϕ, and thermal characteristic length Λ')[11] are directly determined from the meshing of REV.

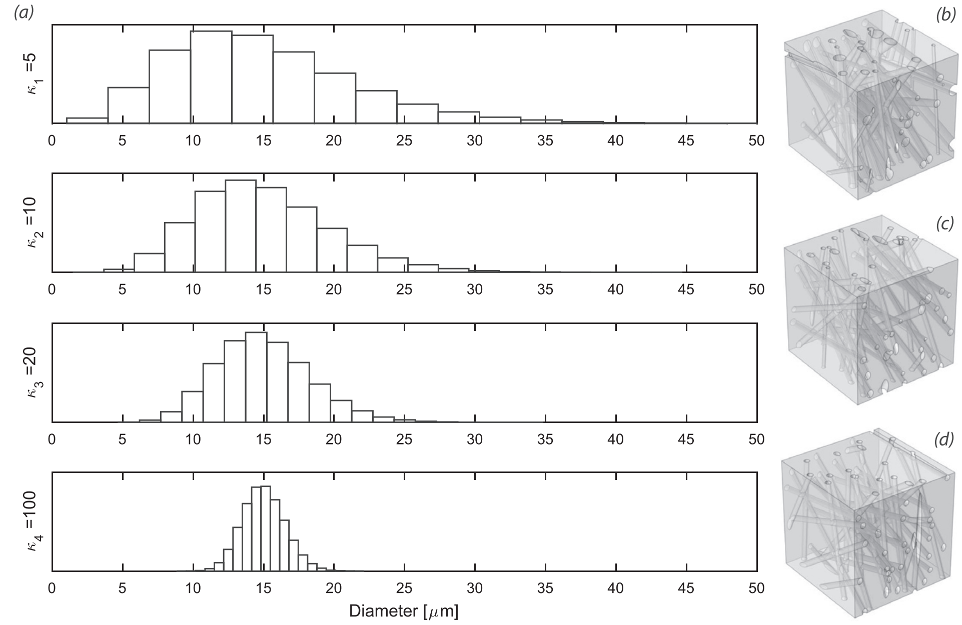

Fig. 1. (a) The evolution of Gamma distribution on the variation of shape parameter κ. The mode of the distribution is given by $(\kappa-1)\theta = 15\,\mu$m. Different REV configurations corresponding to various shape parameters: (b) $\kappa_1 = 5$, (c) $\kappa_2 = 10$, (d) $\kappa_3 = 20$.

The second step of the methodology is to build an equivalent monodisperse fibrous medium (MDFM) for each REV of PDFM and BDFM. The monodisperse REV has the same number of fibers of the corresponding bidisperse or polydisperse REV, where all the fibers have the same effective diameter, that is the weighted averaged diameter of the corresponding distribution, only the orientation of the fibers changes. The effective diameter is $\kappa\theta$ for the Gamma distribution, and $\bar{d} = p_1 d_1 + p_2 d_2$ for the discrete uniform distribution, where p_i is the proportion of fibers with diameter d_i in the bidisperse REV. Once the REVs of monodisperse fibers are reconstructed, the aforementioned numerical calculations at the local scale are performed to deduce the corresponding transport and geometrical properties.

The final step of the methodology is to compare the transport and geometrical properties calculated for each MDFM to its corresponding PDFM or BDFM. The objective is to validate if, or when, an effective fiber diameter is sufficient to model the acoustic properties of polydisperse and bidisperse fiber mixtures. Since the acoustic properties, such as sound absorption coefficient or sound transmission loss, depend on the transport and geometrical properties of the fiber mixture, comparable transport and geometrical properties will lead to comparable acoustic properties.

3. Results and discussion

3.1 PDFM

The PDFM is first considered. As discussed previously, the fiber diameter in PDFM follows a Gamma distribution characterized by shape parameter κ and scale parameter θ. We recall here that $\kappa\theta$ is used as the effective diameter of the corresponding monodisperse fiber mixture (MDFM). To study the sensitivity of the acoustic properties to the shape parameter κ, the effective diameter $\kappa\theta$ is fixed at a given value ($\kappa\theta = 15\,\mu$m) while the shape parameter takes three different values: $\kappa_1 = 5$, $\kappa_2 = 10$, and $\kappa_3 = 20$. Note that in this study, during the reconstruction of the REVs, the target open porosity is $\phi = 0.900$, with a tolerance ε of 0.003 (ε is defined in Ref. 5).

As discussed in step 1 of the methodology, the construction of a REV for a polydisperse fiber mixture is realized with the following input information: the target open porosity ϕ, and the parameters θ and κ of the Gamma distribution. For the corresponding monodisperse fiber mixture, the input information are the open porosity ϕ and the effective constant diameter $\kappa\theta$. Some REVs corresponding to polydisperse fiber mixtures with shape parameters κ_1, κ_2, and κ_3 are depicted in Figs. 1(b), 1(c), and 1(d). As mentioned above, the numerical simulations were applied on the reconstructed REVs in order to calculate the transport and geometrical properties. The numerically calculated properties (average and standard deviation) are plotted in Fig. 2. Note that the statistics for a given set of input information are obtained from the solution on 10

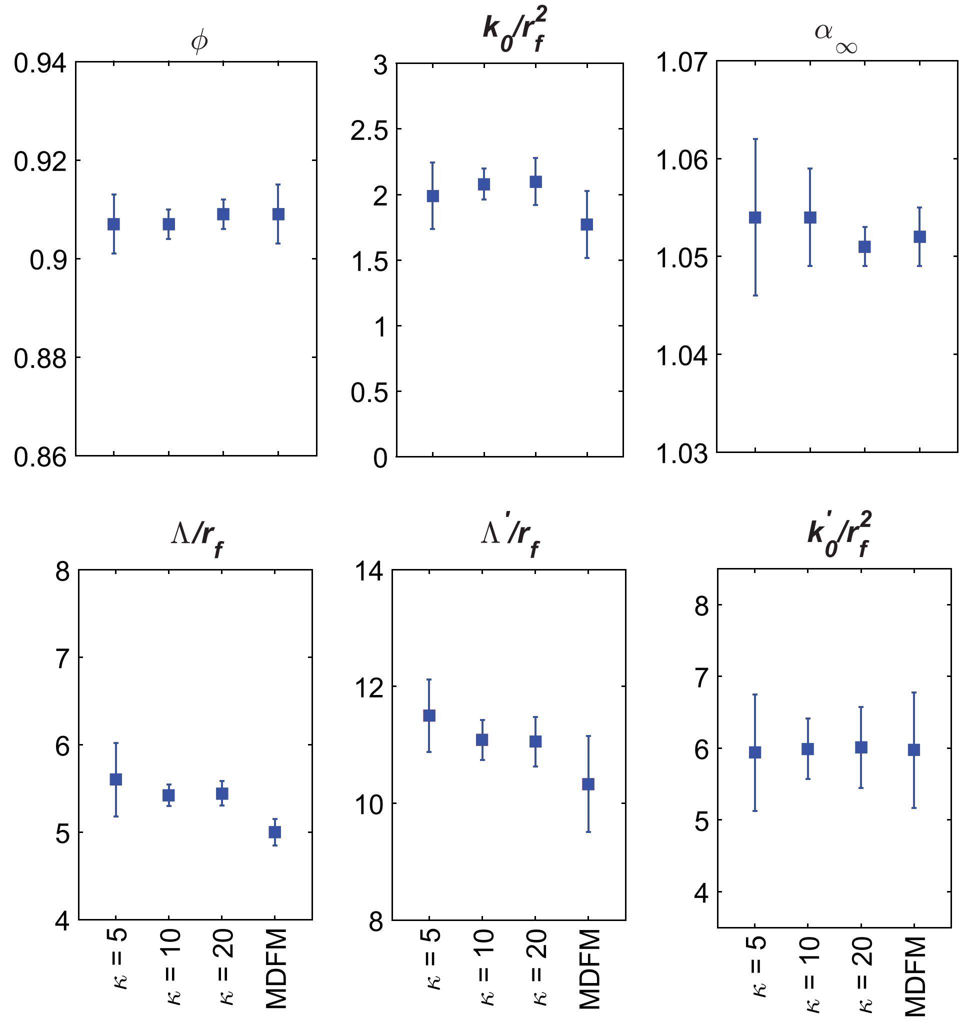

Fig. 2. (Color online) Normalized transport and geometrical properties of PDFM and corresponding MDFM.

REVs numerically reconstructed by the aforementioned algorithm. Except for porosity and tortuosity, the other properties in Fig. 2 are scaled with respect to the mean fiber radius r_f or its square to obtain dimensionless parameters.

The results shown in Fig. 2 indicate that the transport and geometrical properties found with the effective MDFM are statistically equivalent to those obtained with the PDFM. The variations found between the different shape parameters κ are an artefact of the three-dimensional (3D) numerical reconstructions of the REVs. In fact, to simplify the 3D reconstructions for the finite element simulations, fibers are allowed to intersect. This has the effects of overestimating the porosity, and slightly affecting the other properties. This explains why the open porosities for the numerically reconstructed REVs oscillate around 0.908 in Fig. 2 instead of being constant at the target porosity of 0.900. To reduce these variations, intersections should be avoided during the 3D reconstruction of the REVs. This would require considerable reconstruction efforts to reach the same conclusions. While not shown here, but when using the MDFM properties with a given effective fiber diameter in the predictive Johnson-Lafarge model,[12] the found sound absorption coefficient and transmission loss are statistically similar to those obtained using the BDFM properties whatever the shape parameter κ (5, 10, or 20).

3.2 BDFM

The BDFM is now considered. This time, the fiber mixtures are constituted of two groups of fibers with different diameters d_1 and d_2. The percentages of each group of fibers in each mixture are p_1 and p_2, respectively. In the scope of this work, a typical ratio of 3:7 is considered, this yields $p_1 = 30\%$ and $p_2 = 70\%$. This ratio is typical for nonwoven fibrous materials made up from two fiber components: the first component is the principal fiber (e.g., a natural fiber like milkweed floss) occupying 70% of the total number of fibers in the mixture, and the second one is the binding fiber (e.g.,

Table 1. Different combinations of fiber mixtures for BDFM. Effective diameters are for the corresponding MDFM.

	Group 1		Group 2		
Combination	d_1 (μm)	p_1 (%)	d_2 (μm)	p_2 (%)	Effective diameter $\bar{d}$ (μm)
1	10	30	20	70	17
2	10	30	30	70	24
3	10	30	40	70	31
4	40	30	10	70	19
5	40	30	20	70	26
6	40	30	30	70	33

bicomponent fiber) occupying 30% of the mixture. This type of mixture is similar to the nonwoven fibrous material investigated in a previous paper.[5] Note that in this study, during the reconstruction of the REVs, the target open porosity is $\phi = 0.900$, with a tolerance ε of 0.003 (ε is defined in Ref. 5).

Following the methodology described in Sec. 2, REVs are reconstructed for different bidisperse fiber mixtures. Each fiber mixture is characterized by the following input information: the target open porosity ϕ, and a combination of diameters d_1 and d_2, and their ratio. Table 1 presents the six studied combinations of diameters. Similarly, REVs for the corresponding effective monodisperse fiber mixtures are also reconstructed with the following input information: the target open porosity ϕ, and the weighted average diameter $\bar{d}$ (or effective diameter). Once the REVs are reconstructed, the numerical solutions of the governing problems at the local scale provide all their transport and geometrical properties. The numerically calculated properties (average and standard deviation) are plotted in Fig. 3. Note that the statistics for a given set of input information are obtained from the solution on 10 REVs reconstructed by the aforementioned algorithm. Except for porosity and tortuosity, the other properties in Fig. 3 are scaled with respect to the effective fiber radius or its square to obtain dimensionless parameters.

The results shown in Fig. 3 indicate that for most of the studied combinations, the use of an effective MDFM modeling of a BDFM leads to statistically identical results. The only exception is for the tortuosity for combination four where the tortuosities of MDFM and BDFM statistically differ by less than 0.01. They are perhaps not statistically identical, but their values are very close. While not shown here, but

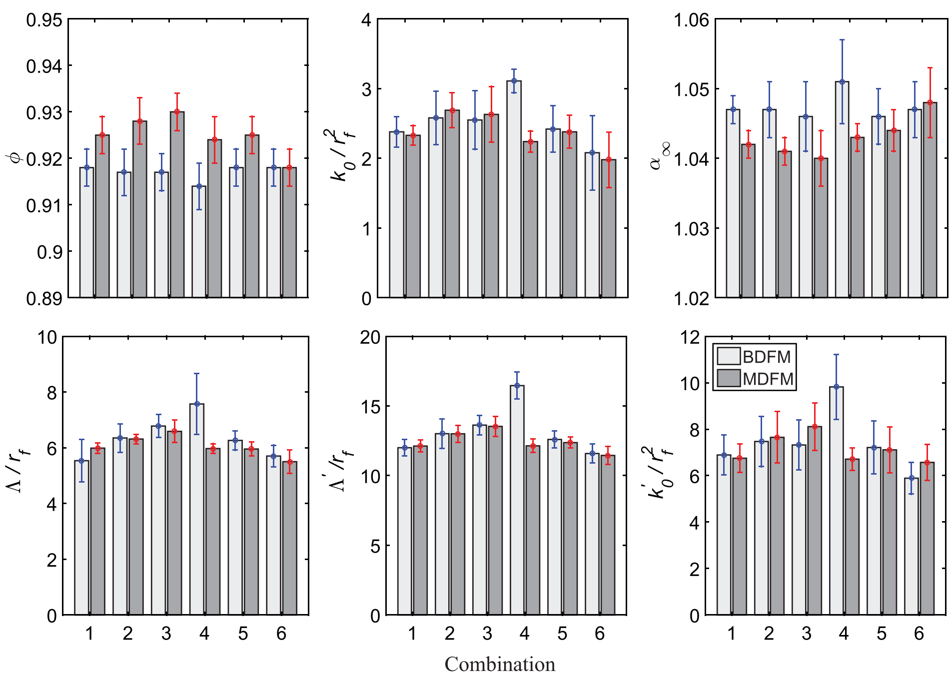

Fig. 3. (Color online) Transport and geometrical properties of BDFM (light gray) and corresponding MDFM (dark gray) for the different combinations described in Table 1.

one can show that these differences found in the transport and geometrical properties would not lead to significant differences on acoustic properties such as sound absorption and transmission coefficients. Consequently, these results showed that the use of an effective diameter (here the weighted average diameter) was sufficient to replicate the acoustic properties of the studied typical bidisperse fibrous media.

4. Conclusions

Some PDFM and BDFM were considered and compared with corresponding monodisperse fiber media (MDFM) of a single effective fiber diameter. The fiber diameters in a PDFM follow a Gamma distribution around an average diameter. The fiber diameters in a BDFM are a combination of two discrete values.

For PDFM, it was shown that for an average diameter of the Gamma distribution, modeling PDFM by MDFM, where all fibers are of the same effective diameter equal, yields mostly to statistical identical results. Here the effective diameter is the average diameter of the Gamma distribution.

For BDFM, it was shown that the use of an effective diameter (here the weighted average diameter) in MDFM was sufficient to replicate the acoustic properties of the studied typical bidisperse fibrous media. Here, only a fiber proportion ratio 3:7 (binding and principal fibers, resp.) was investigated for different combinations of two diameters. This ratio is typical for nonwoven fibrous materials.

Acknowledgments

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