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Multilevel stochastic reduced-order model in computational structural dynamics with an experimental validation for an automobile

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Abstract
This paper deals with the broad-band frequency analysis of complex systems, characterized by the presence of numerous structural scales (flexible parts) attached to the skeleton of the structure (stiff part) and for which numerous local displacements – which are very sensitive to uncertainties – are then coupled with the usual global displacements. Due to this overlap of several scales of displacements, there is an overlap of the low-, medium-, and high-frequency regimes (LF, MF, HF). Hence the introduction of a multilevel reduced-order model (ROM), whose vector basis gathers LF-, MF-, and HF-like families of displacements, for which the separation proceeds from a given filtering strategy. Integrating the nonparametric probabilistic approach of uncertainties, the obtained multilevel stochastic ROM allows for assigning a specific statistical dispersion to each scale. The stochastic ROM allows for (1) tackling the dimensionality induced by the local elastic modes and (2) taking into account the heterogeneous uncertainties associated with the frequency regimes.

1 Introduction

The low-frequency (LF) band is characterized by frequency response functions (FRFs) exhibiting sharp peaks (isolated resonances), which are associated with the corresponding eigenfrequencies and eigenvectors (elastic modes) of the dynamical system. For this band, modal analysis [1, 2, 3, 4, 5] is a widely known method that provides an effective and efficient small-dimension reduced-order model (ROM) through the use of a reduced-order basis (ROB) composed of the first elastic modes (associated with the smallest eigenfrequencies). In contrast, the high-frequency (HF) band is characterized by rather smooth FRFs and by a high modal density. As opposed to the large-wavelength global displacements of the LF band, the numerous elastic modes in HF are composed of small-wavelength displacements. For this band, statistical energy analysis [6] is a well established method. The intermediate band, the medium-frequency (MF) band, exhibits large variations of the modal density, associated with overlapping resonances corresponding to more or less small-wavelength displacements [7]. For this band, several approaches have been investigated [7, 8, 9, 10, 11, 12].

This paper deals with the broad-band frequency analysis of complex systems, characterized by the presence of numerous structural scales (flexible parts) attached to the skeleton of the structure (stiff part) and for which numerous local displacements – which are very sensitive to uncertainties – are then coupled with the usual global displacements. The local displacements are associated with predominant vibrations of the flexible sub-parts, whereas the global displacements involve the deformation of the whole structure, supported by the stiff skeleton. Due to the small dimension of the flexible parts, the local displacements (of small wavelength) of the flexible parts are associated with local elastic modes of low eigenfrequency. As a consequence, there are numerous local elastic modes in the LF band, and consequently, in higher frequencies too. For such complex structures, it is well recognized that the broad-band frequency analysis must be improved by taking into account uncertainties introduced by modeling errors. For this, the nonparametric probabilistic approach of uncertainties has been proposed [13]. Parametric probabilistic approaches [14, 15], despite very effective besides, cannot in general represent the modeling errors. In addition, the variability of such complex systems (concerning automotive vehicle, see [16, 17, 18]) is more important in the HF band than in the LF band. For such complex structures, due to the overlap of several scales of displacements, there is an overlap of the LF, MF, and HF regimes. Since the local displacements (HF-like displacements), which are coupled with the robust global displacements (LF-like displacements), are very sensitive to uncertainties (as it is well known), the model of uncertainties should be able to take into account this heterogeneous behavior. In addition, for such structures, modal analysis then yields ROMs of high dimension (due to the presence of the numerous local elastic modes). Not only are the local displacements highly sensitive to uncertainties, they also have little contribution to the dynamical response of the skeleton of the structure, compared to the global displacements. For obtaining a smaller ROM, one solution would consist in sorting the elastic modes depending on whether they be global or local. Unfortunately, the elastic modes are in general combinations of both global and local displacements. In addition, due to the large amplitude of the local displacements in comparison to the global displacements, it is difficult to distinguish the global displacements based on the mode shapes (this becomes even more difficult for higher frequencies). In order to filter the local displacements, substructuring techniques [19, 20] (component mode synthesis) could be used: the component modes associated with the flexible parts shall be discarded. Unfortunately, for the complex structures considered, there is no clear separation between the stiff and flexible parts (due to the complexity of their geometry). Few research has been devoted to this particular case of a high modal density [21, 22]. Recently, a methodology has been proposed [23] for constructing a stochastic ROM whose ROB is composed of two subsets of either global or local displacements vectors, and for which uncertainties are introduced for the local displacements. The present paper is a continuation of this work. In [17] the original methodology has been applied to an automobile. In [24] it has been generalized and a multilevel ROM has been introduced. In this paper, the multilevel ROM is used and the integration of the nonparametric probabilistic approach of uncertainties allows for obtaining a multilevel stochastic ROM for which specific statistical dispersion hyperparameters are assigned to each scale. The stochastic ROM allows for (1) tackling the dimensionality induced by the local elastic modes and (2) taking into account the heterogeneous uncertainties associated with the frequency regimes.

The paper is organized as follows. First, in Section 2 the basic equations are given and the stochastic ROM of the nonparametric approach, which is based on modal analysis, is presented. Then in Section 3 a filtering methodology, devoted to the seperation between the local displacements and the global displacements, is presented. In Section 4 the developments of Section 3 are used in order to obtain a multilevel ROM composed of three families of LF-, MF-, and HF-like displacements, followed by the introduction of the associated stochastic ROM using the nonparametric probabilistic approach. Section 5 is devoted to an application to an automobile, for which the classical and proposed ROMs are compared, with respect to experimental measurements.

2 Classical reduced-order model

In this section we first summarize very well known results, after which we introduce the basic elements of the nonparametric probabilistic approach.

Using the finite element method [25], the reference computational model is given by positive-definite symmetric $(m \times m)$ real matrices: $[\mathbb{M}]$ (mass), $[\mathbb{D}]$ (damping), and $[\mathbb{K}]$ (stiffness), with m the number of degrees of freedom (DOFs). For all angular frequency ω belonging to the frequency band of analysis, vector $\mathbb{U}(\omega)$

of displacements is the solution of the following matrix equation,

$$(-\omega^2[\mathbb{M}] + i\omega[\mathbb{D}] + [\mathbb{K}])\, \mathbb{U}(\omega) = \mathbb{F}(\omega)\,, \tag{1}$$

in which $\mathbb{F}(\omega)$ is the vector of the prescribed forces. For the complex structures considered, dimension m of the finite element model can be large. Hence the introduction of a ROM, using modal analysis.

Using the first n elastic modes $\boldsymbol{\varphi}_\alpha$ with associated eigenvalues λ_α solutions of the generalized eigenvalue problem,

$$[\mathbb{K}]\boldsymbol{\varphi}_\alpha = \lambda_\alpha[\mathbb{M}]\boldsymbol{\varphi}_\alpha\,, \tag{2}$$

the displacements are approximated as

$$\mathbb{U}(\omega) \simeq \sum_{\alpha=1}^{n} \mathfrak{q}_\alpha(\omega)\, \boldsymbol{\varphi}_\alpha\,, \tag{3}$$

or, introducing $[\Phi] = [\boldsymbol{\varphi}_1 \ldots \boldsymbol{\varphi}_n]$, latter equation can be rewritten as

$$\mathbb{U}(\omega) \simeq [\Phi]\boldsymbol{\mathfrak{q}}(\omega)\,. \tag{4}$$

In latter equations, $\boldsymbol{\mathfrak{q}}(\omega)$ is a vector composed of the generalized coordinates $\mathfrak{q}_\alpha(\omega)$, which is then the solution of the following reduced-matrix equation,

$$(-\omega^2[\mathcal{M}] + i\omega[\mathcal{D}] + [\mathcal{K}])\, \boldsymbol{\mathfrak{q}}(\omega) = \mathfrak{f}(\omega)\,, \tag{5}$$

with $\mathfrak{f}(\omega) = [\Phi]^T\mathbb{F}(\omega)$, $[\mathcal{M}] = [\Phi]^T[\mathbb{M}][\Phi]$, $[\mathcal{D}] = [\Phi]^T[\mathbb{D}][\Phi]$, and $[\mathcal{K}] = [\Phi]^T[\mathbb{K}][\Phi]$. Latter matrices are called the generalized matrices associated with the projection basis (or ROB) $[\Phi]$. In the rest of this paper, several ROBs will be introduced. For each ROB, it is straightforward to obtain the associated ROM in a similar way.

Using the nonparametric probabilistic approach of uncertainties, latter deterministic generalized matrices are replaced by random matrices. For $[\mathcal{A}] = [\mathcal{M}], [\mathcal{D}],$ or$[\mathcal{K}]$ the construction of the associated random matrix, $[\boldsymbol{\mathcal{A}}]$, proceeds from the application of the maximum entropy principe [26, 27] under the constraints:

- Matrix $[\boldsymbol{\mathcal{A}}]$ is a positive-definite symmetric $(n \times n)$ real matrix.
- $E\{[\boldsymbol{\mathcal{A}}]\} = [\mathcal{A}]$, with E the mathematical expectation.
- $E\{||[\boldsymbol{\mathcal{A}}]^{-1}||_F^2\} < +\infty$, with $||.||_F$ the Frobenius norm.

The construction of $[\boldsymbol{\mathcal{A}}]$ is such that

$$[\boldsymbol{\mathcal{A}}] = [L_{\mathcal{A}}]^T[\mathbf{G}(\delta_{\mathcal{A}})][L_{\mathcal{A}}]\,, \tag{6}$$

in which $[L_{\mathcal{A}}]$ is upper-triangular such that $[\mathcal{A}] = [L_{\mathcal{A}}]^T[L_{\mathcal{A}}]$ (Cholesky decomposition) and where the construction of the random matrix $[\mathbf{G}(\delta_{\mathcal{A}})]$ is given in [13], with $\delta_{\mathcal{A}}$ a dispersion hyperparameter such that

$$\delta_{\mathcal{A}}^2 = \frac{1}{n}\, E\{||[\mathbf{G}(\delta_{\mathcal{A}})] - [I_n]||_F^2\}\,. \tag{7}$$

As a consequence, Eq. (5) is replaced by a random-matrix equation whose solution is a random vector. The random solution is statistically estimated using the Monte-Carlo simulation method [28].

Due to the presence of the numerous local displacements, dimension n of the classical stochastic ROM (C-SROM) may be large. As previously introduced, not only are the local displacements highly sensitive to uncertainties, they also have little contribution to the dynamical response of the skeleton of the structure, compared to the global displacements. We thus propose the construction of an unusual ROM, which is based on the use of global-displacements basis vectors.

3 Global-displacements reduced-order model

The construction of the global vectors relies on the use of a suitable approximation (reduced kinematics) for the mass matrix. Instead of using the finite element basis (which yields consistent mass matrix $[\mathbb{M}]$), a vector subspace is introduced. In original work [23], this subspace is given by the set of vectors that are constant within given subdomains that partition the whole structure. Using this approximation, no local displacement is permitted within a given subdomain and the filtering between the global and the local displacements is parameterized by the characteristic dimension of the subdomains (which then have to be of a homogeneous size). The construction of homogeneous domain partitionings of complex geometries is not a straightforward task, and for this a methodology based on the Fast Marching Method [29, 30] has been extended and implemented in [17] for an automobile structure. Instead, in this paper, we choose to use polynomial shape functions (with support the whole structure) in order to span the approximation subspace. This way, the degree of the polynomial approximation allows for efficiently controlling the filtering between the global displacements and the local displacements. Using the multivariate monomials (of the $3D$ physical space) obtained from all the possible combinations up to a given degree, a basis of orthogonal polynomials is calculated by performing a QR decomposition of the matrix of the monomials. It should be noted that the orthogonality is defined with respect to the inner-product given by mass matrix $[\mathbb{M}]$. Using this vector basis, the reduced-kinematics mass matrix, $[\mathbb{M}^r]$, is obtained via the orthogonal-projection matrix associated with this orthogonal basis (with respect to latter inner-product). The dimension of $[\mathbb{M}^r]$ is m while its rank is $r \leq m$. Stiffness matrix $[\mathbb{K}]$ is left unchanged (the elastic energy is kept exact).

Replacing $[\mathbb{M}]$ by $[\mathbb{M}^r]$ in Eq. (2) leads us to an unusual generalized eigenvalue problem, whose eigenvectors consist in more or less global displacements, depending on the polynomial degree used. It should be noted that $[\mathbb{M}^r]$ is only used in order to carry out the filtering between the global displacements and the local displacements. The global-displacements ROM is obtained by projecting Eq. (1) onto the global-displacements ROB. Concerning the global-displacements ROB, it is actually not exactly constituted of latter eigenvectors, these being not orthogonal with respect to $[\mathbb{M}]$. First, only a subset of the eigenvectors is considered (first truncation). Then, projecting the usual generalized eigenvalue problem of Eq. (2) onto these eigenvectors, the components of the global-displacements ROB are given by the newly obtained eigenvectors associated with the lowest eigenfrequencies (second truncation).

We now define the local-displacements subspace as the orthogonal complement of the global-displacements subspace, with respect to the inner-product given by matrix $[\mathbb{M}]$. The ROB of this subspace is constituted of orthogonal vectors with respect to both $[\mathbb{K}]$ and $[\mathbb{M}]$ (the orthogonalization is achieved solving some eigenvalue problem). Concerning computational aspects, for the construction of the global-displacements ROB and of the local-displacements ROB, a double projection method allows for avoiding the need of having $[\mathbb{K}]$ and only requires access to a diagonally-lumped approximation of $[\mathbb{M}]$. It is based on the projection of the dynamics equation onto the subspace spanned by the n elastic modes $[\Phi]$. In addition, it allows for the efficient calculation of the local-displacements ROB, whose associated local-displacements subspace allows the subspace spanned by the elastic modes to be decomposed as the internal orthogonal direct sum of the global-displacements subspace and the local-displacements subspace.

4 Multilevel reduced-order model

In Section 3 we have presented the proposed methodology dedicated to the filtering between the global displacements and the local displacements. This filtering is defined upon the polynomial degree used for the reduced kinematics. The construction of the multilevel ROM is based on this filtering methodology.

The frequency band is decomposed into the LF, MF, and HF bands. Since the HF band exhibits small-wavelength displacements in comparison to the LF band, in order to obtain a sufficient accuracy of the global-displacements ROM, a higher polynomial degree is necessary for analyzing the dynamical response up to HF, compared to the one that would be necessary for LF. We thus introduce three parameters (polynomial degrees) on which the construction of the multilevel ROM is based, $\mathcal{D}_\mathcal{L}$, $\mathcal{D}_\mathcal{M}$, and $\mathcal{D}_\mathcal{H}$, for which the associated reduced kinematics allow sufficient approximations to be obtained respectively up to LF, MF, and HF.

First filtering First, using degree $\mathcal{D}_\mathcal{H}$ and projecting the computational model onto the n elastic modes, the filtering methodology yields a global-displacements subspace and a local-displacements subspace. Assumed $\mathcal{D}_\mathcal{H}$ is high enough for representing the HF band, only the global-displacements are kept, whereas the local-displacements subspace is neglected. This allows the final dimension of the multilevel ROM to be reduced. The global-displacements subspace, which includes the totality of the remaining displacements considered for the multilevel ROM, is denoted as $\mathcal{S}_t$.

Second filtering Second, using degree $\mathcal{D}_\mathcal{M}$ and projecting the computational model onto $\mathcal{S}_t$, the filtering methodology yields a global-displacements subspace, $\mathcal{S}_{\mathcal{L}\mathcal{M}} \subseteq \mathcal{S}_t$, and a local-displacements subspace, $\mathcal{S}_\mathcal{H} \subseteq \mathcal{S}_t$.

Third filtering Third, using degree $\mathcal{D}_\mathcal{L}$ and projecting the computational model onto $\mathcal{S}_{\mathcal{L}\mathcal{M}}$, the filtering methodology yields a global-displacements subspace, $\mathcal{S}_\mathcal{L} \subseteq \mathcal{S}_{\mathcal{L}\mathcal{M}}$, and a local-displacements subspace, $\mathcal{S}_\mathcal{M} \subseteq \mathcal{S}_{\mathcal{L}\mathcal{M}}$.

The three ROBs associated with subspaces $\mathcal{S}_\mathcal{L}$, $\mathcal{S}_\mathcal{M}$, and $\mathcal{S}_\mathcal{H}$, and respectively constituted of LF-, MF-, and HF-like displacements, allow the ROB of the multilevel ROM to be obtained. As a result, each generalized matrix of the multilevel ROM is constituted of $3 \times 3 = 9$ matrix blocks. The purpose of the multilevel ROM is to allow for adapting the level of statistical fluctuations to each frequency regime (to each type of displacement). To this end, a random matrix composed of $3 \times 3 = 9$ blocks (whose dimensions match those of the blocks of the multilevel ROM matrices) is used, for which only the 3 diagonal blocks are non-zero. These blocks are given by the random matrix involved in Eq. (7). For each block, a dedicated dispersion hyperparameter is used. For instance, for the stiffness matrix: dispersion $\delta_K^\mathcal{L}$ is used for the random matrix block associated with the LF-like displacements, dispersion $\delta_K^\mathcal{M}$ is used for the random matrix block associated with the MF-like displacements, and dispersion $\delta_K^\mathcal{H}$ is used for the random matrix block associated with the HF-like displacements. Performing the Cholesky decomposition of each deterministic generalized matrix of the multilevel ROM, the multilevel stochastic ROM (ML-SROM) is obtained in a similar way than the C-SROM (see Eq. (6)), using latter block-diagonal random matrix.

5 Application to an automobile structure

Measurements of FRFs of 20 nominally identical cars are carried out. For each car, the same excitation force is applied (at the engine fasteners) and the acceleration is measured at another point (located somewhere at the back of the car). On the other hand, a finite element model (associated with the model of the measured cars) is given. It is a very detailed model, with about $m = 8,000,000$ DOFs. For this structure, the LF, MF, and HF bands are defined as $\mathcal{B}_\mathcal{L} = 2\pi\times]10, 70]$ Hz , $\mathcal{B}_\mathcal{M} = 2\pi\times]70, 300]$ Hz , and $\mathcal{B}_\mathcal{H} = 2\pi\times]300, 900]$ Hz . There are approximately 2.5 modes per Hz in the LF band, 5 modes per Hz in the MF band and 10 modes per Hz in the HF band. This high modal density is due to the fact that, as soon as low frequencies, numerous local displacements are intertwined with the usual global displacements.

The C-SROM is built upon the use of the first $n = 8,450$ elastic modes, with a maximum eigenfrequency of $1,000$ Hz. Hyperparameters $\delta_{\mathcal{K}}$ and $\delta_{\mathcal{M}}$ of the C-SROM are identified solving a statistical inverse problem (we use a modal damping model and consequently, each random realization of the diagonal damping matrix is deduced from the realization of the random generalized mass and stiffness matrices, hence the disappearance of hyperparameter $\delta_{\mathcal{D}}$). The statistical inverse problem consists in finding hyperparameters $\delta_{\mathcal{K}}$ and $\delta_{\mathcal{M}}$ that maximize the overlap between the computed random response and the experiments. The cost function associated with latter optimization problem is defined frequency-by-frequency using the overlapping coefficient [31] between the distribution of the experimental measurements and the distribution of the computed responses. These probability density functions are estimated using kernel density estimation. For the computational model, $n_{\rm sim} = 40$ Monte-Carlo realizations are considered, enough for reaching convergence of the cost function. More precisely, the analyzed FRF is the modulus of the acceleration of the observed point. Using an irregular $2D$ grid constituted of about 300 sampling points, the C-SROM is identified. In Fig. 1 the random FRF given by the identified C-SROM is plotted in log-log scale, in addition to the experimental measurements. Throughout this paper, by random FRF we mean a 95% confidence interval that is estimated using $n_{\rm sim} = 10,000$ Monte-Carlo realizations, sufficient for its convergence. It can be seen in Fig. 1 that the confidence region is not sufficiently large in the LF and MF bands (especially in the LF band).

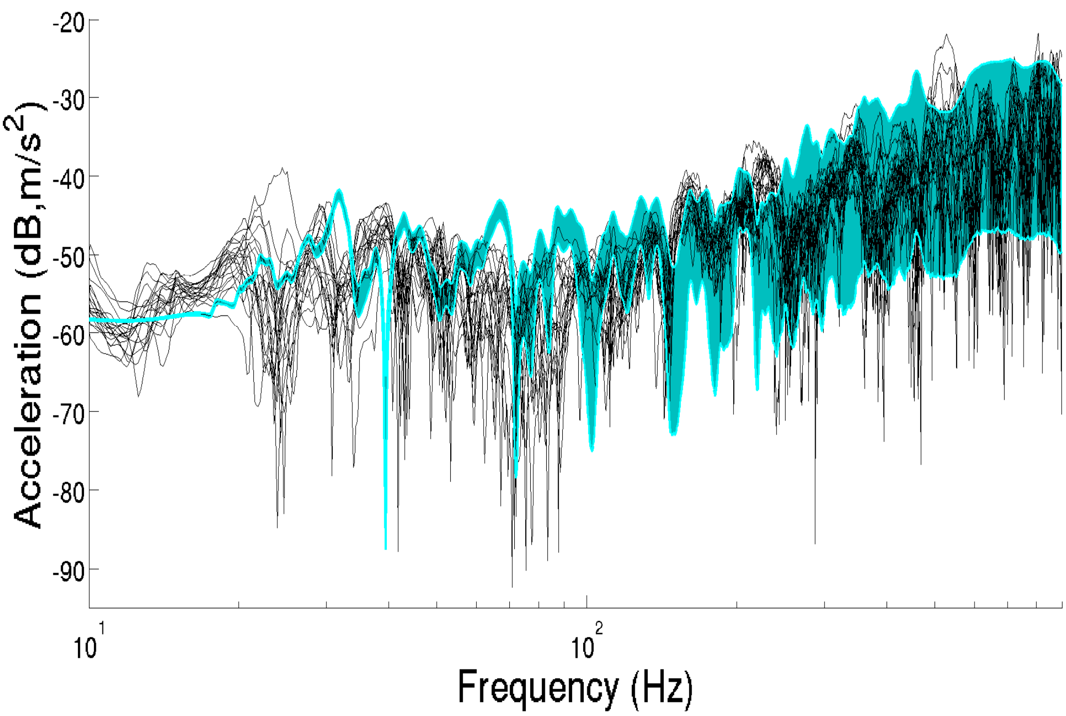

Figure 1: Experimental FRF measurements (black lines) and random FRF using the identified C-SROM (colored region)

For constructing the ML-SROM, filtering parameters $\mathcal{D}_{\mathcal{L}}$, $\mathcal{D}_{\mathcal{M}}$, and $\mathcal{D}_{\mathcal{H}}$ have to be chosen. If these parameters were to tend toward infinity, no dimension reduction would be obtained with respect to a classical modal analysis, and in addition, the families of LF-, MF-, and HF-like displacements would consist in the elastic modes present in the corresponding bands. In addition, the ML-SROM is defined upon 6 dispersions hyperparameters: $\delta_K^{\mathcal{L}}$, $\delta_M^{\mathcal{L}}$, $\delta_K^{\mathcal{M}}$, $\delta_M^{\mathcal{M}}$, $\delta_K^{\mathcal{H}}$, $\delta_M^{\mathcal{H}}$. A sensitivity analysis shows that we can consider $\delta_K^{\mathcal{L}} = \delta_M^{\mathcal{L}} = \delta^{\mathcal{L}}$, $\delta_K^{\mathcal{M}} = \delta_M^{\mathcal{M}} = \delta^{\mathcal{M}}$, and $\delta_K^{\mathcal{H}} = \delta_M^{\mathcal{H}} = \delta^{\mathcal{H}}$. First, the filtering parameters are identified in a deterministic context and fixed. Then, a coarse $3D$ grid allows for identifying $\delta^{\mathcal{L}}$, $\delta^{\mathcal{M}}$, and $\delta^{\mathcal{H}}$ (using about 500 sampling points). The identification is then refined: parameters $\mathcal{D}_{\mathcal{H}}$ and $\delta^{\mathcal{H}}$, whose effect in HF is quite independent of the other parameters, are identified concurrently (with the other parameters fixed). At last, parameters $\delta^{\mathcal{L}}$ and $\delta^{\mathcal{M}}$ are identified concurrently (using a $2D$ grid). The random response given by the identified ML-SROM is plotted in Fig. 2. It can be seen that the prediction is improved in the LF and MF bands. This is explained by the increased capability of the ML-SROM to adapt the level of statistical fluctuations to each of the frequency bands (correlated with each type of displacement). In addition, the dimension of obtained ML-SROM is $n_t = 4,232$, which constitutes a non-negligible reduction.

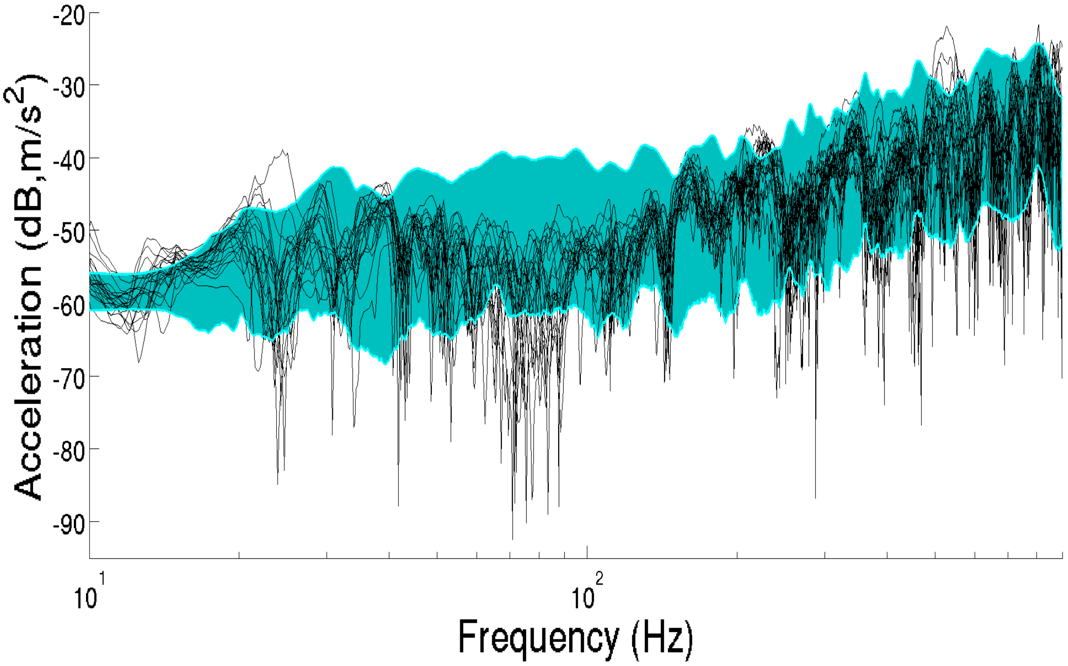

Figure 2: Experimental FRF measurements (black lines) and random FRF using the identified ML-SROM (colored region)

6 Conclusions

We have presented a general method for constructing a multilevel stochastic ROM adapted to the broadband frequency analysis of complex structures. The overlap of the LF, MF, and HF regimes, induced by the intertwining of numerous local displacements with the global displacements, brings up two difficulties, one that is related to uncertainty quantification and the other to computational efficiency. First, in order to filter the local displacements from the global displacements, we have presented a general methodology, for which the implementation is non-intrusive and efficient. It is based on the introduction of a reduced kinematics for the kinetic energy, using polynomial shape functions. Second, in order to separate the LF-, MF-, and HF-like displacements associated with the LF, MF, and HF regimes, the filtering methodology is used several times and the associated multilevel ROM is obtained. Using the nonparametric probabilistic approach of uncertainties, a multilevel stochastic ROM is constructed, which allows specific statistical dispersion levels to be assigned to each frequency regime. The proposed methodology has been efficiently applied to a complex finite element model of an automobile, for which the multilevel stochastic ROM has been identified with respect to experimental measurements. Compared to a classical stochastic ROM based on modal analysis, the proposed multilevel stochastic ROM allows for obtaining an improved FRF prediction for a lower dimension.

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