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NONLINEAR GEOMETRIC ANALYSIS OF A MISTUNED BLADED DISK

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Abstract: This paper deals with the dynamical analysis and uncertainty quantification of a mistuned industrial rotating integrally bladed disk, for which the operating regime under consideration takes into account the nonlinear geometric effects induced by large displacements and deformations. First, a dedicated mean nonlinear reduced-order model of the tuned structure is explicitly constructed using three-dimensional solid finite elements. The random nature of the mistuning is then modeled by using the nonparametric probabilistic approach extended to the nonlinear geometric context. Such a computational strategy provides an efficient tool, which is applied to a computational model of an industrial centrifugal compressor with a large number of degrees of freedom. This allows for putting in evidence some new complex dynamical behaviors.

Keywords: *vibrations, geometric non-linearities, mistuning, uncertainty quantification. . . (max 5 keywords)*

1 Introduction

One essential aspect in the field of turbomachinery is to consider the geometric nonlinear effects in the computational models occurring when exceptional operating speeds of bladed disks are analyzed due to geometric nonlinearities induced by large deformations and large displacements (1; 2). Such situation is realistic when considering a flutter kind phenomenon induced by unsteady aerodynamic coupling and yielding low damping levels. Since the unsteady aerodynamic coupling is not considered in this paper, the nonlinear domain is simulated by increasing the magnitude of the external load, while performing forced response calculations.

The present paper proposes a methodology adapted to geometric nonlinear analysis of mistuned bladed disks combined with an industrial realistic application. The first part is devoted to the development of an adapted nonlinear reduced-order computational model for the tuned structure, referred as the mean NL-ROM. It is explicitly constructed in the context of three-dimensional solid finite elements (3) by using an appropriate projection basis (4) obtained here by a linear tuned eigenvalue analysis. Once the mean NL-ROM is established, mistuning is taken into account by implementing uncertainties through the nonparametric probabilistic framework (5; 6). The application, consists in a finite-element model of an industrial integrated bladed disk with about 2, 000, 000 dof. The geometric nonlinear effects are analyzed and quantified through the dynamic analysis of the magnification factor in both tuned and mistuned cases.

2 Equations

The mean computational model of the tuned bladed-disk, which is constructed by the finite element method (FEM) is written as

$$[M\,]\,\ddot{\mathbf{u}} + (\,[C(\Omega)] + [D\,]\,)\,\dot{\mathbf{u}} + [K(\Omega)]\,\mathbf{u} + \mathbf{f}^{\mathrm{NL}}(\mathbf{u}) \quad = \quad \mathbf{f} \quad , \tag{1}$$

in which the $\mathbb{R}^n$-vector $\mathbf{f}$ corresponds to the finite element discretization of the external force fields. In Eq. (1), the $\mathbb{R}^n$-vector $\mathbf{u}$ is the finite element discretization of the unknown displacement field. In Eq. (1), the matrices $[M\,]$, $[D\,]$, and $[K(\Omega)]$ are the mass, damping, global linear stiffness real matrices with positive definiteness property. The gyroscopic coupling matrix $[C(\Omega)]$ has antisymmetry property. The geometric nonlinearities effects are taken into account through the $\mathbb{R}^n$-vector $\mathbf{f}^{\mathrm{NL}}(\mathbf{u})$ which includes both quadratic and cubic stiffness terms. For large computational models, a reduced-order model strategy is used, and is adapted to the geometric nonlinear context under consideration (see (7; 3) and (4) for a complete overview). Let a given vector basis be represented by the $(n \times P)$ real matrix $[\Phi]$. The nonlinear response $\mathbf{u}$ is expanded as

$$\mathbf{u} \,=\, [\Phi]\,\mathbf{q} \quad , \tag{2}$$

in which $\mathbf{q}$ is the $\mathbb{R}^P$-vector of the generalized coordinates. The random nature of the mistuning is then considered by implementing the nonparametric probabilistic approach, which presents the ability to include both the system-parameter uncertainties and the model uncertainties induced by modeling errors (see (5) for a complete review on the subject). It consists in replacing the operators of the mean NL-ROM by random operators, whose probability distribution is derived from the maximum entropy principle.

3 Numerical results

For fixed $\nu/\nu_0 \in \mathbb{B}$, in which ν_0 is the first linear eigenfrequency, let $Y(2\pi\nu)$ be the random dynamic amplification factor. Figure 1 shows the confidence region of the nonlinear observation $Y^{\,\mathrm{NL}}(2\pi\nu)$ for a moderate mistuning rate. It is seen that the extreme values related to $Y^{\,\mathrm{NL}}(2\pi\nu)$ yield moderate amplification even if the confidence region remains relatively broad. Although not presented in the present paper, it can be shown that, on the contrary, the linearized assumption clearly increases this amplification. It can then be deduced that the geometric nonlinear effects clearly inhibit the amplification of the random response.

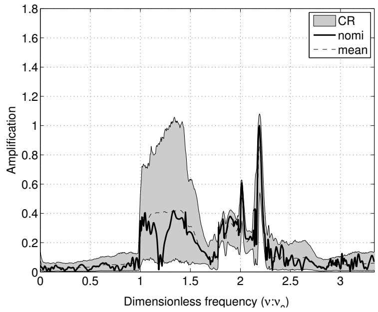

Figure 1: Stochastic analysis: frequency domain observation $Y^{\,\mathrm{NL}}(2\pi\nu)$ related to the nonlinear case for a moderate mistuning rate mean model (thick line), confidence region (gray region).

4 Conclusion

The paper has presented an analysis of the geometric nonlinear effects of an uncertain mistuned rotating industrial integrated bladed disk subjected to a high loading level. Considering the nonlinear mistuned response with uncertainties, the geometric nonlinear effects play an important role for the propagation of uncertainties. In particular, the robustness of the random response with respect to uncertainties remains strong in the frequency band of excitation, yielding reasonable amplification levels. However, such robustness suddenly falls in the sub-harmonic frequency range giving rise to consequent local amplification levels.

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