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GLOBAL REDUCED-ORDER MODEL ADAPTED TO THE LOW- AND MEDIUM-FREQUENCY ANALYSIS OF COMPLEX DYNAMICAL STRUCTURES

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Abstract. In structural dynamics, the use of the vibration eigenmodes (elastic modes) allows for obtaining an accurate small-dimension reduced-order model (ROM) for the low-frequency range analysis. For some complex structures with distinct structural levels (presence of flexible parts attached to a stiff master part), numerous local elastic modes are intertwined with the usual global elastic modes, yielding high-dimension ROM. To circumvent this difficulty, a general method is proposed in order to construct a small-dimension ROM whose reduction vector basis is constituted of the global displacements only. The method is applied to an automobile complex structure.

1 Introduction

In structural dynamics, the low-frequency (LF) range is classically characterized by frequency response functions (FRF) exhibiting well separated resonance peaks that are associated with large-wavelength global shapes. Instead, the high-frequency range, for which the FRF are rather smooth, presents a constant high modal density, which is associated with the presence of numerous elastic modes that are constituted of small-wavelength displacements. An intermediate band, the medium-frequency (MF) band, exists for complex structures [1]. The modal analysis method [2, 3, 4] consists in projecting the dynamics equations onto the elastic modes associated with the first eigenfrequencies. Since in general the LF band presents only a few elastic modes, classical modal analysis is then an efficient tool for constructing a predictive small-dimension reduced-order model (ROM). In this work, we are interested in complex structures for which the modal density can be very high, as soon as low frequencies, due to the presence of numerous local elastic modes intertwined with the usual global elastic modes. This feature is related to the presence of small-scale flexible parts, such as panels, that are attached

to a stiff master part. Subsequently, such flexible parts are responsible for the presence of elastic modes dominated by local displacements. Furthermore, in such complex structures, small geometrical heterogeneities, although not clearly identified as flexible structural elements, are also responsible for the apparition of numerous local elastic modes.

Recently, some researches [5, 6, 7] have been carried out for dealing with this problematic, whose major objective is to reduce the dimension of the ROM. The present work accounts for the last developments made, which constitute a general framework for the construction of a ROM that is adapted, for the LF and MF bands, to complex structures, and whose reduced dimension is related to its controlled accuracy. Since the contributions of the local displacements are not necessarily significant for predicting the response of the stiff master part (in this work, we are not interested in predicting the small-scale local displacements), we propose a general method for constructing a small-dimension ROM whose reduction vector basis is constituted only of the global displacements. The extraction of the global displacements is based on a filtering strategy that is constituted of two steps. Firstly, an approximation subspace is introduced. This subspace is associated with displacements of reduced kinematics. Secondly, the usual generalized eigenvalue problem, associated with the homogeneous conservative system and spanning the reduction vector basis, is modified: the mass matrix is modified such that it corresponds to the reduced kinematics. It should be noted that, in this unusual eigenvalue problem, the elastic energy is kept exact. The method, whose implementation is well adapted to an efficient use in the context of commercial softwares, is applied to the automobile complex structure introduced in [6].

2 Theory

2.1 Context

We are interested in calculating the frequency response functions (FRF) $\mathbb{U}(\omega)$ of a fixed linear structure, for ω in the frequency band of analysis $\mathcal{B} = [\omega_{\min}, \omega_{\max}]$, with $\omega_{\min} > 0$. In the context of finite element analysis [8], vector $\mathbb{U}(\omega)$ is the discretization of the displacement field and is the solution, for all ω in $\mathcal{B}$, of the following matrix equation,

$$(\, -\omega^2 \, [\, \mathbb{M} \,] + i\omega \, [\, \mathbb{D} \,] + [\, \mathbb{K} \,] \,) \, \mathbb{U}(\omega) = \mathbb{F}(\omega) \,, \tag{1}$$

where $[\, \mathbb{M} \,]$, $[\, \mathbb{D} \,]$, and $[\, \mathbb{K} \,]$ are the mass, damping, and stiffness matrices, with $\mathbb{F}(\omega)$ the discretization of the external forces. These matrices are $(m \times m)$ real positive-definite matrices, where m denotes the number of degrees of freedom (DOF) of the finite element model. The first n elastic modes $\{\boldsymbol{\varphi}^\alpha\}_\alpha$, which are associated with the n smallest eigenvalues $\{\lambda_\alpha\}_\alpha$ such that $\lambda_\alpha = \omega_\alpha^2$, are the solutions of the following generalized eigenvalue problem,

$$[\, \mathbb{K} \,] \, \boldsymbol{\varphi}^\alpha = \lambda_\alpha \, [\, \mathbb{M} \,] \, \boldsymbol{\varphi}^\alpha \,, \tag{2}$$

for obtaining the $(m \times n)$ real matrix $[\, \Phi \,] = [\, \boldsymbol{\varphi}^1 \, \ldots \, \boldsymbol{\varphi}^n \,]$, which constitutes the reduction vector basis used in the classical modal analysis method, for which vector $\mathbb{U}(\omega)$ is written, for

ω in $\mathcal{B}$ and with $n \ll m$, as

$$\mathbb{U}(\omega) \simeq \mathbb{U}^{\text{elas}}(\omega) = \sum_{\alpha=1}^{n} q_\alpha(\omega)\, \boldsymbol{\varphi}^\alpha = [\,\Phi\,]\,\mathbf{q}(\omega)\,, \tag{3}$$

in which the n-dimensional complex vector $\mathbf{q}(\omega)$ of generalized coordinates is the solution of the following reduced-order matrix equation,

$$(\,-\omega^2\,[\,M\,] + i\omega\,[\,D\,] + [\,K\,]\,)\,\mathbf{q}(\omega) = \boldsymbol{\mathcal{F}}(\omega)\,, \tag{4}$$

in which $\boldsymbol{\mathcal{F}}(\omega) = [\,\Phi\,]^{\top}\mathbb{F}(\omega)$, and where, for A in $\{M, D, K\}$ and $\mathbb{A}$ in $\{\mathbb{M}, \mathbb{D}, \mathbb{K}\}$, we have $[\,A\,] = [\,\Phi\,]^{\top}\,[\,\mathbb{A}\,]\,[\,\Phi\,]$.

For the complex structures dealt with, dimension n for obtaining convergence in $\mathcal{B}$ can be very high. In addition to the increased cost required for computing, in such a case, the first n elastic modes, the main issue is the high dimension of the classical ROM.

2.2 Methodology proposed

The first step of the method consists in introducing, for the kinetic energy, an approximation (reduced kinematics) that is adapted to the filtering of the local displacements. In the next section, the construction of the mass matrix associated with such a reduced kinematics is presented.

2.2.1 Reduced-kinematics mass matrix

Let $\mathcal{A}_g$ denote a given vector subspace of $\mathbb{R}^m$ and let N_g denote its dimension. This subspace defines a reduced kinematics. Let also $[\,B\,]$ be a $(m \times N_g)$ real matrix whose columns span $\mathcal{A}_g$. For all $\mathbb{V}$ in $\mathbb{R}^m$, the associated vector $\mathbb{V}^{\mathcal{A}_g}$ in $\mathcal{A}_g$ is defined as the orthogonal projection of $\mathbb{V}$ onto $\mathcal{A}_g$. Using the inner-product $<\mathbf{x}, \mathbf{y}>_{\mathbb{M}} = \mathbf{y}^{\top}\,[\,\mathbb{M}\,]\,\mathbf{x}$, the orthogonal-projection matrix $[\,\mathbb{P}_{\mathcal{A}_g}\,]$, which is such that

$$\mathbb{V}^{\mathcal{A}_g} = [\,\mathbb{P}_{\mathcal{A}_g}\,]\,\mathbb{V}\,, \tag{5}$$

is written as

$$[\,\mathbb{P}_{\mathcal{A}_g}\,] = [\,B\,]\,\Big(\,[\,B\,]^{\top}\,[\,\mathbb{M}\,]\,[\,B\,]\,\Big)^{-1}\,[\,B\,]^{\top}\,[\,\mathbb{M}\,]\,. \tag{6}$$

Introducing the approximation of Eq. (5) for the kinetic energy, the associated reduced-kinematics mass matrix $[\,\mathbb{M}_{\mathcal{A}_g}\,]$ is written as

$$[\,\mathbb{M}_{\mathcal{A}_g}\,] = [\,\mathbb{P}_{\mathcal{A}_g}\,]^{\top}\,[\,\mathbb{M}\,]\,[\,\mathbb{P}_{\mathcal{A}_g}\,]\,. \tag{7}$$

The $(m \times m)$ real matrix $[\,\mathbb{M}_{\mathcal{A}_g}\,]$ is symmetric positive semidefinite, of rank N_g. This matrix is generally not sparse.

2.2.2 Global-displacements ROM

The first n_g global eigenvectors $\{\boldsymbol{\psi}^\alpha\}_\alpha$, associated with the n_g smallest eigenvalues $\{\sigma_\alpha\}_\alpha$, are the solutions of the following generalized eigenvalue problem,

$$[\,\mathbb{K}\,]\,\boldsymbol{\psi}^\alpha = \sigma_\alpha\,[\,\mathbb{M}_{\mathcal{A}_g}\,]\,\boldsymbol{\psi}^\alpha\,, \tag{8}$$

for obtaining the $(m \times n_g)$ real matrix $[\,\Psi\,] = [\,\boldsymbol{\psi}^1 \,\ldots\, \boldsymbol{\psi}^{n_g}\,]$, which constitutes the reduction vector basis used in the proposed method, for which vector $\mathbb{U}(\omega)$ is written, for ω in $\mathcal{B}$ and with $n_g \leqslant n$, as

$$\mathbb{U}(\omega) \simeq \mathbb{U}^{\rm glob}(\omega) = \sum_{\alpha=1}^{n_g} q_\alpha^g(\omega)\,\boldsymbol{\psi}^\alpha = [\,\Psi\,]\,\mathbf{q}^g(\omega)\,, \tag{9}$$

and in which the n_g-dimensional complex vector $\mathbf{q}^g(\omega)$ of generalized coordinates is the solution of the following small-dimension matrix equation,

$$(\,-\omega^2\,[\,M^g\,] + i\omega\,[\,D^g\,] + [\,K^g\,]\,)\,\mathbf{q}^g(\omega) = \boldsymbol{\mathcal{F}}^g(\omega)\,, \tag{10}$$

in which $\boldsymbol{\mathcal{F}}^g(\omega) = [\,\Psi\,]^\top \mathbb{F}(\omega)$, and where, for A in $\{M, D, K\}$ and $\mathbb{A}$ in $\{\mathbb{M}, \mathbb{D}, \mathbb{K}\}$, we have $[\,A^g\,] = [\,\Psi\,]^\top\,[\,\mathbb{A}\,]\,[\,\Psi\,]$.

2.2.3 Computational aspects

Although $(m \times m)$ matrix $[\,\mathbb{M}_{\mathcal{A}_g}\,]$ is full, its assembly can be avoided by using a subspace iteration algorithm for solving Eq. (8). Nevertheless, access to stiffness matrix $[\,\mathbb{K}\,]$ may not be possible in the context of the use of commercial softwares. In order to circumvent both these difficulties, a method (that we will call the *indirect method*) is proposed. For all α in $\{1, \ldots, n_g\}$, eigenvector $\boldsymbol{\psi}^\alpha$ is approximated such that

$$\boldsymbol{\psi}^\alpha = [\,\Phi\,]\,\mathbf{s}^\alpha\,, \tag{11}$$

in which $\{\mathbf{s}^\alpha\}_\alpha$ are n_g-dimensional real vectors, which are the solutions of the following reduced-order generalized eigenvalue problem,

$$[\,\Lambda\,]\,\mathbf{s}^\alpha = \sigma_\alpha [\,M_{\mathcal{A}_g}\,]\,\mathbf{s}^\alpha\,, \tag{12}$$

in which $[\,\Lambda\,] = [\,\Phi\,]^T\,[\,\mathbb{K}\,]\,[\,\Phi\,]$ and $[\,M_{\mathcal{A}_g}\,] = [\,\Phi\,]^T\,[\,\mathbb{M}_{\mathcal{A}_g}\,]\,[\,\Phi\,]$. Since the contributions of the discarded local displacements are neglected, a residual error is introduced in the global-displacements ROM. A convergence analysis with respect to subspace $\mathcal{A}_g$ allows for reaching a compromise between dimension n_g and accuracy of the ROM. Using the indirect method, the overall numerical cost for performing such a convergence analysis is then nearly reduced to the computation of the n elastic modes.

3 Application to an automobile complex structure

We use the indirect method proposed, in order to construct a small-dimension ROM for the automobile complex structure whose computational model is displayed in Fig. 1 (in which the gray intensity is related to the level of rigidity).

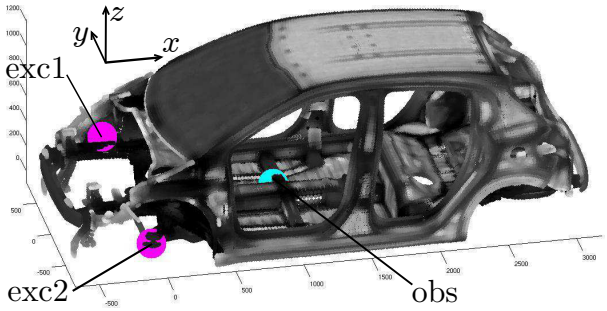

Figure 1: Computational model, with location of excitation nodes exc1 and exc2, and of observation node obs. Gray intensity is related to the level of rigidity (the darker is the stiffer).

There are well identified flexible parts (such as the roof and the floor panels) as well as numerous structural heterogeneities (see for example in the front of the car) that are both responsible for the presence of numerous local elastic modes. The computational model has $m = 1,462,698$ DOF and there are $1,048$ elastic modes in the frequency band of analysis $\mathcal{B} = 2\pi\times]0,500]$ rad/s, while the classical ROM is converged with $n = 1,457$. Figure 2 displays, on the left, a purely local elastic mode and, on the right, a purely global elastic mode.

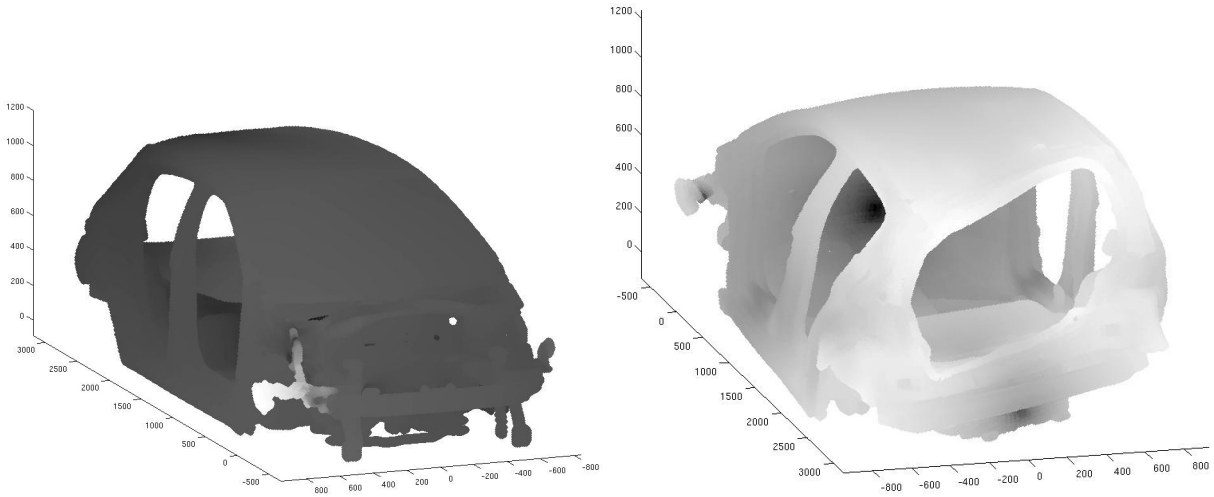

Figure 2: Left: elastic mode $\boldsymbol{\varphi}^1$ (24 Hz), right: elastic mode $\boldsymbol{\varphi}^3$ (39 Hz). Gray intensity is related to the level of amplitude of the displacements (the greater amplitude is the lighter)

In the method proposed, the global-displacements basis, represented by matrix $[\,\Psi\,]$, is entirely

defined upon the subspace $\mathcal{A}_g$ chosen, as well as upon truncation order n_g. In this application, $\mathcal{A}_g$ is spanned by vectors that consist in discrete multivariate Legendre polynomials whose degree, N_d, allows for controlling the filtering of the local displacements. The ROM convergence is analyzed in studying the FRF obtained under the application of unit forces in the x- and z-directions and unit moments around the x- and y- axes, relative to the excitation nodes exc1 and exc2 (depicted in Fig. 1). This convergence analysis of the global-displacements ROM is done with respect to N_d, and, for fixed N_d, with n_g chosen as the smallest value such that the associated highest eigenfrequency is greater than 525 Hz. Figure 3 displays the FRF of observation node obs (norm of the displacements of the node, in log scale) obtained using Eq. (2) for the reference, and using Eq. (9) for the small-dimension ROM in which N_d is successively chosen as equal to 5, 10, 15, and 20. The resulting dimensions n_g are 168, 355, 479, and 624.

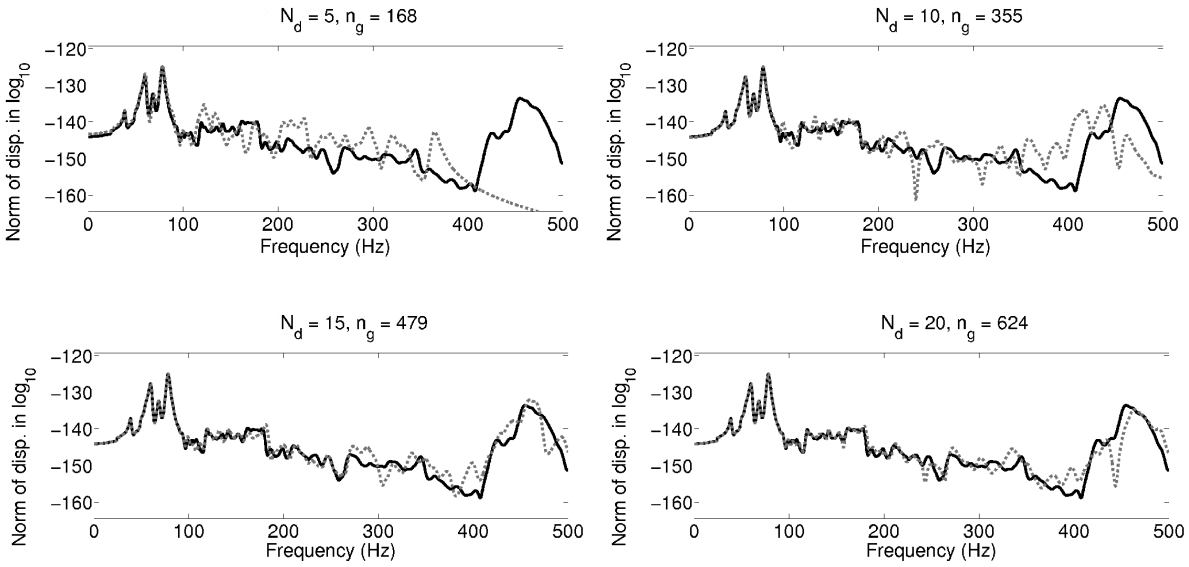

Figure 3: Logarithm of the norm of the displacements (m) versus frequency (Hz): reference $\mathbb{U}_{\text{obs}}^{\text{elas}}$ (black solid line), ROM response $\mathbb{U}_{\text{obs}}^{\text{glob}}$ for N_d equal to either 5, 10, 15, or 20 (gray dashed line).

It can be seen in Fig. 3 that the differences with respect to the reference are decreasing with respect to the increase of N_d (and thus n_g). The differences are larger in the high part of the frequency band, while it is known that the experimental variabilities are generally larger in this band, too. For $n_g = 479$ or $n_g = 624$, the error introduced in the computation of the response is not as important as the experimental variabilities may be [9, 10].

4 Conclusions

A general and efficient method for constructing a reduced-order computational model, which is adapted to predicting the global displacements of complex structures for which there are numerous local elastic modes intertwined with the global elastic modes, has been presented.

The strategy relies on the filtering of the local displacements in order to construct a small-dimension ROM. The filtering is obtained by reducing the kinematics of the kinetic energy in the usual eigenvalue problem. A convergence analysis with respect to the reduced-kinematics responsible for the filtering of the local displacements allows to control, with a reduced cost, the compromise between the ROM accuracy and dimension.

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