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Abstract: In this paper, the robust design with an uncertain model of a vibro-impact electromechanical system is done. The electromechanical system is composed of a cart, whose motion is excited by a DC motor (motor with continuous current), and an embarked hammer into this cart. The hammer is connected to the cart by a nonlinear spring component and by a linear damper, so that a relative motion exists between them. A linear flexible barrier, placed outside of the cart, constrains the hammer movements. Due to the relative movement between the hammer and the barrier, impacts can occur between these two elements. Some system parameters are uncertain, such as the stiffness and the damping coefficients of the flexible barrier. The objective of the paper is to perform an optimization of this electromechanical system with respect to design parameters (spring component, and barrier gap) in order to maximize the impact power under the constraint that the electric power consumed by the DC motor is lower than a maximum value. This optimization is formulated and solved in the framework of robust design due to the presence of uncertainties in the model.

Keywords: electro-mechanical systems, vibro-impact, robust design, nonlinear dynamics.

NOMENCLATURE

ν = source voltage, V
c = electric current, A
$\dot{\alpha}$ = angular speed of the motor, rad/s
l = electric inductance, H
j_m = inertia moment, Kg m^2
b_m = damping ratio in the transmission of the torque, Nm/(rad/s)
k_e = constant of the motor electromagnetic force, (rad/s) /V
r = electrical resistance, Ω
Δ = eccentricity of the pin, m
m_c = cart mass, Kg
m_h = hammer mass, Kg
h = hammer displacement, m
k_{h1} = linear hammer stiffness, N/m
k_{h3} = cubic hammer stiffness, N/m^2
c_{pin} = friction coefficient between the pin and the slot, Ns/m
c_{ext} = friction coefficient between the cart and the rail, Ns/m
c_{int} = friction coefficient between the cart and the hammer, Ns/m
ς_{int} = damping ratio of friction between the cart and the hammer, dimensionless
k_i = barrier stiffness, N/m
c_i = barrier damping coefficient, Ns/m
g = gap barrier, m
π_{imp} = sum of the averages of the impact power, W
π_{elec}= average of the electric power, W

INTRODUCTION

The design of electro-mechanical systems is of a great interest in many areas. Many works have been done in this topic, as Zhankui and Sun (2013), Sadeghian and Rezazadeh (2009) and Lee, Cho and Chang(2006), trying to characterize the mutual interaction between electrical and mechanical parts. This interaction leads us to analyze very interesting nonlinear dynamical systems (see for instance Cartmell (1990)) , in which the nonlinearities vary with the coupling conditions, and also affects the two most important variables used to evaluate the performance of electro-mechanical systems, related to the power consumed by the electrical part, and the power used into the movement of the mechanical part. As the mutual interaction between electrical and mechanical parts affects the two powers used to evaluate the system performance, the coupling effects must be analyzed in the design optimization problem for electro-mechanical systems.

The present work deals with the robust design optimization of a vibro-impact electric-mechanical system in order to improve its performance. The electrical part of the system is a DC motor, and the mechanical part is a vibro-impact system. It should be noted that, in Lima and Sampaio (2012), the equations and the numerical integration were presented for a similar electric-mechanical system for which the embarked mass was replaced by a pendulum and for which there was no impact. This first work has allowed the electro-mechanical coupling to be analyzed as a function of the mass of the mechanical system.

The analysis of vibro-impact systems is not a new subject, and is frequently encountered in technical applications of mechanisms. The interest of analyzing the optimization of their performance is reflected by the increasing amount of research in this area (see for instance Luo et al (2008), Ostasevicius, Gaidys and Dauksevicius (2009), Yue, Xu and Wang (2013)), and also the book Ibrahim (2009), which is completely devoted to this problem).

The objective of the paper is to perform an optimization of this electro-mechanical system with respect to design parameters. The optimization consists in maximizing the impact power under the constraint that the electric power consumed by the DC motor is lower than a maximum value. This optimization is formulated in the framework of robust design due to the presence of uncertainties in the computational nonlinear dynamics model of the electro-mechanical system.

This paper is organized as follows. In Section 1 , the elements (motor, cart, hammer, and barrier) of the electro-mechanical system are presented and the initial value problem is formulated for the vibro-impact electro-mechanical system. In Section 2, we define the variables of interest for the design optimization. The construction of the probabilistic models of the uncertain parameters, and the formulation of the robust design optimization problem are given in Section 3. The robust design optimization consists in finding the optimal design point using the computational model in presence of uncertainties. The numerical results of the robust design optimization problem are presented in Section 4.

1 DYNAMIC OF THE COUPLED SYTEM

1.1 Electrical component: DC motor

The modeling of DC motors is based on the Kirchhoff law (see Karnopp (2006)), which is written as

$$\begin{aligned} l\,\dot{c}(t) + r\,c(t) + k_e\,\dot{\alpha}(t) &= \nu\,, \\ j_m\,\ddot{\alpha}(t) + b_m\,\dot{\alpha}(t) - k_e\,c(t) &= -\tau(t)\,, \end{aligned} \tag{1}$$

where t is time, ν is the source voltage, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the inertia moment of the motor, b_m is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, k_e is the constant of the motor electromagnetic force, and r is the electrical resistance. Figure 1 shows a sketch of the DC motor. The available torque delivered to the mechanical component, in the z-direction, is represented by τ (see Fig. 1).

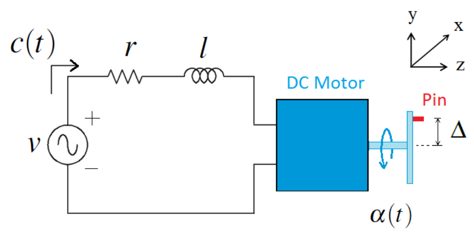


Figure 1 – Sketch of the DC motor.

1.2 Mechanical component: cart and hammer

As described in the introduction, the mechanical component is composed by a cart whose movement is driven by the DC motor, and by a hammer that is embarked into the cart. The motor is coupled to the cart through a pin that slides into a slot machined in an acrylic plate that is attached to the cart, as shown in Fig. 2. The off-center pin is fixed on the disc at distance Δ of the motor shaft, so that the motor rotational motion is transformed into a cart horizontal movement. To model the coupling between the motor and the mechanical system, the motor shaft is assumed to be rigid. Thus, the

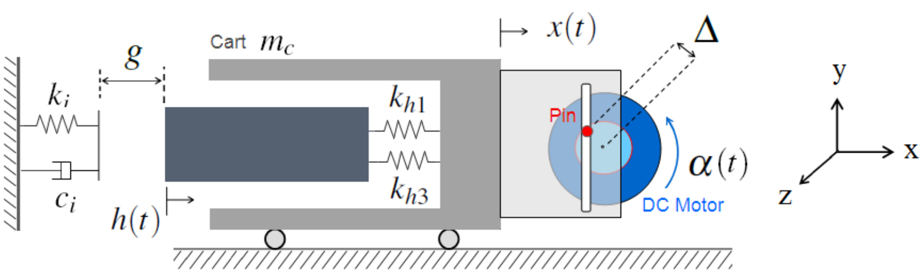


Figure 2 – Vibro-impact electro-mechanical system. The nonlinear component spring is drawn as a linear spring with constant k_{h1} and a nonlinear cubic spring with constant k_{h3}.

available torque vector to the coupled mechanical system, $\boldsymbol{\tau}$, can be written as

$$\boldsymbol{\tau}(t) = \boldsymbol{\Delta}(t) \times \mathbf{f}(t)\,, \tag{2}$$

where $\mathbf{\Delta} = (\Delta\ \cos\alpha(t),\ \Delta\ \sin\alpha(t),\ 0)$ is the vector related to the eccentricity of the pin, and where $\mathbf{f}$ is the coupling force between the DC motor and the cart. Assuming that there is a viscous friction between the pin and the slot, the vector $\mathbf{f}$ has two components: the horizontal force that the DC motor exerts in the cart, f_x, and the vertical force, f_y, induced by the viscous friction. The available torque τ and vertical force f_y are written as

$$\tau(t) = f_y(t)\ \Delta\ \cos\alpha(t) - f_x(t)\ \Delta\ \sin\alpha(t)\,, \tag{3}$$

$$f_y(t) = c_{pin}\ \Delta\ \dot{\alpha}(t)\ \cos\alpha(t)\,, \tag{4}$$

where c_{pin} is defined in Fig. 2. The embarked hammer is modeled as a rigid body of mass m_h and its relative displacement is h with respect to the cart. In the adopted model, the constitutive equation of the spring component between the hammer and the cart is written as $f_s(t) = k_{h1}\ h(t) + k_{h3}\ h(t)^3$. The rate of nonlinearity of the hammer stiffness is defined as $r_h = k_{h3}/k_{h1}$. The horizontal cart displacement is represented by x. Due to constraints, the cart is not allowed to move in the vertical direction. The spring-damper element modeling the medium on which the impacts occur, is constituted of a linear spring with stiffness coefficient k_i and a damper with damping coefficient c_i. The equations of the mechanical component are

$$\ddot{x}(t)\ (m_c + m_h) + \ddot{h}(t)\ m_h + c_{ext}\ \dot{x}(t) = -f_{\text{imp}}(t) + f_x(t)\,, \tag{5}$$

$$\ddot{x}(t)\ m_h + \ddot{h}(t)\ m_h + c_{int}\ \dot{h} + k_{h1}\ h(t) + k_{h3}\ h^3(t) = -f_{\text{imp}}(t)\,, \tag{6}$$

where, c_{ext} is the viscous friction coefficient between the cart and the rail and $c_{int} = 2\varsigma_{int}\sqrt{m_h k_{h1}}$ is the viscous friction coefficient between the cart and the hammer (ς_{int} is the damping ratio), and where f_{imp} is the impact force between the hammer and the barrier, which is written as

$$f_{\text{imp}}(t) = -\phi(t)\left(\ k_i\ (x(t) + h(t) + g) +\ c_i\ (\dot{x}(t) + \dot{h}(t))\right)\,, \tag{7}$$

where

$$\phi(t) = \begin{cases} 1, & \text{if} \quad x(t) + h(t) + g < 0 \quad \text{and} \quad \dot{h}(t) + \dot{x}(t) < 0\,, \\ 0, & \text{in all other cases}\,, \end{cases} \tag{8}$$

in which g is defined as the horizontal distance from the hammer (when $\alpha = \pi/2$ rad) to the equilibrium position of the barrier. In the model defined by Eq. (8), an impact starts when $x(t)+h(t)$ is negative and equal to $-g$ and, $\dot{h}(t)-\dot{x}(t) < 0$. During an impact, the action of the barrier on the hammer stops as soon as the total velocity $\dot{h}(t) + \dot{x}(t)$ becomes positive (the return of the hammer).

1.3 Coupled vibro-impact electro-machanical system

Due to the system geometry, we have the following constraint

$$x(t) = \Delta\ \cos(\alpha(t))\,. \tag{9}$$

Substituting Eqs. (3) to (9) into Eq. (1), we obtain the initial value problem for the vibro-impact electro-machanical system that is written as follows. Given a constant source voltage ν, find (α, c, h) such that, for all $t > 0$,

$$l\dot{c}(t) + rc(t) + k_e\dot{\alpha} = \nu\,, \tag{10}$$

$$\begin{aligned} \ddot{\alpha}(t)\left[j_m + (m_c + m_h)\Delta^2 \sin(\alpha(t))^2\right] - \ddot{h}(t)\left[m_h\Delta\ \sin(\alpha(t))\right] - k_e c(t) \\ +\dot{\alpha}(t)\left[b_m + \dot{\alpha}(t)(m_c + m_h)\Delta^2\ \cos(\alpha(t))\sin(\alpha(t)) + c_{pin}\Delta^2\cos(\alpha(t))^2 - c_{ext}\Delta^2\sin(\alpha(t))^2\right] \\ = \phi\left(k_i(\Delta\ \cos(\alpha(t)) + h + g) + c_i(-d\dot{\alpha}(t)\sin(\alpha(t)) + \dot{h}(t))\right)\Delta\ \sin(\alpha(t))\,, \end{aligned} \tag{11}$$

$$\begin{aligned} \ddot{h}(t)m_h - \ddot{\alpha}(t)\left[m_h\Delta\ \sin(\alpha(t))\right] - \dot{\alpha}(t)\left[m_h\Delta\,\dot{\alpha}(t)\cos(\alpha(t))\right] + \dot{h}(t)c_{int} + k_{h1}h(t) + k_{h3}h^3(t) \\ = \phi(t)\left(k_i(\Delta\ \cos(\alpha(t)) + h + g) + c_i(-\Delta\,\dot{\alpha}(t)\sin(\alpha(t)) + \dot{h}(t))\right)\,, \end{aligned} \tag{12}$$

$$\text{where} \quad \phi(t) = \begin{cases} 1, & \text{if} \quad \Delta\cos\alpha(t) + h(t) + g < 0 \quad \text{and} \quad \dot{h}(t) - \Delta\,\dot{\alpha}(t)\cos(\alpha(t)) < 0 \\ 0, & \text{in all other cases}\,, \end{cases} \tag{13}$$

with the initial conditions, $\alpha(0) = 0$, $c(0) = \nu/r$, and $h(0) = 0$.

2 POWERS OF THE SYSTEM

The average of the electric power consumed in an interval $[0,T]$ is written as

$$\pi_{\text{elec}} = \frac{1}{T}\int_0^T \nu\, c(t)\, \mathrm{d}t. \tag{14}$$

Let t_b^j and t_e^j be the instants of begin and end of the j-th impact, such that for all t belonging to $[t_b^j\,, t_e^j]$, we have $\dot{x}(t) + \dot{h}(t) < 0$. At time t, the impact power, $\pi_{\text{imp}}^j(t)$, is then written as

$$\pi_{\text{imp}}^j(t) = k_i\,(x(t)+h(t))\,(\dot{x}(t)+\dot{h}(t)), \qquad t_b^j \le t \le t_e^j\,. \tag{15}$$

The time average of the impact power during the j-th impact, $\underline{\pi}_{\text{imp}}^j$, is written as

$$\underline{\pi}_{\text{imp}}^j = \frac{1}{t_e^j - t_b^j}\int_{t_b^j}^{t_e^j} \pi_{\text{imp}}^j(t)\, dt\,. \tag{16}$$

Let N_{imp} be the total number of impacts that occur during time interval $[0,T]$. The sum of the averages of the impact power is

$$\pi_{\text{imp}} = \sum_{j=1}^{N_{\text{imp}}} \underline{\pi}_{\text{imp}}^j\,. \tag{17}$$

The variables π_{imp} and π_{elec} are considered for measuring the system performance. More π_{imp} is large and smaller is π_{elec}, better will be the system performance.

3 ROBUST DESIGN OPTIMIZATION PROBLEM

The three parameters that are assumed to be uncertain are $kh1$, k_i and c_i, which are modeled by the independent random variables K_{h1}, K_i and C_i. The probability distribution of each one is constructed using the maximum entropy principle. In order to formulate the robust design problem, the set of all the system parameters are divided into three subsets. The first one is the family of the fixed parameters that are represented by the vector $\mathbf{p}_{\text{fix}} = \{\,\nu,\ l,\ r,\ j_m,\ k_e,\ b_m,\ c_{pin},\ c_{ext},\ \varsigma_{int},\ r_h,\ m_c,\ m_h,\ \Delta\,\}$. The second one is the family of the design parameters that are represented by the vector $\mathbf{p}_{\text{des}} = \{\underline{K}_{h1}/m_h,\ g\}$. The third one is the family of the uncertain parameters that are represented by the random vector $\mathbf{P}_{\text{unc}} = \{K_i,\ C_i,\ K_{h1}\}$. Since $\mathbf{P}_{\text{unc}}$ is a random vector, the outputs of the electro-mechanical system are stochastic processes, and consequently, $\pi_{\text{imp}}(\mathbf{p}_{\text{des}}, \mathbf{p}_{\text{unc}})$ and $\pi_{\text{elec}}(\mathbf{p}_{\text{des}}, \mathbf{p}_{\text{unc}})$, become random variables $\Pi_{\text{imp}}(\mathbf{p}_{\text{des}}) = \pi_{\text{imp}}(\mathbf{p}_{\text{des}}, \mathbf{P}_{\text{unc}})$ and $\Pi_{\text{elec}}(\mathbf{p}_{\text{des}}) = \pi_{\text{elec}}(\mathbf{p}_{\text{des}}, \mathbf{P}_{\text{unc}})$. The cost function of the robust design optimization problem is defined by

$$J(\mathbf{p}_{\text{des}}) = E\{\Pi_{\text{imp}}(\mathbf{p}_{\text{des}})\}\,. \tag{18}$$

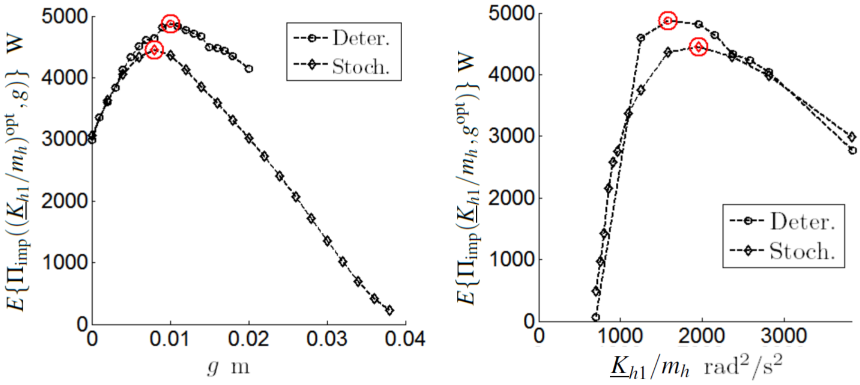
The robust design optimization problem is written as

$$\mathbf{p}_{\text{des}}^{\text{opt}} = \arg\max_{\mathbf{p}_{\text{des}} \in \mathcal{C}_{ad}} J(\mathbf{p}_{\text{des}})\,, \tag{19}$$

in which $\mathcal{C}_{ad} = \{\mathbf{p}_{\text{des}} \in \mathcal{P}_{\text{des}};\ E\{\Pi_{\text{elec}}(\mathbf{p}_{\text{des}})\} \le c_{\text{elec}}\}$, where $\mathcal{P}_{\text{des}}$ is the admissible set of the values of $\mathbf{p}_{\text{des}}$, and where c_{elec} is an upper bound.

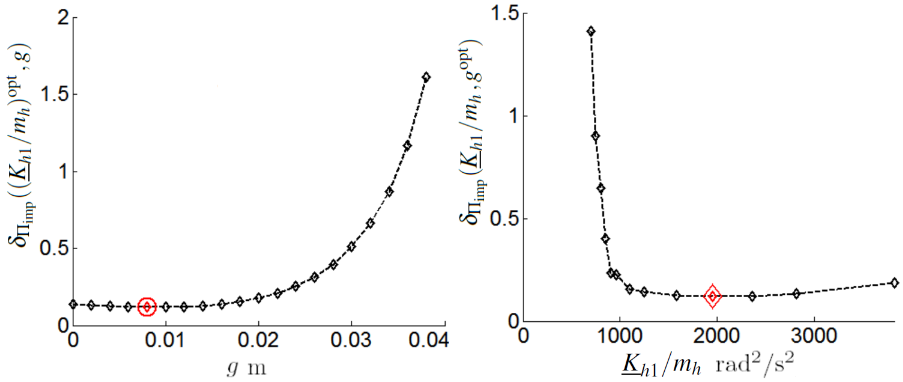
4 RESULTS OF THE ROBUST OPTIMIZATION PROBLEM

The hyperparameters δ_{K_i}, δ_{C_i} and $\delta_{k_{h1}}$, which control the level of uncertainties for K_i, C_i and K_{h1} are fixed to 0.1. The optimization problem is also considered without uncertainties in the systems parameters, that is, the deterministic case ($\delta_{K_{h1}} = \delta_{K_i} = \delta_{C_i} = 0$). For $\mathbf{p}_{\text{des}} \in \mathcal{C}_{ad}$, the cost function is estimated by the Monte Carlo simulation method using 100 independent realizations of random vector $\mathbf{P}_{\text{unc}}$ following its probability distribution. The optimization problem (defined by Eq. (19)) is solved using the trial method for which the admissible set $\mathcal{C}_{ad}$ is meshed as follows: for $\underline{K}_{h1}/m_h$, 13 values are non-uniformly selected in the interval $[703\,, 3, 830]$, and for g, 20 nonuniform values in $[0\,, 0.038]$. For computation, the initial value problem defined by Eqs. (10) to (13) has been rewritten in a dimensionless form. Duration is chosen as $T = 10.0$ s. The 4th-order Runge-Kutta method is used for the time integration scheme for which we have implemented a varying time-step. The time-step is adapted to the state of the dynamical system according to the occurrence or the not occurrence of impacts. When the hammer is not impacting the barrier, the time step used is 10^{-4} s, but when the hammer is approaching the barrier and when it is impacting it, the time step is chosen as the value 10^{-5} s. The values used for the motor parameters, were obtained from the specifications of the motor Maxon DC brushless number $411, 678$. The others elements of $\mathbf{p}_{\text{fix}}$ are: $\nu = 2.4$ V, $m_c = 0.30$ Kg, $m_h = 0.50$ Kg, $r_h = 0.30$ 1/m^2, $c_{pin} = c_{ext} = 5.00$ Ns/m, $\varsigma_{int} = 0.05$, and $\Delta = 0.01$ m. Upper bound c_{elec} is 6.00 W. For the deterministic case, the components of the optimal solution $\mathbf{p}_{\text{des}}^{\text{opt}}$



(a) (b)

Figure 3 – (a) Cost function as function of g with $(\underline{K}_{h1}/m_h)^{\text{opt}}$. (b) Cost function as function of $\underline{K}_{h1}/m_h$ with g^{opt}. In both graphs, the $E\{\Pi_{\text{imp}}(\mathbf{p}_{\text{des}}^{\text{opt}})\}$ is highlighted for the deterministic and stochastic cases with markers .



(a) (b)

Figure 4 – (a) Coefficient variation of Π_{imp} as function of g with $(\underline{K}_{h1}/m_h)^{\text{opt}}$ (b) Coefficient variation of Π_{imp} as function of $\underline{K}_{h1}/m_h$ with g^{opt}. In both graphs, the $\delta_{\Pi_{\text{imp}}}(\mathbf{p}_{\text{des}}^{\text{opt}})$ is highlighted with markers.

are $(\underline{K}_{h1}/m_h)^{\rm opt} = 1,580$ rad^2/s^2 and $g^{\rm opt} = 0.011$ m. For case with uncertainties, it is $\underline{K}_{h1}/m_h = 1,950$ rad^2/s^2 and $g = 0.008$ m. The role played by uncertainties on the optimal values of the design parameters can be analyzed through Fig. 3, which display the graphs $g \mapsto E\{\Pi_{\rm imp}((\underline{K}_{h1}/m_h)^{\rm opt}, g)\}$, and $\underline{K}_{h1}/m_h \mapsto E\{\Pi_{\rm imp}(\underline{K}_{h1}/m_h, g^{\rm opt})\}$. The robustness of the optimal design point, $\mathbf{p}_{\rm des}^{\rm opt}$, can be analyzed in studying the evolution of the coefficient variation, $\delta_{\Pi_{\rm imp}}(\mathbf{p}_{\rm des}^{\rm opt})$, of random variable $\Pi_{\rm imp}(\mathbf{p}_{\rm des}^{\rm opt})$ as a function of the uncertainty level. However, in order to better analyze the sensitivity of the responses with respect to the uncertainty level, we have constructed Fig. 4 that displays the graphs $g \mapsto \delta_{\Pi_{\rm imp}}((\underline{K}_{h1}/m_h)^{\rm opt}, g)$ and $\underline{K}_{h1}/m_h \mapsto \delta_{\Pi_{\rm imp}}(\underline{K}_{h1}/m_h, g^{\rm opt})$. It can be seen that the value $\delta_{\Pi_{\rm imp}}(\mathbf{p}_{\rm des}^{\rm opt})$ occurs in a region for which the two following functions $g \mapsto \delta_{\Pi_{\rm imp}}((\underline{K}_{h1}/m_h)^{\rm opt}, g)$ and $\underline{K}_{h1}/m_h \mapsto \delta_{\Pi_{\rm imp}}(\underline{K}_{h1}/m_h, g^{\rm opt})$ are minima. This means the optimal design point is robust with respect to uncertainties.

5 CONCLUSIONS

In this paper, the formulation and the solution of a robust design optimization problem have been presented for a nonlinear vibro-impact electro-mechanical system in presence of uncertainties in the computational model. Since this nonlinear electro-mechanical system is devoted to the vibro-impact optimization, the time responses exhibit numerous shocks that have to be identified with accuracy, and consequently, a very small time step is required. We have thus chosen an explicit time-integration scheme and not an implicit one. Nevertheless, due to the presence of low-frequency contributions in the time responses, a long time duration is required, which will imply a huge number of integration time step if the time step were chosen constant. This is the reason why we have implemented an adaptive integration time step. It was one of the difficulties encountered for the solver implementation. The design optimization problem of the dynamical system without uncertainties yields an optimal design point that differs from the nominal values, and which can not be determined, *a priori*, without solving the design optimization problem. In addition, the robust analysis that has been presented demonstrates the interest that there is to take into account the uncertainties in the computational model. The optimal design point that has been identified in the robust design framework significantly differs from design point obtained with the computational model without uncertainties. For this electro-mechanical system, it has been seen that, the minimum value of the dispersion of the random output occurs in the region of the optimal design parameters, which means that the optimal design point is robust with respect to uncertainties.

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