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# Identification of a stochastic computational dynamical model using experimental modal data - Application on an industrial built-up structure

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## Abstract

This research is devoted to the identification of a stochastic computational model using experimental eigenfrequencies and mode shapes measured on several specimens. The objective here is to construct a one-to-one correspondence between the results provided by the stochastic computational model and the experimental data that takes into account the random modes crossing and veering phenomena that may occur from one realization to another one. Then such a correspondence allows the computed modal quantities to be compared with the experimental data in order to identify the parameters of the stochastic computational model. The methodology is applied to a booster pump of thermal units.

## 1 Introduction

We consider the random context for which the available experimental data are related to a family of several experimental configurations of a given dynamical structure. The observed variability between the experimental configurations of this family is induced (1) by the uncontrolled differences that can appear during the manufacturing process (manufacturing tolerances) and during the life cycle of the structure (natural damage, incidents, etc) and (2) by some slight differences which are controlled and are related, for instance, to the boundary conditions, the embedded equipments, etc. These two types of variability induce differences for the data measured on two configurations of the given dynamical structure.

In such a random context, we have to construct a stochastic computational model (SCM) for which two sources of uncertainties have to be taken into account (see for instance [11]): (1) the uncertainties relative to some model parameters of the nominal computational model (NCM) and (2) the modeling errors. The stochastic computational model which is constructed with these two sources of uncertainties (and with additional input and output noises if measurement errors are significant) must have the capability of representing the variability of all the measured configurations.

In this paper, the uncertainties are taken into account using a probabilistic approach and then the SCM is constructed including both the model-parameter uncertainties and the model uncertainties in a separate way (using the *generalized probabilistic approach of uncertainties* proposed in [10, 11]). Usually, a SCM is

controlled by a set of hyperparameters such as mean values, coefficients of variation, and so on. These hyperparameters have to be identified using experimental data and realizations of the SCM. Several types of observation can be used in order to perform such an identification. The objective of this paper consists in identifying the hyperparameters of a SCM using natural frequencies and mass-normalized mode shapes measured for a family of structures. The methodology proposed introduces a random transformation of the computational observations (computational eigenfrequencies and computational mode shapes) in order to match them to the experimental observation of each measured structure. This methodology automatically takes into account the mode crossings and the mode veerings which can appear between two experimental configurations or between two computational realizations of the SCM. In Section 2, the construction of the SCM is summarized. Section 3 is devoted to the identification of the hyperparameters of the SCM. Finally, in Section 4, an application devoted to an industrial pump of a thermal unit is presented.

## 2 Construction of the stochastic computational model

The NCM is constructed using the FE method and the boundary conditions of the structure are such that there are no rigid body modes. In this section, a parameterized SCM is constructed using of the *generalized probabilistic approach of uncertainties* proposed in [10, 11], for which both the model-parameter uncertainties and the model uncertainties are taken into account and are separately identified.

### 2.1 Construction of the probabilistic model of uncertain model parameters

The NCM exhibits  $n_p$  uncertain model parameters denoted  $h_1, \dots, h_{n_p}$ . Let be  $\mathbf{h} = (h_1, \dots, h_{n_p})$ . The probabilistic model of uncertain model parameters is constructed by replacing vector  $\mathbf{h}$  of the uncertain model parameters by the random vector  $\mathbf{H}$  with values in  $\mathbb{R}^{n_p}$ , defined on a probability space  $(\Theta^{\text{par}}, \mathcal{T}^{\text{par}}, \mathcal{P}^{\text{par}})$ . The first  $n$  random eigenvalues  $0 < \Lambda_1^{\text{par}} \leq \dots \leq \Lambda_n^{\text{par}}$  associated with the random mode shapes  $\phi_1^{\text{par}}, \dots, \phi_n^{\text{par}}$  are the solutions of the following random generalized eigenvalue problem,

$$[\mathbb{K}(\mathbf{H})] \phi^{\text{par}} = \Lambda^{\text{par}} [\mathbb{M}(\mathbf{H})] \phi^{\text{par}}. \quad (1)$$

Let  $[\Phi^{\text{par}}]$  be the  $m \times n$  random matrix whose columns are the first  $n$  random mode shapes. We then introduce the  $n \times n$  random mass and stiffness reduced matrices  $[\mathbf{M}^{\text{par}}] = [\Phi^{\text{par}}]^T [\mathbb{M}(\mathbf{H})] [\Phi^{\text{par}}]$  and  $[\mathbf{K}^{\text{par}}] = [\Phi^{\text{par}}]^T [\mathbb{K}(\mathbf{H})] [\Phi^{\text{par}}]$ . The random mode shapes are normalized (almost surely) with respect to the random mass matrix such that

$$[\mathbf{M}^{\text{par}}] = [I_n], \quad (2)$$

and thus, the random diagonal  $n \times n$  real matrix  $[\mathbf{K}^{\text{par}}]$  is written as

$$[\mathbf{K}^{\text{par}}] = \text{diag}(\Lambda_1^{\text{par}}, \dots, \Lambda_n^{\text{par}}). \quad (3)$$

By construction, the random matrices  $[\mathbf{M}^{\text{par}}]$  and  $[\mathbf{K}^{\text{par}}]$  are positive definite (almost surely) and therefore, their Cholesky decompositions yield,

$$[\mathbf{M}^{\text{par}}] = [\mathbf{L}_M]^T [\mathbf{L}_M], \quad [\mathbf{K}^{\text{par}}] = [\mathbf{L}_K]^T [\mathbf{L}_K]. \quad (4)$$

Let  $\alpha_{\text{par}}$  be the vector whose components are the hyperparameters of the pdf  $p_{\mathbf{H}}(\mathbf{h})$  which is then rewritten as  $p_{\mathbf{H}}(\mathbf{h}; \alpha_{\text{par}})$ .

### 2.2 Construction of the generalized probabilistic model of uncertainties

Let  $(\Theta^{\text{mod}}, \mathcal{T}^{\text{mod}}, \mathcal{P}^{\text{mod}})$  be another probability space. To take into account model uncertainties (induced by modeling errors), the dependent random matrices  $[\mathbf{M}^{\text{par}}]$  and  $[\mathbf{K}^{\text{par}}]$  are replaced by the dependent random

matrices  $[\mathbf{M}^{\text{tot}}]$ ,  $[\mathbf{K}^{\text{tot}}]$ , defined on a probability space ( $\Theta = \Theta^{\text{par}} \times \Theta^{\text{mod}}$ ,  $\mathcal{T} = \mathcal{T}^{\text{par}} \otimes \mathcal{T}^{\text{mod}}$ ,  $\mathcal{P} = \mathcal{P}^{\text{par}} \otimes \mathcal{P}^{\text{mod}}$ ), such that for all  $\theta = (\theta^{\text{par}}, \theta^{\text{mod}})$  in  $\Theta = \Theta^{\text{par}} \times \Theta^{\text{mod}}$ ,

$$\begin{aligned} [\mathbf{M}^{\text{tot}}(\theta)] &= [\mathbf{L}_M(\theta^{\text{par}})]^T [\mathbf{G}_M(\theta^{\text{mod}})] [\mathbf{L}_M(\theta^{\text{par}})], \\ [\mathbf{K}^{\text{tot}}(\theta)] &= [\mathbf{L}_K(\theta^{\text{par}})]^T [\mathbf{G}_K(\theta^{\text{mod}})] [\mathbf{L}_K(\theta^{\text{par}})], \end{aligned} \quad (5)$$

in which the probability distributions of the random matrices  $[\mathbf{G}_M]$  and  $[\mathbf{G}_K]$ , defined on  $(\Theta^{\text{mod}}, \mathcal{T}^{\text{mod}}, \mathcal{P}^{\text{mod}})$ , are explicitly given in [9] in the context of the nonparametric probabilistic approach of uncertainties. The probability distributions of  $[\mathbf{G}_M]$  and  $[\mathbf{G}_K]$  depend on the dispersion parameters  $\delta_M$  and  $\delta_K$  respectively. Let  $\alpha_{\text{mod}}$  be the vector of the dispersion parameters such that  $\alpha_{\text{mod}} = (\delta_M, \delta_K)$ .

The random matrices  $[\mathbf{M}^{\text{tot}}]$  and  $[\mathbf{K}^{\text{tot}}]$  are not diagonal. In order to calculate the random eigenfrequencies and the random mode shapes of the SCM with both the model-parameter uncertainties and the model uncertainties, the following small-dimension random generalized eigenvalue problem is introduced. Let  $0 < \Lambda_1 \leq \dots \leq \Lambda_n$  be the first  $n$  random eigenvalues associated with the random eigenvectors  $\phi_1^{\text{tot}}, \dots, \phi_n^{\text{tot}}$  which are the solutions of the following random reduced generalized eigenvalue problem

$$[\mathbf{K}^{\text{tot}}] \phi^{\text{tot}} = \Lambda [\mathbf{M}^{\text{tot}}] \phi^{\text{tot}}. \quad (6)$$

Let  $[\Phi^{\text{tot}}] = [\phi_1^{\text{tot}}, \dots, \phi_n^{\text{tot}}]$ . These random modes are normalized with respect to the random mass matrix  $[\mathbf{M}^{\text{tot}}]$ ,

$$[\mathbf{M}] = [\Phi^{\text{tot}}]^T [\mathbf{M}^{\text{tot}}] [\Phi^{\text{tot}}] = [I_n], \quad (7)$$

and we have

$$[\mathbf{K}] = [\Phi^{\text{tot}}]^T [\mathbf{K}^{\text{tot}}] [\Phi^{\text{tot}}] = \text{diag}(\Lambda_1, \dots, \Lambda_n). \quad (8)$$

Then the first  $n$  random eigenvalues of the SCM, with both the model-parameter uncertainties and the model uncertainties, are  $0 < \Lambda_1 \leq \dots \leq \Lambda_n$  and the associated random vectors are  $\phi_1, \dots, \phi_n$  such that the  $m \times n$  random matrix  $[\Phi] = [\phi_1, \dots, \phi_n]$  is written as

$$[\Phi] = [\Phi^{\text{par}}] [\Phi^{\text{tot}}]. \quad (9)$$

Finally the SCM is parameterized by the vector-valued hyperparameter  $\alpha = (\alpha_{\text{par}}, \alpha_{\text{mod}})$  which has to be identified using experimental modal data. The admissible space for vector  $\alpha$  is denoted by  $\mathcal{C}$ .

### 3 Identification of the SCM using experimental modal data

The objective of this section is to identify the parameter  $\alpha$  of the SCM using experimental modal data (eigenfrequencies and mass-normalized mode shapes) and realizations of the modal data calculated using the SCM.

#### 3.1 Experimental modal data as observations

It is assumed that  $n_{\text{exp}}$  experimental configurations of the structure have been tested. For each configuration  $j$ ,  $n_j$  modes have been experimentally identified using an experimental modal analysis method. For two given configurations, the number of modes, the number and locations of the sensors can be different. For each configuration  $j$ ,  $n_j$  experimental eigenfrequencies  $\omega_1^{\text{exp},j}, \dots, \omega_{n_j}^{\text{exp},j}$  associated with  $n_j$  mass-normalized experimental mode shapes  $\hat{\varphi}_1^{\text{exp},j}, \dots, \hat{\varphi}_{n_j}^{\text{exp},j}$  have been identified for  $m_j$  degrees of freedom (DOFs). Let  $[\hat{\Phi}^{\text{exp},j}] = [\hat{\varphi}_1^{\text{exp},j} \dots \hat{\varphi}_{n_j}^{\text{exp},j}]$  be the  $m_j \times n_j$  matrix of the  $n_j$  experimental mode shapes of the configuration  $j$ . It is assumed that  $n_j < n < m_j < m$  for all  $j$  in  $\{1, \dots, n_{\text{exp}}\}$ . The experimental reduced mass matrix and the experimental reduced stiffness matrix are then written as

$$[\hat{M}^{\text{exp},j}] = [I_{n_j}] \quad , \quad [\hat{K}^{\text{exp},j}] = \text{diag}(\lambda_1^{\text{exp},j}, \dots, \lambda_{n_j}^{\text{exp},j}), \quad (10)$$

in which  $\lambda_i^{\text{exp},j} = (\omega_i^{\text{exp},j})^2$ .

### 3.2 Transformation of the modal data

For all  $j$  in  $\{1, \dots, n_{exp}\}$ , let  $[P^j]$  be the  $m_j \times m$  matrix that performs the projection from the  $m$  DOFs of the SCM to the  $m_j$  DOFs of the experimental configuration  $j$ . Then the projected random modal basis  $[\tilde{\Phi}^j]$  of the SCM is defined by

$$[\tilde{\Phi}^j] = [P^j] [\Phi]. \quad (11)$$

The experimental modes  $[\hat{\Phi}^{exp,j}]$  cannot directly be compared to the computational modes  $[\tilde{\Phi}^j]$  because, in general, there is not a one-to-one correspondence between the experimental modes and the computational modes. Indeed, some modes may be missing in the experimental modal basis or in the computational modal basis. Furthermore, due to the experimental variability (variability of the configurations) and the computational randomness (uncertainties), some mode crossing and/or mode veering [7, 8, 6] phenomena may occur.

Therefore, the projected computational reduced-order model (ROM),  $\{[\tilde{\Phi}^j], [\mathbf{K}], [\mathbf{M}]\}$  has to be transformed into the ROM,  $\{[\hat{\Phi}^j], [\hat{\mathbf{K}}^j], [\hat{\mathbf{M}}^j]\}$ , such that

$$[\hat{\Phi}^j] = [\tilde{\Phi}^j] [\mathbf{Q}^{opt,j}], \quad (12)$$

$$[\hat{\mathbf{K}}^j] = [\mathbf{Q}^{opt,j}]^T [\mathbf{K}] [\mathbf{Q}^{opt,j}], \quad (13)$$

$$[\hat{\mathbf{M}}^j] = [\mathbf{Q}^{opt,j}]^T [\mathbf{M}] [\mathbf{Q}^{opt,j}], \quad (14)$$

in which  $[\mathbf{Q}^{opt,j}]$  is a random  $n \times n_j$  real matrix for which each realization  $[Q] = [\mathbf{Q}^{opt,j}(\theta)]$ , for  $\theta$  in  $\Theta$ , must belong to the Stiefel manifold,  $\mathcal{OSt}(n, n_j)$ , defined by

$$\mathcal{OSt}(n, n_j) = \{[Q] \in \mathbb{R}^{n \times n_j}, [Q]^T [Q] = [I_{n_j}]\}. \quad (15)$$

For all  $\theta$  in  $\Theta$ , orthogonal matrix  $[\mathbf{Q}^{opt,j}(\theta)]$  is calculated by minimizing the distance between the computational modal basis  $[\tilde{\Phi}^j(\theta)]$  and the experimental modal basis  $[\hat{\Phi}^{exp,j}]$  (see [1]),

$$[\mathbf{Q}^{opt,j}(\theta)] = \arg \min_{[Q] \in \mathcal{OSt}(n, n_j)} \| [\tilde{\Phi}^j(\theta)] [Q] - [\hat{\Phi}^{exp,j}] \|_F. \quad (16)$$

The minimization problem (11) is referred as a Procruste problem [4, 5] for which a solution can be calculated iteratively (see [4]). It should be noted that, in [2], the authors have proposed an alternative solution to take into account mode crossings and mode veerings in the identification of the hyperparameters of a SCM.

### 3.3 Identification of hyperparameter $\alpha$

Hyperparameter  $\alpha$  of the SCM is identified using the maximum likelihood method and experimental modal data. Then the optimal values  $\alpha^{opt}$  is solution of the following optimization problem

$$\alpha^{opt} = \arg \max_{\alpha \in \mathcal{C}} \sum_{j=1}^{n_{exp}} \log(p_{\mathbf{w}^j}(\mathbf{W}^{exp,j}; \alpha)), \quad (17)$$

where  $p_{\mathbf{w}^j}(\mathbf{w}; \alpha)$  is the probability density function (pdf) of the random observation vector  $\mathbf{W}^j$  which is constructed using the random transformed ROM  $\{[\hat{\Phi}^j], [\hat{\mathbf{K}}^j], [\hat{\mathbf{M}}^j]\}$  (see [3]).



Figure 1: Industrial mechanical system.

## 4 Application

### 4.1 Industrial mechanical system and experimental modal data

We are interested in the dynamical behavior of a one-stage booster pump used by Electricité de France (EDF) company in its thermal units (see Fig. 1). This pump is made up of a diffuser and a volute, with axial suction and vertical delivery, and is mounted on a metallic frame. It has been designed forty years ago by Sulzer Pumps. An experimental modal analysis has been carried out on two specimens of this pump located at two different thermal units. Therefore, there are  $n_{exp} = 2$  experimental configurations (denoted as Pump 1 and Pump 2) which are measured. There are slight differences between Pump 1 and Pump 2 concerning the joints between the pumps and their metallic frame. The experimental meshes for the two pumps are not the same. An experimental modal analysis has been carried for each pump. For Pump 1,  $n_t = 6$  modes have been identified. The three six mode shapes for Pump 1 and Pump 2 are plotted in Figs. 2 and 3 respectively.



Figure 2: Pump 1: First three experimental mode shapes (Thick black line).



Figure 3: Pump 2: First three experimental mode shapes (Thick black line).

## 4.2 Construction of the stochastic computational model

### 4.2.1 Construction of the nominal computational model

The finite element mesh of the NCM is plotted in Fig. 4. The nominal finite element model is made up of 3D

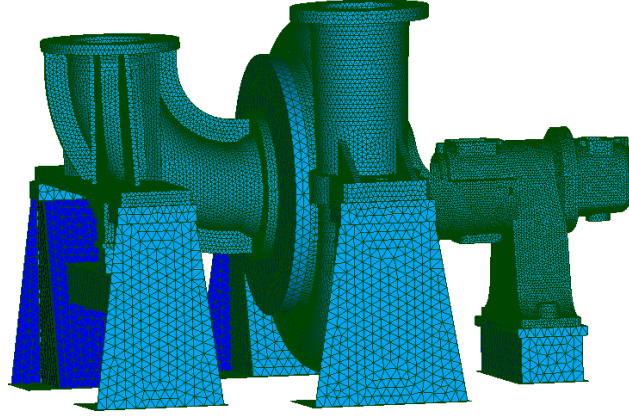


Figure 4: Finite element mesh of the NCM.

solid elements, Kirchhoff plate elements and spring elements. The assembled model has 488,220 DOFs. The uncertain model parameters of the NCM are the Young modulus  $y_s$  of the steel, the Young's modulus  $y_c$  of the cast iron, the thicknesses  $t_1$ ,  $t_2$  and  $t_3$  of the plates 1, 2 and 3 of the metallic frame and the stiffnesses  $k_1$ ,  $k_2$ ,  $k_3$  and  $k_4$  of the discrete springs normal to the metallic frame. Let  $\mathbf{h} = (y_s, y_c, t_1, t_2, t_3, k_1, k_2, k_3, k_4)$  be the vector of the uncertain model parameters. For the updated NCM of Pump 1 and Pump 2, the updated values of  $\mathbf{h}$  are denoted by  $\mathbf{h}^1$  and  $\mathbf{h}^2$ . For each updated NCM,  $n = 20$  modes are calculated. For  $\mathbf{h} = \mathbf{h}^1$ , the first 3 projected mode shapes are plotted in Fig. 5. For  $\mathbf{h} = \mathbf{h}^2$ , the first 3 projected mode shapes are



Figure 5: First 3 projected mode shapes for Pump 1.

plotted in Fig. 6.



Figure 6: First 3 projected mode shapes for Pump 2.

### 4.2.2 Construction of the stochastic computational model

The vector  $\mathbf{h}$  is modeled by a random vector  $\mathbf{H} = (Y_s, Y_c, T_1, T_2, T_3, K_1, K_2, K_3, K_4)$ . The Maximum Entropy principle has been used for constructing the probability distribution of  $\mathbf{H}$ . Taking into account the

available information, it can be deduced that (1) all the components of  $\mathbf{H}$  are independent random variables; (2) positive-valued Young moduli  $Y_s$  and  $Y_c$  are Gamma random variables parameterized by their mean values  $m_{Y_s}$  and  $m_{Y_c}$ , and by their coefficients of variation (standard deviation divided by the mean value)  $\delta_{Y_s}$  and  $\delta_{Y_c}$ ; (3) positive-valued random thicknesses  $T_1$ ,  $T_2$  and  $T_3$  are uniform positive-valued random variables parameterized by their mean values  $m_{T_1}$ ,  $m_{T_2}$  and  $m_{T_3}$ , and by their coefficients of variation  $\delta_{T_1}$ ,  $\delta_{T_2}$  and  $\delta_{T_3}$ ; (4) positive-valued stiffnesses  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  are Gamma random variables parameterized by their mean values  $m_{K_1}$ ,  $m_{K_2}$ ,  $m_{K_3}$  and  $m_{K_4}$ , and by their coefficients of variation  $\delta_{K_1}$ ,  $\delta_{K_2}$ ,  $\delta_{K_3}$  and  $\delta_{K_4}$ . Then  $\alpha_{\text{par}} = (m_{Y_s}, \delta_{Y_s}, m_{Y_c}, \delta_{Y_c}, m_{T_1}, \delta_{T_1}, m_{T_2}, \delta_{T_2}, m_{T_3}, \delta_{T_3}, m_{K_1}, \delta_{K_1}, m_{K_2}, \delta_{K_2}, m_{K_3}, \delta_{K_3}, m_{K_4}, \delta_{K_4})$  has 18 components to be identified and  $\alpha = (\alpha_{\text{par}}, \alpha_{\text{mod}})$  has 20 components to be identified.

#### 4.2.3 Identification of the optimal hyperparameter $\alpha^{\text{opt}}$ .

The vector  $\alpha^{\text{opt}}$  is given by the optimization problem defined by Eq. (17) which is solved using a genetic algorithm. For each value of  $\alpha$ , the probability density functions  $p_{\mathbf{w}_1}(\mathbf{w}; \alpha)$  and  $p_{\mathbf{w}_2}(\mathbf{w}; \alpha)$  are estimated using 800 realizations of the observation vectors  $\mathbf{W}^1$  and  $\mathbf{W}^2$  calculated with the SCM. The components of  $\alpha_{\text{par}}^{\text{opt}}$  are given in Table 1. The two components of  $\alpha_{\text{mod}}^{\text{opt}}$  are  $\delta_M^{\text{opt}} = 0.52$  and  $\delta_K^{\text{opt}} = 0.43$ . These optimal dispersions of the probability distributions for model uncertainties are relatively high due to the experimental variability and due to modeling errors introduced in the NCM. These dispersions could be partly decreased by constructing a more accurate NCM.

	Mean value	Coefficient of variation
Young's modulus steel $Y_s$	$m_{Y_s} = 1.33 \times 10^{11}$ Pa	$\delta_{Y_s} = 0.2$
Young's modulus cast iron $Y_c$	$m_{Y_c} = 7.57 \times 10^{10}$ Pa	$\delta_{Y_c} = 0.17$
Thickness $T_1$	$m_{T_1} = 0.013$ m	$\delta_{T_1} = 0.12$
Thickness $T_2$	$m_{T_2} = 0.011$ m	$\delta_{T_2} = 0.57$
Thickness $T_3$	$m_{T_3} = 0.01$ m	$\delta_{T_3} = 0.09$
Stiffness $K_1$	$m_{K_1} = 8.01 \times 10^8$ N/m	$\delta_{K_1} = 0.19$
Stiffness $K_2$	$m_{K_2} = 5.74 \times 10^9$ N/m	$\delta_{K_2} = 0.13$
Stiffness $K_3$	$m_{K_3} = 6.30 \times 10^7$ N/m	$\delta_{K_3} = 0.34$
Stiffness $K_4$	$m_{K_4} = 2.68 \times 10^8$ N/m	$\delta_{K_4} = 0.42$

Table 1: Components of  $\alpha_{\text{par}}^{\text{opt}}$ .

### 4.3 Validation of the results

For  $\alpha = \alpha^{\text{opt}}$ , the marginal pdf of the first three eigenvalues of the matrices  $[\hat{\mathbf{K}}^1]$  and  $[\hat{\mathbf{K}}^2]$  are shown in Figs. 7 and 8 respectively. It can be seen in these figures that the first 3 experimental eigenvalues for Pump 1 and Pump 2 are predicted by the SCM with a high probability level (very high for Pump 1).

For Pump 1, the mean value of the MAC matrix between the random modal basis  $[\tilde{\Phi}^1]$  (before transformation) of the SCM and the experimental modal basis  $[\hat{\Phi}^{\text{exp},1}]$  is plotted in Fig. 9, while for Pump 2, the mean value of MAC matrix is plotted in Fig. 10. Figures 9 and 10 show that the randomness of the SCM introduces random mode crossings and random mode veerings which modify the correspondence. For Pump 1, the mean value of the MAC matrix between the random modal basis  $[\hat{\Phi}^1]$  (after transformation) of the SCM and the experimental modal basis  $[\hat{\Phi}^{\text{exp},1}]$  is plotted in Fig. 11 while for Pump 2, the mean value of MAC matrix is plotted in Fig. 12. In Figs. 11 and 12, it can be seen that the random transformation of the random mode shapes of the SCM allows to achieve a good correspondence between the random computational modes of the SCM and the experimental modes.



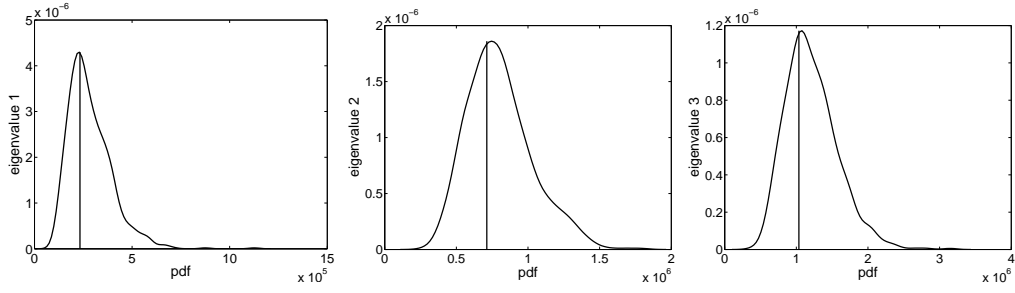


Figure 7: Probability density function for the first three eigenvalues of  $\hat{\mathbf{K}}^1$ . Vertical lines: corresponding experimental values (eigenvalues of  $\hat{\mathbf{K}}^{\text{exp},1}$ ).

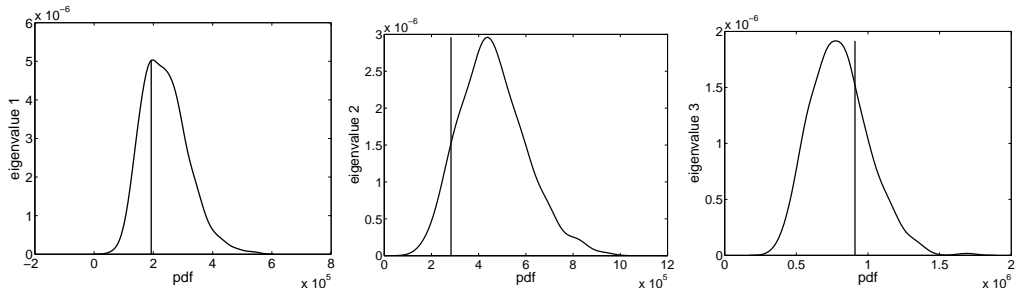


Figure 8: Probability density function for the first three eigenvalues of  $\hat{\mathbf{K}}^2$ . Vertical lines: corresponding experimental values (eigenvalues of  $\hat{\mathbf{K}}^{\text{exp},2}$ ).

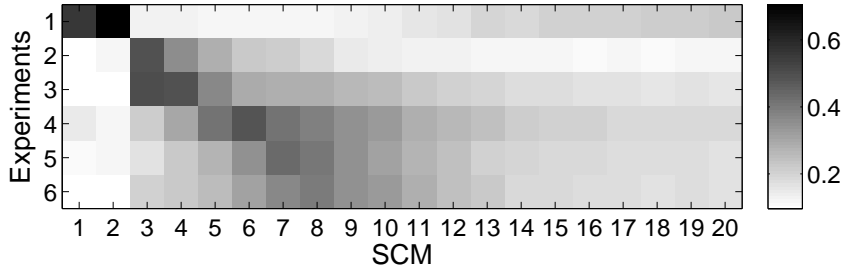


Figure 9: For Pump 1, mean value of the MAC matrix between the random mode shapes of the SCM and the experimental mode shapes before transformation.

## 5 Conclusion

A methodology for the construction and the identification of a stochastic computational model using experimental eigenfrequencies and mode shapes has been presented. The model-parameter uncertainties and the modeling errors are taken into account in the framework of a generalized probabilistic approach of uncertainties. A transformation of the computational modal quantities is introduced in order to construct a correspondence between the experimental modal data and the computational modal quantities. This method allows us to take into account mode crossings and mode veerings that may occur. The methodology has been applied to the construction a stochastic computational model representing a family of booster pumps of thermal units.

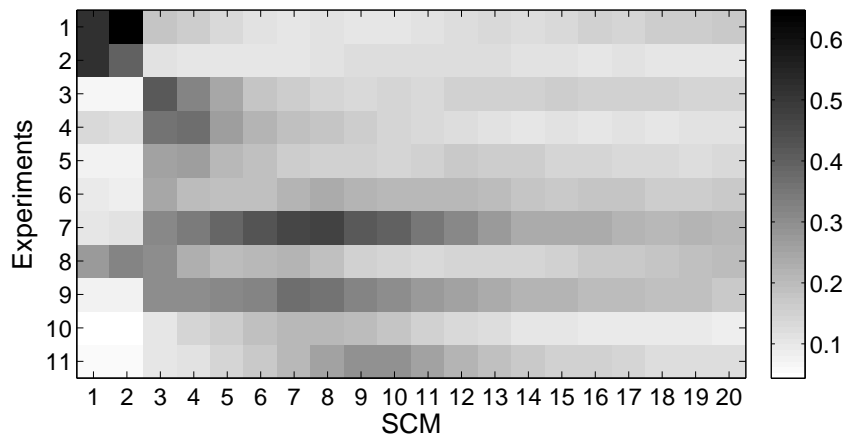


Figure 10: For Pump 2, mean value of the MAC matrix between the random mode shapes of the SCM and the experimental mode shapes before transformation.

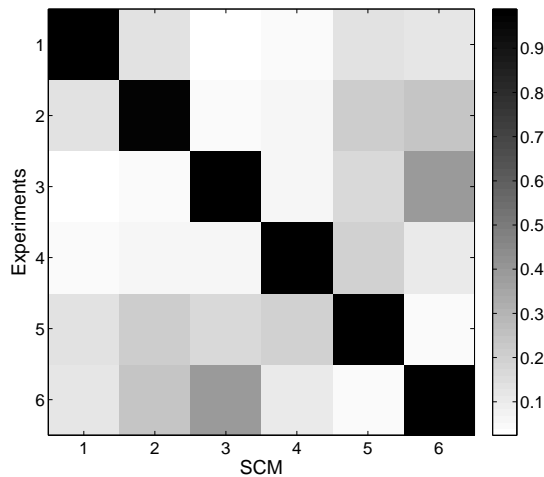


Figure 11: For Pump 1, mean value of the MAC matrix between the random mode shapes of the SCM and the experimental mode shapes after transformation.

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## References

- [1] D. Amsallem, C. Farhat, *An online method for interpolating linear parametric reduced-order models*, SIAM Journal on Scientific Computing. Vol. 33, No. 5 (2011), pp. 2169-2198.
- [2] J. Avalos, E. D. Swenson, M. P. Mignolet, N. J. Lindsley, *Stochastic modeling of structural uncertainty/variability from ground vibration modal test data*, Journal of Aircraft. Vol. 49, No. 3 (2012), pp. 870-884.

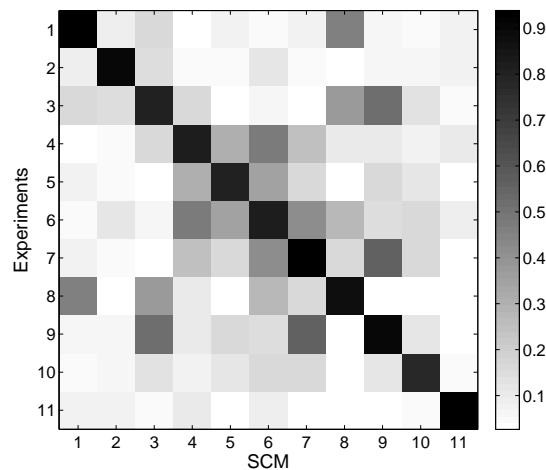


Figure 12: For Pump 2, mean value of the MAC matrix between the random mode shapes of the SCM and the experimental mode shapes after transformation.

- [3] A. Batou, C. Soize, S. Audebert, *Model identification in computational stochastic dynamics using experimental modal data*, Mechanical Systems and Signal Processing, *accepted for publication* (2014).
- [4] J.M.F.T. Berge, D. L. Knol, *Orthogonal rotations to maximal agreement for two or more matrices of different column orders*, Psychometrika. Vol. 49, No. 1 (1984), pp. 49-55.
- [5] A. Bojanczyk, A. Lutoborski, *The procrustes problem for orthogonal Stiefel matrices*, SIAM Journal on Scientific Computing. Vol. 21, No. 4 (1999), pp. 1291-1304.
- [6] H.J.-P. Morand, R. Ohayon, *Fluid-Structure Interaction*, Wiley, New York, 1995.
- [7] N. C. Perkins, C. D. Mote Jr, *Comments on a curve veering in eigenvalue problems*, Journal of Sound and Vibration. Vol. 106, No. 3 (1986), pp. 451-463.
- [8] C. Pierre, *Mode localization and eigenvalue loci veering phenomena in disordered structures*, Journal of Sound and Vibration. Vol. 126, No. 3 (1988), pp. 485-502.
- [9] C. Soize, *A nonparametric model of random uncertainties for reduced matrix models in structural dynamics*, Probabilistic Engineering Mechanics. Vol. 15, No. 3 (2000), pp. 277-294.
- [10] C. Soize, *Generalized probabilistic approach of uncertainties in computational dynamics using random matrices and polynomial chaos decomposition*, International Journal for Numerical Methods in Engineering. Vol. 81, No. 8 (2010), pp. 939-970.
- [11] C. Soize, *Stochastic modeling of uncertainties in computational structural dynamics - Recent theoretical advances*, Journal of Sound and Vibration. Vol. 332, No. 10 (2013), pp. 2379-2395.