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REDUCED-ORDER MODEL FOR THE DYNAMICAL ANALYSIS OF COMPLEX STRUCTURES WITH A HIGH MODAL DENSITY

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Modal analysis is a widely used model-reduction method in structural dynamics for the low-frequency (LF) band. This LF band is classically characterized by the presence of relatively well separated resonances associated with global elastic modes (non local elastic modes). In this case, a basis constituted of such elastic modes is efficient for the construction of a small-size and predictive reduced-order model. In this work, we are interested in predicting the dynamical response of complex structures presenting several structural scales (for instance, the presence of flexible panels connected to a stiff master structure). For such structures, a high modal density can be observed in the low- and the medium-frequency bands. This high modal density for the low-frequency band is not the usual case considered by the modal analysis which would require a large number of elastic modes to represent the response with a good accuracy. In this context, a new methodology is introduced for constructing a small-size reduced-order basis adapted to span the global displacements space. This construction is performed from the computational dynamics model by solving a non usual generalized eigenvalue problem for which the rank of the mass matrix is modified allowing the local elastic modes (small-wavelength) to be removed (filtered). The so-built global basis allows the frequency response in the stiff part to be predicted with a good accuracy. The efficiency and utility of this methodology is demonstrated and a numerical application is presented for a heterogeneous thin plate constituted of two structural scales.

1. Introduction

In this research, we are interested in predicting the dynamical response of complex structures which are characterized by the presence of a stiff master structure supporting numerous flexible components. Even though the highly detailed computational models arising from such structures are very large, the calculation of thousands of elastic modes from finite element models having millions of degrees of freedom is nowadays carried out effectively. The medium- and high-frequency ranges are classically characterized by a higher modal density (see [1]), related to small-wavelength vibrations. Localized heterogeneities, such as flexible components connected to the master structure, can have their fundamental natural frequencies belonging to the low-frequency band. In such a case, modal density rapidly increases as soon as such local elastic modes appear. The number of local elastic modes coupled with the first few global elastic modes can be found to be high, thus resulting in high-dimension reduced-order models constructed by the modal analysis method (see [2, 1] concerning

this widely used tool) in low-frequency dynamics.
In this work, we are interested in constructing a small-dimension reduced-order model adapted to this particular case of high modal density in the low-frequency band, for predicting the global dynamical behavior of such complex structures, that is the response in their stiff master part. To achieve this objective, we consider the fact that, in contrast with global elastic modes, local elastic modes, i.e. elastic modes whose displacements are localized on a delimited region of the structure (typically displacements of the flexible components), have a more or less negligible impact on the response of the stiff master structure. That is to say, assuming the elastic modes were either exclusively global or exclusively local, the small-dimension basis constituted of the global ones would allow the global response to be represented in an approximate but still accurate way. The energy stored by the local elastic modes is responsible for an apparent damping (as explained in [3, 4, 5]) at the resonances associated with global displacements. Besides, local elastic modes with non-negligible kinetic energy can result in the presence of corresponding small resonances among the main global resonances in the response of the stiff part. On top of that, in fact, the elastic modes cannot in general be separately defined as global or local elastic modes, as their shape is a combination of global and local displacements.
There is few research concerning the filtering of local displacements for the construction of reduced-order models adapted to the dynamical analysis of complex structures. Concerning the vibration analysis of automotive vehicles in the low-frequency range, the common methods are based on the use of modal analysis with sub-structuring techniques (see [1, 6, 7]).
This paper is a continuation of the method proposed in [8] and presents an original methodology which allows us to separate the local displacements from the global displacements. The direct sum of the subspace of global displacements with the subspace of local displacements constitutes the admissible displacements space. These two subspaces are separately spanned by the eigenvectors of two distinct eigenvalue problems for which the kinetic energy is modified while the elastic energy is kept exact. The global eigenvalue problem is obtained reducing the kinematics of the kinetic energy, without altering the elastic energy. The local eigenvalue problem is built upon the residual kinetic energy.
The reference matrix model and its reduction on the elastic modes are first introduced. Then, the methodology for the construction of both the global and local displacements spaces, from which the original reduced-order model is constructed, is exposed. Finally, a numerical validation involving a heterogeneous thin plate is presented.

2. Theory

2.1 Reference Dynamical Model

We are interested in predicting the dynamical response in the frequency band of analysis $\mathcal{B} = [\omega_{\min}, \omega_{\max}]$ of a tridimensional linear damped structure (without rigid body displacements) occupying a bounded domain Ω and subjected to external loads. The reference computational model is constructed using the finite element method (see [9, 10]) and the complex vector $\mathbb{U}(\omega)$ of the m degrees of freedom is solution of the following matrix equation, for all ω in $\mathcal{B}$,

$$(-\omega^2\,[\mathbb{M}] + i\omega\,[\mathbb{D}] + [\mathbb{K}])\mathbb{U}(\omega) = \mathbb{F}(\omega)\,, \tag{1}$$

where $[\mathbb{M}]$, $[\mathbb{D}]$ and $[\mathbb{K}]$ are the symmetric positive-definite $(m \times m)$ real mass, damping and stiffness matrices, and where complex vector $\mathbb{F}(\omega)$ corresponds to the external forces.

2.2 Modal Analysis Method

The eigenfrequencies ω_α and the associated elastic modes $\boldsymbol{\varphi}_\alpha$ in $\mathbb{R}^m$ are obtained solving the generalized eigenvalue problem corresponding to the conservative dynamical model,

$$[\mathbb{K}]\,\boldsymbol{\varphi}_\alpha = \lambda_\alpha\,[\mathbb{M}]\,\boldsymbol{\varphi}_\alpha\,, \tag{2}$$

with $\alpha = 1, \ldots, m$ and where the real eigenvalues $\lambda_\alpha = \omega_\alpha^2$ are such that $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_m$. The elastic modes form a vector basis of $\mathbb{R}^m$ and the modal analysis method consists in approximating vector $\mathbb{U}(\omega)$ in the subspace of $\mathbb{C}^m$ spanned by the first n elastic modes, with $n \ll m$, such that

$$\forall \omega \in \mathcal{B}, \quad \mathbb{U}(\omega) \simeq \mathbb{U}_n(\omega) = \sum_{\alpha=1}^{n} \widetilde{q}_\alpha(\omega) \boldsymbol{\varphi}_\alpha = [\Phi] \, \widetilde{\mathbf{q}}(\omega) \,, \tag{3}$$

where $[\Phi] = [\boldsymbol{\varphi}_1 \ldots \boldsymbol{\varphi}_n]$ is the $(m \times n)$ real matrix of the first n elastic modes, that is the modes associated with the first n (smallest) eigenvalues. The vector $\widetilde{\mathbf{q}}(\omega)$ in $\mathbb{C}^n$ is the complex vector of the generalized coordinates. For all ω in $\mathcal{B}$, the classical reduced-order model constructed with the modal analysis method allows the generalized coordinates to be solved,

$$(-\omega^2 [\widetilde{M}] + i\omega [\widetilde{D}] + [\widetilde{K}]) \, \widetilde{\mathbf{q}}(\omega) = \widetilde{\mathcal{F}}(\omega) \,, \tag{4}$$

where $[\widetilde{M}] = [\Phi]^T \, [\mathbb{M}] \, [\Phi]$, $[\widetilde{D}] = [\Phi]^T \, [\mathbb{D}] \, [\Phi]$ and $[\widetilde{K}] = [\Phi]^T \, [\mathbb{K}] \, [\Phi]$ are the symmetric positive-definite $(n \times n)$ real generalized mass, damping and stiffness matrices and $\widetilde{\mathcal{F}}(\omega) = [\Phi]^T \, \mathbb{F}(\omega)$ is the generalized force. Then, $\mathbb{U}_n(\omega)$ is derived using Eq. (3). The modal contributions of elastic modes corresponding to higher eigenfrequencies are neglected. To ensure its accuracy, a convergence analysis of $\mathbb{U}_n(\omega)$ with respect to n must be carried out, for all ω in $\mathcal{B}$.

2.3 Reduced Displacements Space

Let $\mathbb{H}_r$ be a subspace of $\mathbb{R}^m$ of dimension d_r and let us note $|| \cdot ||_{\mathbb{M}}$ the norm associated with the Euclidean inner product $\langle \mathbf{v}, \mathbf{w} \rangle_{\mathbb{M}} = \mathbf{w}^T \, [\mathbb{M}] \, \mathbf{v}$, for $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^m$. For all $\mathbb{U}$ in $\mathbb{R}^m$, we then have to find $\mathbb{U}^r$ in $\mathbb{H}_r$ such that

$$||\mathbb{U} - \mathbb{U}^r||_{\mathbb{M}} = \inf_{\widetilde{\mathbb{U}}^r \in \mathbb{H}_r} ||\mathbb{U} - \widetilde{\mathbb{U}}^r||_{\mathbb{M}} \,, \tag{5}$$

where the reduced displacements vector $\mathbb{U}^r$ is the orthogonal projection of $\mathbb{U}$ on $\mathbb{H}_r$ given by $\mathbb{U}^r = [H^r] \, \mathbb{U}$, where $[H^r]$ is a projection matrix. The complementary displacements vector $\mathbb{U}^c = \mathbb{U} - \mathbb{U}^r$ is such that $[H^r] \, \mathbb{U}^c = 0$. Thus, vector $\mathbb{U}^r$ is constructed such that the residual kinetic energy is minimized. The construction of the projection matrix $[H^r]$ is relative to the corresponding reduced displacements space $\mathbb{H}_r$. In order to be able to spatially control the kinematics reduction, we perform a domain partitioning of the structure using the Fast Marching Method (FMM - see [11, 12] and see [13] for homogeneous domain partitioning of the complex mesh of an automotive vehicle). Each subdomain kinematics is reduced on a few dynamic degrees of freedom so that small-wavelength vibrations can no longer be represented in a given subdomain, provided the displacement kinematics of its kinetic energy is sufficiently reduced.

2.4 Reduced-Order Model

2.4.1 Global Displacements Space

The following generalized eigenvalue problem corresponding to the conservative global dynamical model is introduced,

$$[\mathbb{K}] \, \boldsymbol{\varphi}_\alpha^g = \lambda_\alpha^g \, [\mathbb{M}^r] \, \boldsymbol{\varphi}_\alpha^g \,, \tag{6}$$

where the mass matrix $[\mathbb{M}^r]$ of rank d_r, defined as $[\mathbb{M}^r] = [H^r]^T \, [\mathbb{M}] \, [H^r]$, is positive semidefinite such that the d_r real eigenvectors $\{\boldsymbol{\varphi}_\alpha^g \,, \alpha = 1, \ldots, d_r\}$ associated with the d_r finite positive eigenvalues $0 < \lambda_1^g \leq \lambda_2^g \leq \ldots \leq \lambda_{d_r}^g$ are used as a vector basis for spanning the global displacements space, $\mathbb{V}_g$. We denote as $[\Phi^g]$ the $(m \times n_g)$ real matrix of the first n_g global eigenvectors such that $[\Phi^g] = [\boldsymbol{\varphi}_1^g \ldots \boldsymbol{\varphi}_{n_g}^g]$ with $n_g \leq d_r$. The global elastic modes $\boldsymbol{\psi}^g$ are then defined as

$$\boldsymbol{\psi}^g = [\Phi^g]\widetilde{\boldsymbol{\psi}}^g \,, \tag{7}$$

where the generalized global elastic modes $\widetilde{\boldsymbol{\psi}}^g$ are solutions of the generalized eigenvalue problem

$$[K^{gg}]\widetilde{\boldsymbol{\psi}}^g = \lambda^{m,g}[M^{gg}]\widetilde{\boldsymbol{\psi}}^g \,, \tag{8}$$

in which $[K^{gg}] = [\Phi^g]^T[\mathbb{K}][\Phi^g]$ and $[M^{gg}] = [\Phi^g]^T[\mathbb{M}][\Phi^g]$. The global eigenfrequencies ω^g are such that $\omega^g = \sqrt{\lambda^{m,g}}$ and the global elastic modes $\boldsymbol{\psi}^g$ are obtained using Eq. (7).

2.4.2 Local Displacements Space

We introduce the generalized eigenvalue problem, corresponding to the conservative local dynamical model

$$[\mathbb{K}]\,\boldsymbol{\varphi}_\alpha^\ell = \lambda_\alpha^\ell\,[\mathbb{M}^c]\,\boldsymbol{\varphi}_\alpha^\ell \,, \tag{9}$$

where the mass matrix $[\mathbb{M}^c] = [\mathbb{M}] - [\mathbb{M}^r]$ of rank $d_c = m - d_r$ is positive semidefinite such that the d_c real eigenvectors $\left\{\boldsymbol{\varphi}_\alpha^\ell\,, \alpha = 1, \ldots, d_c\right\}$ associated with the d_c finite positive eigenvalues $0 < \lambda_1^\ell \leq \lambda_2^\ell \leq \ldots \leq \lambda_{d_c}^\ell$ are used as a vector basis for spanning the local displacements space, $\mathbb{V}_\ell$. We denote as $[\Phi^\ell]$ the $(m \times n_\ell)$ real matrix of the first n_ℓ local eigenvectors such that $[\Phi^\ell] = [\boldsymbol{\varphi}_1^\ell \ldots \boldsymbol{\varphi}_{n_\ell}^\ell]$ with $n_\ell \leq d_c$. The local elastic modes $\boldsymbol{\psi}^\ell$ are then defined as

$$\boldsymbol{\psi}^\ell = [\Phi^\ell]\widetilde{\boldsymbol{\psi}}^\ell \,, \tag{10}$$

where the generalized local elastic modes $\widetilde{\boldsymbol{\psi}}^\ell$ are solutions of the generalized eigenvalue problem

$$[K^{\ell\ell}]\widetilde{\boldsymbol{\psi}}^\ell = \lambda^{m,\ell}[M^{\ell\ell}]\widetilde{\boldsymbol{\psi}}^\ell \,, \tag{11}$$

in which $[K^{\ell\ell}] = [\Phi^\ell]^T[\mathbb{K}][\Phi^\ell]$ and $[M^{\ell\ell}] = [\Phi^\ell]^T[\mathbb{M}][\Phi^\ell]$. The local eigenfrequencies ω^ℓ are such that $\omega^\ell = \sqrt{\lambda^{m,\ell}}$ and the local elastic modes $\boldsymbol{\psi}^\ell$ are obtained using Eq. (10).

2.4.3 Reduced-Order Model

The union of the global eigenvectors $\{\boldsymbol{\varphi}_\alpha^g\,, \alpha = 1, \ldots, d_r\}$ spanning space $\mathbb{V}_g$ with the local eigenvectors $\left\{\boldsymbol{\varphi}_\alpha^\ell\,, \alpha = 1, \ldots, d_c\right\}$ spanning space $\mathbb{V}_\ell$ form a vector basis of $\mathbb{R}^m$ such that vector $\mathbb{U}\,(\omega)$ in $\mathbb{C}^m$, which is a solution of Eq. (1), can be expanded as

$$\mathbb{U}\,(\omega) = \sum_{\alpha=1}^{d_r} q_\alpha^g\,(\omega)\,\boldsymbol{\varphi}_\alpha^g + \sum_{\beta=1}^{d_c} q_\beta^\ell\,(\omega)\,\boldsymbol{\varphi}_\beta^\ell \,, \tag{12}$$

where $\mathbf{q}^g\,(\omega)$ in $\mathbb{C}^{d_r}$ is the vector of the global generalized coordinates and $\mathbf{q}^\ell\,(\omega)$ in $\mathbb{C}^{d_c}$ is the vector of the local generalized coordinates.
We denote as $[\Phi^{g\ell}]$ the $(m \times n_t)$ real matrix constituted of the first n_g global eigenvectors and the first n_ℓ local eigenvectors such that $[\Phi^{g\ell}] = [\Phi^g\;\Phi^\ell]$ with $n_t = n_g + n_\ell$. The global-local elastic modes $\boldsymbol{\psi}$ are then defined as

$$\boldsymbol{\psi} = [\Phi^{g\ell}]\widetilde{\boldsymbol{\psi}} \,, \tag{13}$$

where the generalized global-local elastic modes $\widetilde{\boldsymbol{\psi}}$ are solutions of the generalized eigenvalue problem

$$[\widehat{K}]\widetilde{\boldsymbol{\psi}} = \lambda^{m,g\ell}[\widehat{M}]\widetilde{\boldsymbol{\psi}}\,, \tag{14}$$

in which $[\widehat{K}] = [\Phi^{g\ell}]^T[\mathbb{K}][\Phi^{g\ell}]$ and $[\widehat{M}] = [\Phi^{g\ell}]^T[\mathbb{M}][\Phi^{g\ell}]$. The global-local elastic modes are obtained using Eq. (13).
The approximation $\mathbb{U}_{n_g,n_\ell}(\omega)$ of $\mathbb{U}(\omega)$ at order (n_g, n_ℓ) with $n_g \leq d_r$ and $n_\ell \leq d_c$ is, for all ω in $\mathcal{B}$, such that

$$\mathbb{U}_{n_g,n_\ell}(\omega) = \sum_{\gamma=1}^{n_t} q_\gamma(\omega)\,\boldsymbol{\psi}_\gamma = [\Psi]\,\mathbf{q}(\omega)\ , \tag{15}$$

in which the matrix $[\Psi]$ is such that $[\Psi] = [\boldsymbol{\psi}_1 \ldots \boldsymbol{\psi}_{n_t}]$ and where the global-local generalized coordinates vector $\mathbf{q}(\omega)$ is constructed solving

$$(-\omega^2[M] + i\omega[D] + [K])\,\mathbf{q}(\omega) = \mathcal{F}(\omega)\ , \tag{16}$$

where $[M] = [\Psi]^T[\mathbb{M}]\,[\Psi]$, $[D] = [\Psi]^T\,[\mathbb{D}]\,[\Psi]$ and $[K] = [\Psi]^T\,[\mathbb{K}]\,[\Psi]$ are the symmetric positive-definite $(n_t \times n_t)$ real global-local generalized mass, damping and stiffness matrices, and where $\mathcal{F}(\omega) = [\Psi]^T\,\mathbb{F}(\omega)$ is the global-local generalized force. Then, $\mathbb{U}_{n_g,n_l}(\omega)$ is derived from Eq. (15).

3. Application

3.1 Finite Element Model

The dynamical system is a heterogenous $0.26 \times 0.2\,m^2$ plate constituted of a stiff master part supporting 12 flexible panels.

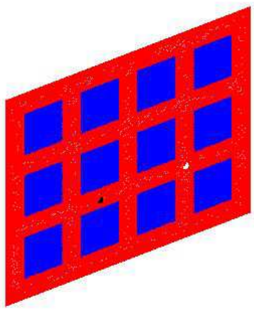

Figure 1: Dynamical System

Each panel is an isotropic and homogeneous square plate with side of $0.04\,m$ and with a constant thickness of $10^{-4}\,m$. The master structure is constituted of isotropic and homogeneous identical plates with a width of $0.02\,m$ and with a constant thickness of $0.005\,m$. The elastic properties of the structure are roughly uniform as only the Young modulus changes depending on the panel. The mass density is of $7,850\,kg/m^3$ and the Poisson ratio is of 0.29. The Young modulus of the master structure is of $210 \times 10^9\,Pa$ and the Young moduli of the 12 panels are of 131, 147, 154, 174, 147, 171, 49, 109, 174, 134, 168, and $61 \times 10^9\,Pa$, going first from the left to the right and then from the bottom to the top. The master structure is modeled with about $40,000$ Kirchhoff plate elements and the flexible panels are modeled with about $6,500$ Kirchhoff plate elements each with a total of about $120,000$ elements for the whole structure. In-plane displacements and rotation are constrained and as boundary conditions the four corner nodes are fixed. The structure has $60,391$ nodes and $181,161$ DOFs. The frequency band of analysis is $\mathcal{B} = 2\pi \times]0, 1400]\,rad/s$.

3.2 Reduced-Order Models

3.2.1 Classical Elastic Modes

The usual elastic modes are computed using Eq. (2). In the frequency band of analysis $\mathcal{B}$, there are 57 elastic modes. The flexible panels are responsible for the presence of numerous associated local elastic modes and the proportion of global elastic modes rapidly decreases with respect to the frequency. The first elastic mode is a global one coupled with local displacements. The second one is a local elastic mode.

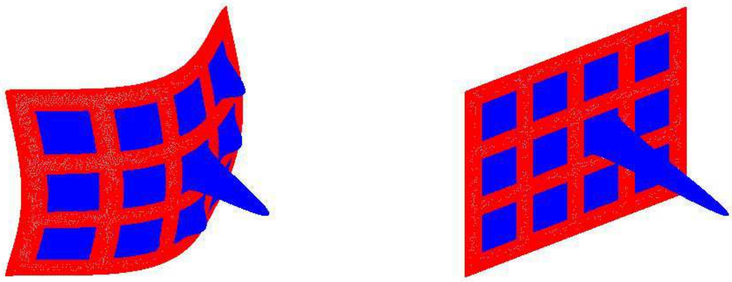

Figure 2: First Elastic Mode (left), Second Elastic Mode (right)

3.2.2 Global Elastic Modes

The domain of the structure is partitionned into 20 subdomains that do not coincide with the flexible panels. We first introduce $\mathbb{H}_r^1$ defined as the space $\mathbb{H}_r$ of vectors whose displacements components are constant in each subdomain and whose rotations components are set to zero. The projection matrix $[H^r]$, denoted as $[H_1^r]$, is constructed accordingly to space $\mathbb{H}_r^1$ of dimension 20 and allows the mass matrix $[\mathbb{M}_1^r]$ to be obtained. Then, the 20 global displacements eigenvectors in $\mathbb{V}_g$, denoted as $\mathbb{V}_g^1$, are computed using Eq. (6) and the associated global elastic modes are computed using Eq. (7). In the frequency band of analysis $\mathcal{B}$, there are 10 so-defined global elastic modes.

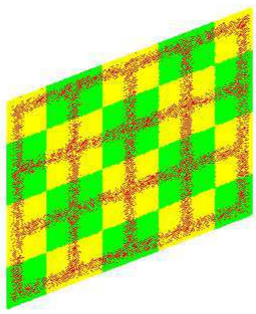

Figure 3: Subdomains

We now introduce $\mathbb{H}_r^2$ defined as the space $\mathbb{H}_r$ of vectors whose displacements components correspond to rigid body displacements in each subdomain and whose rotations components are set to zero. The projection matrix $[H^r]$, denoted as $[H_2^r]$, is constructed accordingly to space $\mathbb{H}_r^2$ of dimension 60 and allows the mass matrix $[\mathbb{M}_2^r]$ to be obtained. Then, the first 40 global displacements eigenvectors in $\mathbb{V}_g$, denoted as $\mathbb{V}_g^2$, are computed using Eq. (6) and the associated global elastic modes are computed using Eq. (7). In the frequency band of analysis $\mathcal{B}$, there are 26 so-defined global elastic modes.
The frequency distributions of such global elastic modes are plotted in Fig. (4) in comparison with the usual elastic modes distribution.

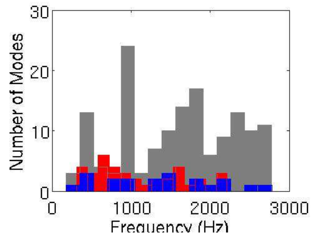

Figure 4: Modal density: usual elastic modes (grey), global elastic modes in $\mathbb{V}_g^1$ (blue or dark-grey in b&w), global elastic modes in $\mathbb{V}_g^2$ (red or mid-grey in b&w) .

3.2.3 Frequency Responses

For all ω in $\mathcal{B}$, the structure is subjected to an external point load of $1N$ (following the normal direction) located in the master structure at the black-marked node depicted in Fig. 1. The damping matrix is constructed using a Rayleigh model associated with a damping rate $\xi = 0.04$ for the frequencies $f_\alpha = 1Hz$ and $f_\beta = 2,300Hz$. The modulus in log scale of the normal displacement of the observation node located in the master structure at the white-marked point depicted in Fig. 1 is calculated using different projection bases: the usual elastic modes until convergence in $\mathcal{B}$ ($n = 400$), the first $n_g^1 = 20$ global elastic modes belonging to $\mathbb{V}_g^1$, and the first $n_g^2 = 40$ global elastic modes belonging to $\mathbb{V}_g^2$. Convergence of the reference constituted of usual elastic modes is slowly reached, especially in the $2\pi \times [1000, 1200]\, rad/s$ band (there are only 57 elastic modes in $\mathcal{B}$). The responses obtained via the usual elastic modes basis with dimensions n_g^1 and n_g^2 are computed for comparison with the global bases.

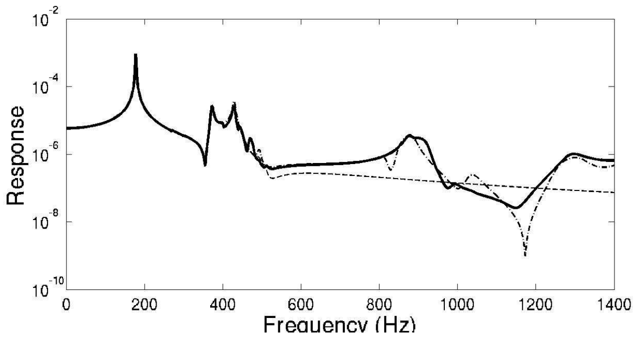

Figure 5: Frequency response in log scale for different projection bases: usual elastic modes ($n = 400$), solid line; $n_g^1 = 20$ global elastic modes in $\mathbb{V}_g^1$, mixed line; first 20 usual elastic modes, dashed line.

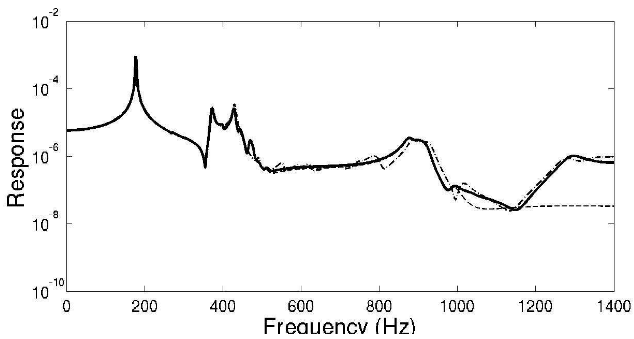

Figure 6: Frequency response in log scale for different projection bases: usual elastic modes ($n = 400$), solid line; $n_g^2 = 40$ first global elastic modes in $\mathbb{V}_g^2$, mixed line; first 40 usual elastic modes, dashed line.

Both the small-dimension global bases allow the frequency response to be predicted with a good approximation on a wide frequency band in comparison with the usual modal basis of same dimension. The global basis constructed using rigid body kinematics for the kinetic energy is more accurate at the expense of its dimension.

4. Conclusions

We have introduced a general methodology which allows the automatic separation between global and local displacements to be performed within the computational dynamical model. The small-dimension basis spanned by the space of global displacements is adapted to predict the response of the master structure, for which local displacements are of lower importance.

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