Dynamic substructuring of structural systems with dissipative physical interface

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DYNAMIC SUBSTRUCTURING OF STRUCTURAL SYSTEMS WITH DISSIPATIVE PHYSICAL INTERFACE

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Abstract. *This paper deals with the theoretical aspects concerning linear elastodynamic of a damped structure composed of two main damped substructures perfectly connected through interfaces by a linking damped substructure. A reduced-order model is constructed using the free interface elastic modes of the two main substructures and an appropriate elastostatic lifting operator related to the linking substructure.*

Keywords: *reduced-order model, ROM, structural dynamics, dynamic substructuring*

1. INTRODUCTION

In this paper, we are interested in the construction of a reduced-order model of a damped structure composed of two main damped substructures perfectly connected through interfaces by a linking damped substructure. Such a reduced-order model allows the frequency response function calculations to be carried out for this structure subjected to prescribed forces. More precisely, this paper is devoted to theoretical aspects of substructure-substructure coupling through a third linking substructure using a dynamic substructuring method and a modal reduction procedure.

For linear structural vibrations, dynamic substructuring techniques have been widely developed in the literature using fixed-interface modes or free-interface modes (completed by static boundary functions, attachment modes, residual flexibility, etc.) of each substructure. For conservative structures, we refer the reader, for example, to Guyan (1965); Hurty (1965); Craig and Bampton (1968); MacNeal (1971); Rubin (1975); Morand and Ohayon (1995); Craig and Kurdila (2006), and for damped structures, to Klein and Dowell (1974); Hale and Meirovitch (1980); Leung (1993); Ohayon and Soize (1998). Some papers are based on a mixed formulation using a Lagrange multiplier in order to impose the linear constraints on the coupling interfaces (Farhat and Geradin, 1994; Ohayon *et al.*, 1997; Park and Park, 2004). A general synthesis of the various techniques can be founded in de Klerk *et al.* (2008). Concerning dynamic substructuring with linking substructures, using simplified hypotheses on the linking substructures behavior, we refer the reader to the stiffness coupling method introduced by Kuhar and Stahle (1974), which is at the origin of the present paper. In addition, linking substructures model correspond to a rough modeling of the real linking systems and uncertainties induced by modeling errors must be introduced (Mignolet *et al.*, 2013).

2. DISPLACEMENT VARIATIONAL FORMULATION FOR TWO SUBSTRUCTURES CONNECTED BY A LINKING SUBSTRUCTURE

2.1 Description of the mechanical system and hypotheses

We consider the linear vibration of a free structure, around a static equilibrium configuration which is considered as a natural state (for the sake of brevity, prestresses are not considered but could be added without changing the presentation), submitted to prescribed external forces which are assumed to be in equilibrium at each instant. The displacement field of the structure is then defined up to an additive rigid body displacement field. We are only interested in the part of the displacement field due to the structural deformation. The structure Ω is composed of two substructures Ω_1 and Ω_2 perfectly connected through interfaces Γ_{1L} and Γ_{2L} by a linking substructure Ω_L (see Fig. 1). We then have $\Omega = \Omega_1 \cup \Omega_L \cup \Omega_2$. The boundaries are such that $\partial\Omega_1 = \Gamma_{1L} \cup \Gamma_1$, $\partial\Omega_2 = \Gamma_{2L} \cup \Gamma_2$, $\partial\Omega_L = \Gamma_{1L} \cup \Gamma_{1L} \cup \Gamma_L$ and $\partial\Omega = \Gamma_1 \cup \Gamma_L \cup \Gamma_2$. Each substructure is a three-dimensional dissipative elastic medium in linear vibration. A frequency domain formulation is used, the convention for the Fourier transform being $\mathbf{u}(\omega) = \int_{\mathbb{R}} e^{-i\omega t}\, \mathbf{u}(t)\, dt$ where ω denotes the circular frequency, $\mathbf{u}(\omega)$ is a vector in $\mathbb{C}^3$ and $\overline{\mathbf{u}}(\omega)$ its conjugate.

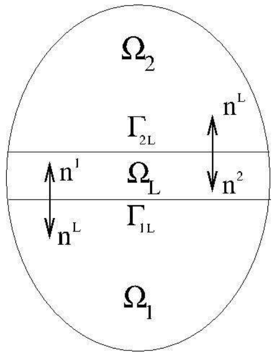

Figure 1. Two substructures Ω_1 and Ω_2 connected with a linking structure Ω_L

2.2 Notation for a substructure Ω_r

For r in $\{1, L, 2\}$, the external prescribed body and surface force fields applied to Ω_r and Γ_r are denoted by $\mathbf{g}_{\Omega_r}$ and $\mathbf{g}_{\Gamma_r}$ respectively. Let $\mathbf{u}^r = (u_1^r, u_2^r, u_3^r)$ be the displacement field at each point $\mathbf{x} = (x_1, x_2, x_3)$ in cartesian coordinates. The set of admissible displacement fields with values in $\mathbb{C}^3$ (resp. in $\mathbb{R}^3$) is denoted by $\mathcal{C}_{\Omega_r}$ (resp. $\mathcal{R}_{\Omega_r}$) and is used for dissipative problems (resp. associated conservative problems). For substructure Ω_r, the test function (weighted function) associated with $\mathbf{u}^r$ is denoted by $\delta\mathbf{u}^r \in \mathcal{C}_{\Omega_r}$ (or in $\mathcal{R}_{\Omega_r}$). From a mathematical point of view, for $r = 1, L, 2$, $\mathcal{R}_{\Omega_r}$ is the real Sobolev space $(H^1(\Omega_r))^3$ and $\mathcal{C}_{\Omega_r}$ is considered as the complexified Hilbert space of $\mathcal{R}_{\Omega_r}$.

The strain tensor is defined by

$$\varepsilon_{ij}(\mathbf{u}^r) = \frac{1}{2}(u_{i,j}^r + u_{j,i}^r)\,, \tag{1}$$

in which $v_{,j}$ denotes the partial derivative of v with respect to x_j. The constitutive equation for substructure Ω_r which is assumed to be made up of an elastic material with linear viscous term is written as

$$\sigma_{\text{tot}}^r = \sigma^r + i\omega\, s^r\,, \tag{2}$$

where σ^r is the elastic stress tensor defined by $\sigma_{ij}^r(\mathbf{u}^r) = a_{ijkh}\,\varepsilon_{kh}(\mathbf{u}^r)$ and where $i\omega\, s^r$ is the viscous part of the total stress tensor such that $s_{ij}^r(\mathbf{u}^r) = b_{ijkh}\,\varepsilon_{kh}(\mathbf{u}^r)$ (using summation over repeated indices). The mechanical coefficients a_{ijkh} and b_{ijkh} depend on $\mathbf{x}$ but are independent of ω and verify the usual properties of symmetry, positiveness and boundedness (lower and upper). The mass density is denoted by ρ^r and depends on $\mathbf{x}$. For this dissipative substructure, three sesquilinear forms on $\mathcal{C}_{\Omega_r} \times \mathcal{C}_{\Omega_r}$ corresponding to the mass, stiffness and damping operators of substructure Ω_r, are introduced as follows

$$m^r(\mathbf{u}^r, \delta\mathbf{u}^r) = \int_{\Omega_r} \rho^r\, \mathbf{u}^r \cdot \overline{\delta\mathbf{u}^r}\, d\mathbf{x}\,, \tag{3}$$

$$k^r(\mathbf{u}^r, \delta\mathbf{u}^r) = \int_{\Omega_r} \sigma_{ij}^r(\mathbf{u}^r)\, \varepsilon_{ij}(\,\overline{\delta\mathbf{u}^r}\,)\, d\mathbf{x}\,, \tag{4}$$

$$d^r(\mathbf{u}^r, \delta\mathbf{u}^r) = \int_{\Omega_r} s_{ij}^r(\mathbf{u}^r)\, \varepsilon_{ij}(\,\overline{\delta\mathbf{u}^r}\,)\, d\mathbf{x}\,. \tag{5}$$

In Eq. (3) and below, the dot denotes the usual Euclidean inner product on $\mathbb{R}^3$ extended to $\mathbb{C}^3$. It should be noted that the hermitian form m^r is positive definite on $\mathcal{C}_{\Omega_r} \times \mathcal{C}_{\Omega_r}$. The hermitian forms k^r and d^r are semi-definite positive since there are rigid body displacement fields. The set $\mathcal{R}_{rig}^r$ of $\mathbb{R}^3$-valued rigid body displacement fields (of dimension 6) is a subset of $\mathcal{C}_{\Omega_r}$. Consequently, for all $\delta\mathbf{u}^r$ in $\mathcal{C}_{\Omega_r}$, $k^r(\mathbf{u}^r, \delta\mathbf{u}^r)$ and $d^r(\mathbf{u}^r, \delta\mathbf{u}^r)$ are equal to zero for any $\mathbf{u}^r$ in $\mathcal{R}_{rig}^r$. The following sesquilinear form z^r is defined on $\mathcal{C}_{\Omega_r} \times \mathcal{C}_{\Omega_r}$ by

$$z^r(\mathbf{u}^r, \delta\mathbf{u}^r) = -\omega^2\, m^r(\mathbf{u}^r, \delta\mathbf{u}^r) + i\omega\, d^r(\mathbf{u}^r, \delta\mathbf{u}^r) + k^r(\mathbf{u}^r, \delta\mathbf{u}^r)\,. \tag{6}$$

Finally, we define the antilinear form $\mathbf{f}^r$ on $\mathcal{C}_{\Omega_r}$ by

$$\ll \mathbf{f}^r\,, \overline{\delta\mathbf{u}^r} \gg = \int_{\Omega_r} \mathbf{g}_{\Omega_r} \cdot \overline{\delta\mathbf{u}^r}\, d\mathbf{x} + \int_{\Gamma_r} \mathbf{g}_{\Gamma_r} \cdot \overline{\delta\mathbf{u}^r}\, ds\,. \tag{7}$$

2.3 Variational formulation in $\mathbf{u}^1$, $\mathbf{u}^L$ and $\mathbf{u}^2$ for structure Ω

The coupling conditions of the linking substructure Ω_L with substructures Ω_1 and Ω_2 on Γ are written as

$$\mathbf{u}^1 = \mathbf{u}^L \quad \text{on} \quad \Gamma_{1L} \quad , \quad \mathbf{u}^2 = \mathbf{u}^L \quad \text{on} \quad \Gamma_{2L} \,. \tag{8}$$

$$\sigma^1_{\text{tot}}\,\mathbf{n}^1 = -\sigma^L_{\text{tot}}\,\mathbf{n}^L \quad \text{on} \quad \Gamma_{1L} \quad , \quad \sigma^2_{\text{tot}}\,\mathbf{n}^2 = -\sigma^L_{\text{tot}}\,\mathbf{n}^L \quad \text{on} \quad \Gamma_{2L} \,, \tag{9}$$

where, for $r = 1, L, 2$, the vector $\mathbf{n}^r$ is the unit normal to $\partial\Omega_r$, external to Ω_r.

The variational formulation in $\mathbf{u}^1$, $\mathbf{u}^L$ and $\mathbf{u}^2$ for structure $\Omega = \Omega_1 \cup \Omega_L \cup \Omega_2$ is the following. For all real ω in $\mathbb{R}$ and for prescribed $(\mathbf{f}^1, \mathbf{f}^L, \mathbf{f}^2)$, find $(\mathbf{u}^1, \mathbf{u}^L, \mathbf{u}^2)$ in $\mathcal{C}_{\Omega_1} \times \mathcal{C}_{\Omega_L} \times \mathcal{C}_{\Omega_2}$ verifying the linear constraints $\mathbf{u}^1 = \mathbf{u}^L$ on Γ_{1L} and $\mathbf{u}^2 = \mathbf{u}^L$ on Γ_{2L}, such that, for all $(\delta\mathbf{u}^1, \delta\mathbf{u}^L, \delta\mathbf{u}^2)$ in $\mathcal{C}_{\Omega_1} \times \mathcal{C}_{\Omega_L} \times \mathcal{C}_{\Omega_2}$ verifying the linear constraints $\delta\mathbf{u}^1 = \delta\mathbf{u}^L$ on Γ_{1L} and $\delta\mathbf{u}^2 = \delta\mathbf{u}^L$ on Γ_{2L}, one has

$$z^1(\mathbf{u}^1, \delta\mathbf{u}^1) + z^L(\mathbf{u}^L, \delta\mathbf{u}^L) + z^2(\mathbf{u}^2, \delta\mathbf{u}^2) = \ll \mathbf{f}^1 , \overline{\delta\mathbf{u}^1} \gg + \ll \mathbf{f}^L , \overline{\delta\mathbf{u}^L} \gg + \ll \mathbf{f}^2 , \overline{\delta\mathbf{u}^2} \gg \;. \tag{10}$$

From the mathematical point of view, the existence and uniqueness of a solution can be proved.

3. DYNAMIC SUBSTRUCTURING USING THE FREE-INTERFACE MODES OF Ω_1 AND Ω_2

The method is based on the use of the variational formulation defined by Eq. (10). The dynamic substructuring is carried out using the Ritz-Galerkin projection on the free-interface modes of each substructure Ω_1 and Ω_2, and using a elastostatic lifting operator for Ω_L.

3.1 Free-interface modes of substructures Ω_1 and Ω_2

For $r = 1, 2$, a free-interface mode of substructure Ω_r is defined as an eigenmode of the conservative problem associated with free substructure Ω_r, subject to zero forces on $\partial\Omega_r$. The real eigenvalues $\omega^2 \geq 0$ and the corresponding eigenmodes $\mathbf{u}^r$ in $\mathcal{R}_{\Omega_r}$ are then the solutions of the following spectral problem: find $\omega^2 \geq 0$, $\mathbf{u}^r \in \mathcal{R}_{\Omega_r} (\mathbf{u}^r \neq \mathbf{0})$ such that for all $\delta\mathbf{u}^r \in \mathcal{R}_{\Omega_r}$, one has

$$k^r(\mathbf{u}^r, \delta\mathbf{u}^r) = \omega^2\, m^r(\mathbf{u}^r, \delta\mathbf{u}^r) \,. \tag{11}$$

It can be shown that there exist six zero eigenvalues $0 = (\omega^r_{-5})^2 = \ldots = (\omega^r_0)^2$ (associated with the rigid body displacement fields) and that the strictly positive eigenvalues (associated with the displacement field due to structural deformation) constitute the increasing sequence $0 < (\omega^r_1)^2 \leq (\omega^r_2)^2, \ldots$. The six eigenvectors $\{\mathbf{u}^r_{-5}, \ldots, \mathbf{u}^r_0\}$ associated with zero eigenvalues span $\mathcal{R}_{rig}$ (space of the rigid body displacement fields). The family $\{\mathbf{u}^r_{-5}, \ldots, \mathbf{u}^r_0; \mathbf{u}^r_1, \ldots\}$ of all the eigenmodes forms a complete set in $\mathcal{R}_{\Omega_r}$. For α and β in $\{-5, \ldots, 0; 1, \ldots\}$, we have the orthogonality conditions

$$m^r(\mathbf{u}^r_\alpha, \mathbf{u}^r_\beta) = \delta_{\alpha\beta}\, \mu^r_\alpha \,, \tag{12}$$

$$k^r(\mathbf{u}^r_\alpha, \mathbf{u}^r_\beta) = \delta_{\alpha\beta}\, \mu^r_\alpha\, {\omega^r_\alpha}^2 \,, \tag{13}$$

in which $\mu^r_\alpha > 0$ is the generalized mass of mode α depending on the normalization of the eigenmodes.

3.2 Introduction of the elastostatic lifting operator S^L

We consider the solution $\mathbf{u}^L_S$ of the elastostatic problem for substructure Ω_L subjected to prescribed displacement fields $\mathbf{u}^1_{\Gamma_{1L}}$ on Γ_{1L} and $\mathbf{u}^2_{\Gamma_{2L}}$ on Γ_{2L}. Let $\mathcal{R}_{\Gamma_{1L},\Gamma_{2L}} = \mathcal{R}_{\Gamma_{1L}} \times \mathcal{R}_{\Gamma_{2L}}$ (from a mathematical point of view, $\mathcal{R}_{\Gamma_{1L},\Gamma_{2L}}$ is the Sobolev space $H^{1/2}(\Gamma_{1L}, \mathbb{C}^3) \times H^{1/2}(\Gamma_{2L}, \mathbb{C}^3)$) and $\mathcal{R}^{\mathbf{u}^1_{\Gamma_{1L}}, \mathbf{u}^2_{\Gamma_{2L}}}_{\Omega_L}$ be the sets of functions such that

$$\mathcal{R}_{\Gamma_{1L}} = \left\{\, \mathbf{x}^1 \mapsto \mathbf{u}^1_{\Gamma_{1L}}(\mathbf{x}^1) \;,\; \forall \mathbf{x}^1 \in \Gamma_{1L} \,\right\} \quad ; \quad \mathcal{R}_{\Gamma_{2L}} = \left\{\, \mathbf{x}^2 \mapsto \mathbf{u}^2_{\Gamma_{2L}}(\mathbf{x}^2) \;,\; \forall \mathbf{x}^2 \in \Gamma_{2L} \,\right\} \,, \tag{14}$$

$$\mathcal{R}^{\mathbf{u}^1_{\Gamma_{1L}}, \mathbf{u}^2_{\Gamma_{2L}}}_{\Omega_L} = \left\{\, \mathbf{u}^L \in \mathcal{C}_{\Omega_L} \quad \middle| \quad \mathbf{u}^L = \mathbf{u}^1_{\Gamma_{1L}} \text{ on } \Gamma_{1L} \quad ; \quad \mathbf{u}^L = \mathbf{u}^2_{\Gamma_{2L}} \text{ on } \Gamma_{2L} \,\right\} \,. \tag{15}$$

From Eq. (15), it can be deduced the definition of $\mathcal{R}^{0,0}_{\Omega_L}$,

$$\mathcal{R}^{0,0}_{\Omega_L} = \left\{\, \mathbf{u}^L \in \mathcal{C}_{\Omega_L} \quad \middle| \quad \mathbf{u}^L = 0 \text{ on } \Gamma_{1L} \quad ; \quad \mathbf{u}^L = 0 \text{ on } \Gamma_{2L} \,\right\} \,, \tag{16}$$

Field $\mathbf{u}^L_S$ satisfies the following variational formulation

$$k^r(\mathbf{u}^L_S, \delta\mathbf{u}^L_S) = 0 \quad , \quad \mathbf{u}^L_S \in \mathcal{R}^{\mathbf{u}^1_{\Gamma_{1L}}, \mathbf{u}^2_{\Gamma_{2L}}}_{\Omega_L} \quad , \quad \forall\, \delta\mathbf{u}^L_S \in \mathcal{R}^{0,0}_{\Omega_L} \,. \tag{17}$$

The unique solution $\mathbf{u}_S^L$ of Eq. (17) defines the linear operator S^L from $\mathcal{R}_{\Gamma_{1L},\Gamma_{2L}}$ into $\mathcal{R}_{\Omega_L}^{\mathbf{u}^1_{\Gamma_{1L}},\mathbf{u}^2_{\Gamma_{2L}}}$ (called the elastostatic lifting operator), such that

$$(\mathbf{u}^1_{\Gamma_{1L}},\mathbf{u}^2_{\Gamma_{2L}}) \mapsto \mathbf{u}_S^L = S^L(\mathbf{u}^1_{\Gamma_{1L}},\mathbf{u}^2_{\Gamma_{2L}})\,. \tag{18}$$

It should be noted that the discretization of S^L by the finite element method is obtained by a classical static condensation procedure of the stiffness matrix of substructure Ω_L with respect to degrees of freedom on $\Gamma_{1L}\cup\Gamma_{2L}$.

3.3 Construction of a reduced-order model

The following reduced-order model can then be constructed using the elastostatic lifting operator and performing a Ritz-Galerkin projection with the free-interface modes of substructures Ω_1 and Ω_2. More precisely, let z_S^L be the sesquilinear form defined on $\mathcal{R}_{\Gamma_{1L},\Gamma_{2L}}\times\mathcal{R}_{\Gamma_{1L},\Gamma_{2L}}$ such that

$$z_S^L((\mathbf{u}^1_{\Gamma_{1L}},\mathbf{u}^2_{\Gamma_{2L}}),(\delta\mathbf{u}^1_{\Gamma_{1L}},\delta\mathbf{u}^2_{\Gamma_{2L}})) = z^L(S^L(\mathbf{u}^1_{\Gamma_{1L}},\mathbf{u}^2_{\Gamma_{2L}}),S^L(\delta\mathbf{u}^1_{\Gamma_{1L}},\delta\mathbf{u}^2_{\Gamma_{2L}}))\,. \tag{19}$$

The reduced-order model of order (m_1,m_2) is then obtained in substituting, in Eq. (10), $z^L(\mathbf{u}^L,\delta\mathbf{u}^L)$ by its approximation $z_S^L((\mathbf{u}^1_{\Gamma_{1L}},\mathbf{u}^2_{\Gamma_{2L}}),(\delta\mathbf{u}^1_{\Gamma_{1L}},\delta\mathbf{u}^2_{\Gamma_{2L}}))$, in which $\mathbf{u}^1_{|\Gamma_{1L}}$ and $\mathbf{u}^2_{|\Gamma_{2L}}$ are the traces of $\mathbf{u}^1$ and $\mathbf{u}^2$ on Γ_{1L} and Γ_{2L}, and then, in projecting the obtained variational equation in $(\mathbf{u}^1,\mathbf{u}^2)$ on the subspace of $\mathcal{C}_{\Omega_1}\times\mathcal{C}_{\Omega_2}$ spanned by the eigenmodes (rigid body modes and elastic modes) $(\{\mathbf{u}^1_{-5},\ldots\mathbf{u}^1_{m_1}\},\{\mathbf{u}^2_1,\ldots\mathbf{u}^2_{m_2}\})$ as follows,

$$\mathbf{u}^{1,m_1} = \sum_{\alpha=-5}^{m_1} q^1_\alpha\,\mathbf{u}^1_\alpha \quad , \quad \mathbf{u}^{2,m_2} = \sum_{\alpha=-5}^{m_2} q^2_\alpha\,\mathbf{u}^2_\alpha\,. \tag{20}$$

We then obtained an complex linear algebraic equation in $(\{q^1_{-5},\ldots,q^1_0,q^1_1\ldots q^1_{m_1}\},\{q^2_{-5},\ldots,q^2_0,q^2_1\ldots q^2_{m_2}\})$ which has, for all fixed real ω, a unique solution.

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