Membranes in the 3D cellular solid model provide the micro-/macro scaling for the long-wavelength acoustics of real foam samples

Camille Perrot, M. T. Hoang, Guy Bonnet, F. Chevillotte, A. Duval

MEMBRANES IN THE 3D CELLULAR SOLID MODEL PROVIDE THE MICRO-/MACRO SCALING FOR THE LONG-WAVELENGTH ACOUSTICS OF REAL FOAM SAMPLES

Camille Perrot[1a)], Minh Tan Hoang[1,2], Guy Bonnet[1], Fabien Chevillotte[3] & Arnaud Duval[2]
[1]*Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME UMR 8208 CNRS, 5 bd Descartes, F-77454 Marne-la-Vallée, France*
[2]*Faurecia Acoustic Excellence Center, Z.I. BP 13, F-08210 Mouzon, France*
[3]*Matelys - Acoustique & Vibrations, 1 rue Baumer, F-69120 Vaulx-en-Velin, France*

Summary The general objective of our work is the optimization of the long-wavelength acoustic properties of real sound proofing foam samples. In this purpose, our general methodology is to use the method of periodic unit cell (PUC) reconstruction of porous media which consists of two main steps. In the first one, the critical local geometry features governing long-wavelengths acoustic wave propagation and dissipation phenomena through porous media are identified. Then, one generates three-dimensional (3D) parameterized PUC suitable for optimization purposes, in which macroscopic properties are obtained by integrating the relevant partial differential equations. This short paper presents recent developments in the field of critical local characteristic sizes identification with application to real polymeric foam samples, where emphasis is put on the techniques which have been devised to account explicitly for the fundamental role played by membranes in the overall transport phenomena.

Membranes real porous media such as polyurethane or metallic foams only account for a very small fraction of material in the overall mass of the porous media. Yet, their role might be of primary importance in the understanding of transport and acoustical properties of these foams. As a long-wavelength wave propagates, the visco-inertial and thermal interactions between the disordered interconnected pores and the surrounding air poses a fundamental physical challenge to the microstructural identification of features which are characteristic of the overall transport phenomena. Part of the solution of this problem lies in the fact that the overall dissipation of the real porous media is expected to be dominated by a few key linkages responsible for the main energy dissipation mechanisms [1]; and in the successful identification of the critical-path responsible for viscous ones [2]. Here, we demonstrate that a complementary part of the solution involves the fluid-structure interaction between the (thermally conducting) air inside the interconnected pores and the membranes that partially close them. Using finite element analysis on a periodic unit-cell local geometry model, experimental estimations of transport parameters, and high resolution imaging of real foam samples, we characterize the closure rate of membranes at the junction between interconnected pores. We find that the presence of non-closed membranes between pores effectively corresponds to the introduction of a second set of critical characteristic sizes, which governs the inertial effects and meanwhile enables a correct description of the thermal ones. For an increasing rate of semi-open membranes, because of the fact that the throat size reduces, then the length Λ which has been identified as a weighted volume-to-surface ratio for the porous medium diminishes, whereas the infinite tortuosity factor α_∞ that traduces strong cross section changes increases [3]. An increasing membranes closure rate promotes the emergence of a stronger contrast between two distinctive critical characteristic sizes inside one periodic unit-cell, the size of a pore itself and the size of the interconnections between pores, which provides a scaling behavior of real polyurethane foam samples for both viscous and inertial effects (Fig. 1 and Tab. I). The presence of membranes also favors surface effects known to have a strong influence in transport phenomena such as diffusion controlled reactions. As characterized from the low frequency asymptotic behavior of thermal exchanges between the solid frame and the surrounding air measured in a standing wave tube, the trapping constants $\Gamma=1/k_0'$ of the real foam samples tend to agree well with the one simulated from the previously identified visco-inertial scaling behavior of the unit-cells. A combination of advanced experiments and detailed numerical modeling of fluid-structure interactions at the pore scale reveals the basic microstructural features behind transport phenomena and shows quantitatively how these thin elements are crucial to the correct microstructural description of long-wavelength acoustic waves propagation and dissipation in real foam samples.

Acknowledgments
This work was part of a project supported by ANRT and Faurecia Acoustics and Soft Trim Division under convention CIFRE No. 748/2009.

References

[1] Sheng P., Zhou M.-Y.: Dynamic permeability in porous media. *Phys. Rev. Lett.* **61**:1591-1594, 1988.
[2] Perrot C., Chevillotte F., Hoang M. T., Bonnet G., Bécot F.-X., Gautron L., Duval A.: Microstructure, transport, and acoustic properties of open-cell foam samples: Experiments and three-dimensional numerical simulations. *J. Appl. Phys.* **111**(1), 2012.
[3] Johnson D. L., Koplik J., Dashen R., Theory of dynamic permeability and tortuosity in fluid-saturated porous media. *J. Fluid Mech.* **176**: 379-402, 1987.
[4] Allard J. F., Atalla N.: Propagation of Sound in Porous Media: modeling sound absorbing materials. Wiley, Chichester 2009.
[5] Lafarge D., Lemarinier P., Allard J. F., Tarnow V., Dynamic compressibility of air in porous structures at audible frequencies. *J. Acoust. Soc. Am.* **102**: 1995-2006, 1997.

a) Corresponding author. Email: camille.perrot@univ-paris-est.fr.

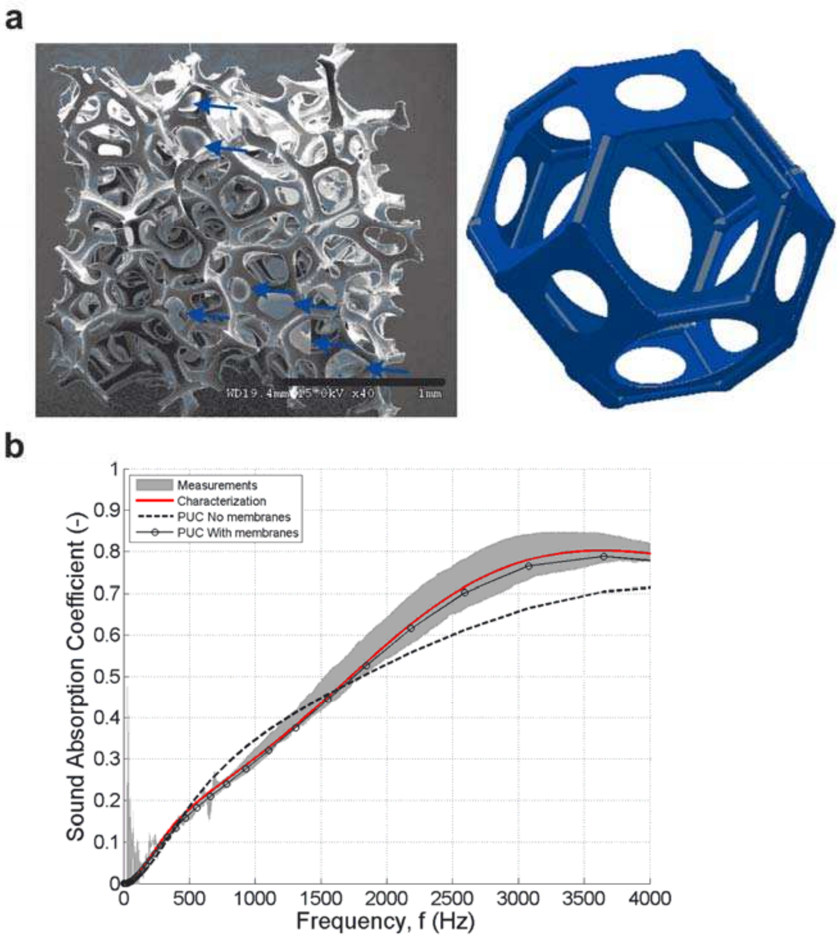

Figure 1. Structure and sound-absorbing properties of a polyurethane foam sample. (**a**) A scanning electron micrograph of a polyurethane foam sample illustrates a number of semi-open or even closed membranes, at the interconnection between pores which are filled with air, a visco-thermal fluid. The calibration bar corresponds to 1 mm. Right: a schematic diagram depicts a single idealized periodic unit cell of polyurethane foam, with membranes at the peripheral of its struts, and a closure rate of 50 % for the smallest windows (the squares in this case). (**b**) Sound absorption coefficient of an acoustically excited polyurethane foam sample in normal incidence with plane waves, without membranes convey the impression of a missing microstructural ingredient governing the asymptotic high-frequency behavior of the porous media, i.e., the inertial part. This is confirmed by the computed sound absorbing behavior with the unit cell containing membranes. Computations were performed using first principles calculations of transport parameters, and Johnson Champoux Allard Lafarge semi-phenomenological model (JCAL) [4]. The thickness of the real polyurethane foam sample is 25 mm.

Table I. Comparison between measured, characterized, and computed multi-scale parameters (with and without membranes). The porosity ϕ and visous permeability k_0 are experimental data of the real foam sample, taken as input parameters for scaling the local geometry model, by solving Stokes equations in the periodic three-dimensional microstructure. This provides initial ligament lengths L and thickness $2r$ of the isotropic unit-cell. Then, the generalized hydraulic radius also known as the thermal characteristic length Λ' is deduced from integration over the scaled unit-cell. All the other macroscopic parameters are derived from first principles calculations [1] [5]. Furthermore, an iterative strategy is used to increase the closure rate of membranes. With non adjustable constant, the iteration counter is stopped when the ligament length of the membrane-based three-dimensional local geometry model is comparable with measurements obtained from standard micrographs.

Method	ϕ(-)	Λ' (μm)	k_0 ($\times 10^{-9}$ m^2)	Λ (μm)	α_∞ (-)	k_0' ($\times 10^{-9}$ m^2)	L (μm)	r (μm)
Measurements	0.98 ± 0.01		2.60 ± 0.08				205 ± 42	15 ± 3
Characterization		440 ± 202		129 ± 23	1.12 ± 0.20	8.30 ± 1.60		
Membranes		295 ± 5		147 ± 4	1.15 ± 0.02	8.06 ± 0.50	202 ± 11	16 ± 6
No Membranes		506 ± 114		297 ± 66	1.02 ± 0.01	5.01 ± 0.22	123 ± 12	10 ± 4