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In this study, we present a novel approach that allows to couple two stochastic continuum models. The coupling strategy is performed in the Arlequin framework [1], which is based on a volume coupling and a partition of the energy between two models. A suitable functional space is chosen for the weak enforcement of the continuity between the two models. The choice of this space ensures that the ensemble average of the two stochastic solutions are equal point-wise in the coupling area, and that appropriate boundary conditions on the stochastic dimension are passed from one model to the other.

We consider two stochastic continuum models, each characterized by a stochastic mechanical parameter $k(x)$ and a solution $u(x)$, and controlled by an elliptic equation : $\nabla \cdot k(x)\nabla u(x) = f$, for a given load f. Both the solution $u_2(x)$ and the mechanical parameter $k_2(x)$ of the so-called micro-scale model fluctuate over small characteristic lengths (typically a correlation length L_2). The stochastic model of the mechanical parameter $k_1(x)$ of the so-called meso-scale model is an upscaled version of the previous one, such that the power spectrum is filtered for a correlation length $L_1 > L_2$ (see [2] for a partial review). Figure 1 illustrates one realization of each of the two random fields.

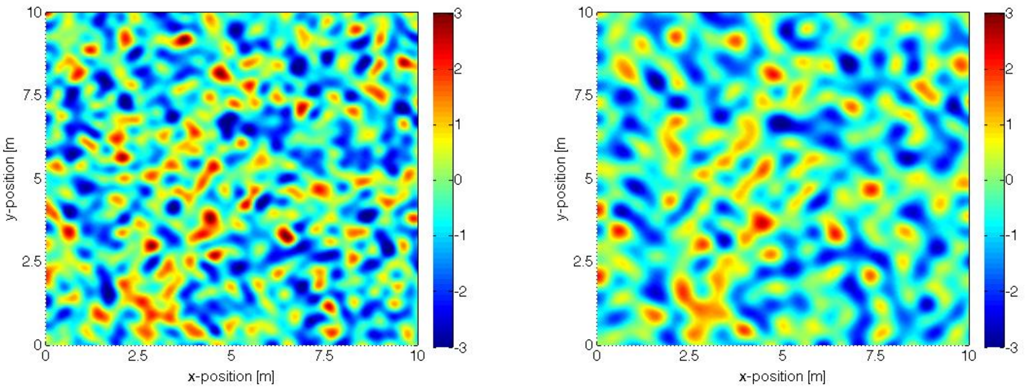

Fig. 1 – One realization of the logarithm of the mechanical parameter $\log k(x)$ of a micro-scale model (left figure), with correlation length $L_2 = 0.5$ m, and of a meso-scale model (right figure) with correlation length filtered to $L_1 = 0.8$ m.

The proposed coupling approach is an extension of a previous work dealing with the coupling of a stochastic model with a deterministic one [3]. It leads (in the particular case of $\Omega_2 \in \Omega_1$ and homogeneous Dirichlet boundary conditions on $\partial\Omega_1$) to the following mixed problem : find $(u_1, u_2, \Phi) \in \mathcal{V}_1 \times \mathcal{V}_2 \times \mathcal{V}_c$ such that

$$\begin{cases} a_1(u_1, v) + C(\Phi, v) = \ell_1(v), & \forall v \in \mathcal{V}_1 \\ a_2(u_2, v) - C(\Phi, v) = \ell_2(v), & \forall v \in \mathcal{V}_2 \\ C(\Psi, u_1 - u_2) = 0, & \forall \Psi \in \mathcal{V}_c \end{cases}, \tag{1}$$

where the bilinear forms $a_1 : \mathcal{V}_1 \times \mathcal{V}_1 \to \mathbb{R}$, $a_2 : \mathcal{V}_2 \times \mathcal{V}_2 \to \mathbb{R}$, and $C : \mathcal{V}_c \times \mathcal{V}_c \to \mathbb{R}$ are defined by $a_1(u, v) = \int_{\Omega_1} \alpha_1 k_1(x)\nabla u \cdot \nabla v \, dx$, $a_2(u, v) = \int_{\Omega_1} \alpha_2 k_2(x)\nabla u \cdot \nabla v \, dx$, and

$$C(u, v) = \mathrm{E}\left[\int_{\Omega_c} (\kappa_0 uv + \kappa_1 \nabla u \cdot \nabla v)\, dx\right], \tag{2}$$

where the linear forms $\ell_1 : \mathcal{V}_1 \to \mathbb{R}$ and $\ell_2 : \mathcal{V}_2 \to \mathbb{R}$ are defined, respectively, by $\ell_1(v) = \int_{\Omega_1} \alpha_1 f \mathrm{E}[v] dx$ and $\ell_2(v) = \int_{\Omega_2} \alpha_2 f \mathrm{E}[v] dx$, and the functional spaces are $\mathcal{V}_1 = \mathcal{L}^2(\Theta, \mathcal{H}_0^1(\Omega_1))$, $\mathcal{V}_2 = \mathcal{L}^2(\Theta, \mathcal{H}^1(\Omega_2))$, and

$$\mathcal{V}_c = \mathcal{H}^1(\Omega_c) \oplus \mathcal{L}^2(\Theta, \mathbb{R}). \tag{3}$$

This choice of functional spaces and coupling operator ensures that the mixed problem (1) has a unique solution that can be approximated by spectral finite elements or a Monte Carlo technique.

Références

[1] H. Ben Dhia. Multiscale mechanical problems : the Arlequin method. *Comptes Rendus de l'Académie des Sciences - Series IIB*, 326(12) :899–904, 1998.

[2] B. Nøetinger, V. Artus, and G. Zargar. The future of stochastic and upscaling methods in hydrogeology. *Hydrogeol. J.*, 13 :184–201, 2005.

[3] R. Cottereau, D. Clouteau, H. Ben Dhia, and C. Zaccardi. A stochastic-deterministic coupling method for continuum mechanics. *Comp. Meths. Appl. Mech. Engr.*, 200 :3280–3288, 2011.