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Abstract
The research addressed here concerns the generation of seismic accelerograms compatible with a given response spectrum and other associated properties. The time sampling of the stochastic accelerogram yields a time series represented by a random vector in high dimension. The probability density function (pdf) of this random vector is constructed using the Maximum Entropy (MaxEnt) principle under constraints defined by the available information. In this paper, a new algorithm, adapted to the high stochastic dimension, is proposed to identify the Lagrange multipliers introduced in the MaxEnt principle to take into account the constraints. This novel algorithm is based on (1) the minimization of an appropriate convex functional and (2) the construction of the probability distribution defined as the invariant measure of an Itô Stochastic Differential Equation in order to estimate the integrals in high dimension of the problem.

Key words: MaxEnt, Accelerogram, Response Spectrum, PGA

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1. Introduction

This research is devoted to the generation of seismic accelerograms which are compatible with some design specifications such as the Velocity Response Spectrum, the Peak Ground Acceleration (PGA), etc. The Maximum Entropy (MaxEnt) principle [Kapur & Kevasan] is a powerful method which allows us to construct a probability distribution of a random vector under some constraints defined by the available information. This method has recently been applied in [Soize] for the generation of spectrum-compatible accelerograms as trajectories of a non-Gaussian non-stationary centered random process represented by a high-dimension random vector for which the probability density function (pdf) is constructed using the MaxEnt principle under constraints relative to (1) the mean value, (1) the variance of the components and (2) the mean value of the Velocity Response Spectrum (VRS).

The objective of this paper is to take into account additional constraints which characterize the natural features of a seismic accelerogram. To achieve this objective, the methodology proposed in [Soize] is extended to take into account constraints relative to statistics on (1) the end values for the velocity and the displacement, (2) the PGA, (3) the Peak Ground Velocity (PGV), (4) the envelop of the random VRS and (5) the Cumulative Absolute Velocity (CAV). The MaxEnt pdf is constructed and a generator of independent realizations adapted to the high-stochastic dimension of an accelerogram is proposed. Furthermore an adapted method for the identification of the Lagrange multipliers is developed.

In Section 2 the MaxEnt principle is used to construct the pdf of the acceleration random vector under constraints defined by the available information. Finally, Section 3 is devoted to an application of the methodology for which the target VRS is constructed following the Eurocode 8.

2. Construction of the probability distribution

The MaxEnt principle is a powerful method to construct the probability distribution of a random vector associated with a sampled stochastic process under some constraints defined by the available information. The random acceleration of the soil is modeled by a second-order centered stochastic process $\{A(t), t \in [0,T]\}$. A time sampling of this stochastic process is introduced yielding a time series $\{A_1,..., A_N\}$ for which the random vector $\mathbf{A} = (A_1,..., A_N)$ is associated with. We then have $T = N\Delta t$ in which Δt is the sampling time step. Finally, we have to construct the probability distribution of random vector $\mathbf{A}$.

It is assumed that the available information is written as

$$E\{\mathbf{g}(\mathbf{A})\} = \mathbf{f}, \tag{1}$$

in which $\mathbf{g}(\mathbf{a})$ is a given function and where $\mathbf{f}$ is a target vector. Equation (1) can be rewritten as

$$\int_{\Re^N} \mathbf{g}(\mathbf{a})\, p_A(\mathbf{a})\, d\mathbf{a} = \mathbf{f}. \tag{2}$$

An additional constraint relative to the normalization of the pdf is introduced such that

$$\int_{\Re^N} p_A(\mathbf{a})\, d\mathbf{a} = 1. \tag{3}$$

The entropy of the pdf $p_A(\mathbf{a})$ is defined by

$$S(p_A) = -\int_{\Re^N} \log(p_A(\mathbf{a}))\, p_A(\mathbf{a})\, d\mathbf{a}. \tag{4}$$

Then the MaxEnt principle consists in constructing the pdf $\mathbf{a}$ a $\mathbf{g}(\mathbf{a})$ as the unique pdf which maximizes the entropy. Then by introducing a Lagrange multiplier $\boldsymbol{\lambda}$ associated with Eq.(2), it can be shown (see [Kapur & Kevasan]) that the MaxEnt solution, if it exists, is defined by

$$p_A(\mathbf{a}) = c_0(\boldsymbol{\lambda}^{sol}) \exp\left(-\left\langle \boldsymbol{\lambda}^{sol}, \mathbf{g}(\mathbf{a}) \right\rangle\right) \tag{5}$$

in which $c_0(\boldsymbol{\lambda}^{sol})$ is a normalization constant and where $\langle .,. \rangle$ denotes the Euclidean inner product. The Lagrange multiplier $\boldsymbol{\lambda}^{sol}$ is calculated by minimizing the following functional (see [Golan et al.])

$$\Gamma(\boldsymbol{\lambda}) = \langle \boldsymbol{\lambda}, \mathbf{f} \rangle - \log(c_0(\boldsymbol{\lambda})). \tag{6}$$

The optimal value is calculated iteratively using the Newton method

$$\boldsymbol{\lambda}^{i+1} = \boldsymbol{\lambda}^{i} - \alpha[H(\boldsymbol{\lambda})]\nabla\Gamma(\boldsymbol{\lambda}), \tag{7}$$

in which α is an under-relaxation parameter and where $\nabla\Gamma(\boldsymbol{\lambda})$ and $[H(\boldsymbol{\lambda})]$ are respectively the gradient vector and the hessian matrix of $\Gamma(\boldsymbol{\lambda})$ with respect to $\boldsymbol{\lambda}$, and are respectively written as

$$\nabla\Gamma(\boldsymbol{\lambda}) = \mathbf{f} - E\{\mathbf{g}(\mathbf{A}_\lambda)\}, \tag{8}$$

$$[H(\boldsymbol{\lambda})] = E\{\mathbf{g}(\mathbf{A}_\lambda)\mathbf{g}(\mathbf{A}_\lambda)^T\} - E\{\mathbf{g}(\mathbf{A}_\lambda)\}E\{\mathbf{g}(\mathbf{A}_\lambda)\}^T. \tag{9}$$

The calculation of $\nabla\Gamma(\boldsymbol{\lambda})$ and $[H(\boldsymbol{\lambda})]$ requires the estimation of integrals in high dimension. In general, these integrals cannot explicitly be calculated and cannot be discretized. In this research, these integrals are estimated using the Monte Carlo simulation method for which independent realizations of the random vector $\mathbf{A}_\lambda$ are generated using an algorithm belonging to the MCMC class which is adapted to the high dimension, as proposed in [Soize]. This algorithm consists in constructing the pdf of random vector $\mathbf{A}_\lambda$ as the density of the invariant measure associated with the stationary solution of a second-order nonlinear Itô Stochastic differential equation (ISDE).

3. Application

The acceleration stochastic process is sampled such that the final time $T = 20$ s. The time step is $\Delta T = 0.0125$ s. We then have $N = 1600$ (we assume $A(0) = 0$ ms^{-2} almost surely). The available information is relative to the variance of the components of the random vector $\mathbf{A}$, the mean value of the VRS (see [Clough & Penzien] for the definition of the VRS), the envelop of the VRS, the variance of the end value of the velocity and displacement random vectors (resulting from two successive numerical integrations of the acceleration random vector $\mathbf{A}$). The methodology for the construction of the MaxEnt pdf introduced in the previous section is applied.

Figure 1 compares the target standard deviation of the components with the estimated one.

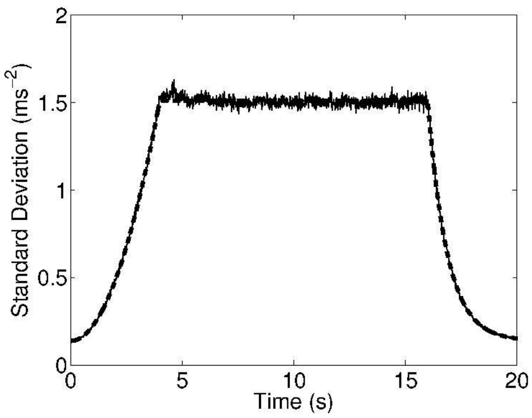

Fig. 1 Standard deviation: Target (thick dashed line) and estimation (thin solid line).

The target VRS in constructed following the Eurocode 8. There are two constraints relative to the VRS. The first one concerns the mean value and the second one concerns the probability for the acceleration trajectories of being between two envelops (target VRS +/- 50%). The results are plotted on Fig.2.

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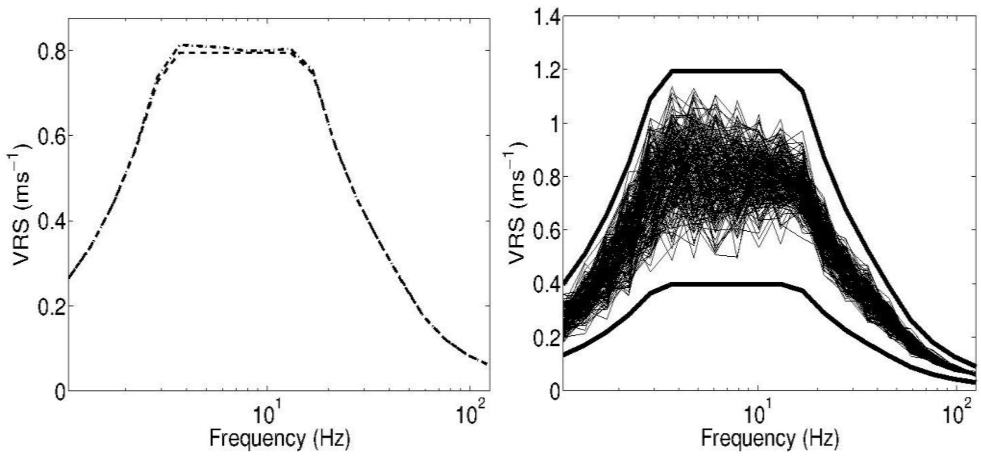

Fig. 2 Random VRS. Left figure: mean value: Target (dashed line) and estimation (mixed line). Right figure: 100 trajectories (thin lines), lower and upper envelop (thick lines).

The end values for the velocity and displacement trajectories are controlled by constraining the variance of the end value of the velocity and displacement random vectors (obtained by two successive numerical integrations) to be zero. Figure 3 shows a simulated accelerogram and the corresponding velocity time series and displacement time series.

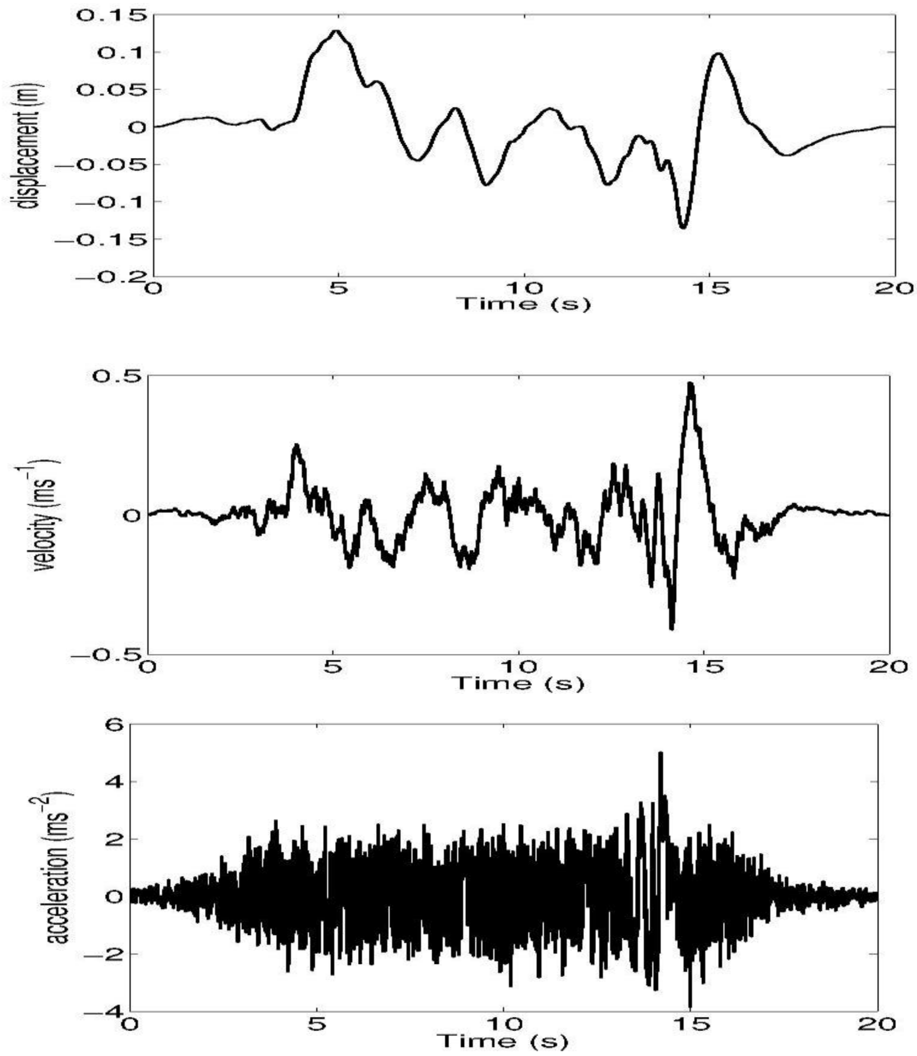

Fig. 3 A realization of the random acceleration, the random velocity and the random displacement.

Finally the mean value for the PGA, the PGV and the CAV are also constrained. The results are reported on Table 1.

Table 1. For the PGA, the PGV, the CAV: comparison of the estimated mean value with the target value

Constraint	Target	Estimation
Mean PGA (ms^{-2})	5	5.08
Mean PGV (ms^{-1})	0.45	0.46
Mean CAV (ms^{-1})	20	19.99

Figures 1 to 3 and Table 1 show a good matching of the estimated values with the target values.

5. Conclusions

We have presented a new methodology for the generation of accelerograms compatible with a given VRS and other properties. If necessary, additional constraints can easily be taken into account in addition to those developed in this paper. The application shows a good matching between the estimated values and the target values.

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