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To cite this version:

Anas Batou, Christian Soize, N. Brie. Reduced-order model for nonlinear dynamical structures having a high modal density in the low-frequency range. MEDYNA 2013, 1st Euro-Mediterranean Conference on Structural Dynamics and Vibroacoustics, Apr 2013, Marrakech, Morocco. pp.1-4. hal-00806396

HAL Id: hal-00806396

https://hal-upec-upem.archives-ouvertes.fr/hal-00806396

Submitted on 30 Mar 2013

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REDUCED-ORDER MODEL FOR NONLINEAR DYNAMICAL STRUCTURES HAVING A HIGH MODAL DENSITY IN THE LOW-FREQUENCY RANGE

A. Batou[1*], C. Soize[1], N. Brie[2]

[1]Laboratoire Modélisation et Simulation Multi-Echelle, MSME UMR 8208 CNRS
Université Paris-Est, 5 Boulevard Descartes, 77454 Marne-la Vallée, France
Email: anas.batou@univ-pairs-est.fr

[2]EDF R&D, Département AMA
1 avenue du général De Gaulle, 92140 Clamart, France

ABSTRACT

This research is devoted to the construction of a reduced-order computational model for nonlinear dynamical structures which are characterized by the presence of numerous local elastic modes in the low-frequency band. Therefore these structures have a high modal density in the low-frequency band and the use of the classical modal analysis method is not suited here. We propose to construct a reduced-order computational model using a small-dimension basis of a space of global displacements, which is constructed a priori by solving an unusual eigenvalue problem. Then the reduced-order computational model allows the nonlinear dynamical response to be predicted with a good accuracy on the stiff part of the structure. The methodology is applied to a complex industrial structure which is made up of a row of seven fuel assemblies with possibility of collisions between grids and which is submitted to a seismic loading.

1 INTRODUCTION

This paper is devoted to the construction of a reduced-order model for nonlinear dynamical structures having numerous local elastic modes in the low-frequency (LF) range. This paper focuses specifically on localized non-linearities such as elastic stops. We are interested in the nonlinear response of structures which are made up of a rigid master structure coupled with several flexible substructures. Such structures are characterized by the fact that they present in the LF band, both classical global elastic modes but also many local elastic modes. Moreover, the structure we consider is modelled with a large finite element model and has several localized nonlinearities (such as elastic stops). As a consequence, the non-linear transient response has to be constructed using a small time step for the integration scheme in order to correctly capture the nonlinear effects in the non-linear transient response. Then, the direct construction of the non-linear transient response is a very challenging issue and therefore the computational model has to be reduced. Due to the high modal density of the structures under consideration, the classical reduction [1] consisting in using the elastic modes of the underlying linear part of the nonlinear dynamical system is not suited here. Since we want to construct a small-dimension reduced-order computational model which has the capability to predict the nonlinear dynamical responses on the stiff part for which the local displacements are negligible, we have to construct the reduced-order computational model using a basis adapted to the prediction of the global displacements and therefore, we have to filter the local displacements in the construction of the basis.

Recently, a new method has been proposed to construct a reduced-order computational model in linear structural dynamics for structures having numerous local elastic modes in the low-frequency band [2]. In this method, a basis of the global displacements space and a basis of the local displacements space are calculated by solving two unusual eigenvalues problems. The elements of these two bases are not constituted of the usual elastic modes. The eigenvalue problem, allowing a basis of the global displacements space to be constructed, is constructed by introducing a kinematic reduction for the kinetic energy while the elastic energy is kept exact. In this paper, this method will be used to construct a basis of the global displacements space and then to deduce the reduced-order computational model of the nonlinear dynamical structure. Therefore, the contributions of the local displacements of the structure observed on the stiff part are neglected in the research presented here.

2 CONSTRUCTION OF THE REDUCED-ORDER COMPUTATIONAL MODEL

We are interested in predicting the transient responses of a three-dimensional nonlinear damped structure, with localized nonlinearities, and occupying a bounded domain Ω.

The methodology proposed in [2] consists in introducing a kinematic reduction of the structural kinetic energy. In a first step, the domain Ω is partitioned into n_J disjoint subdomains Ω_j. In a second step, this decomposition is used to construct the projection linear operator $\boldsymbol{u} \mapsto h^r(\boldsymbol{u})$ defined by

$$\{h^r(\boldsymbol{u})\}(\boldsymbol{x}) = \sum_{j=1}^{n_J} \mathbb{1}_{\Omega_j}(\boldsymbol{x}) \, \frac{1}{m_j} \int_{\Omega_j} \rho(\boldsymbol{x}') \, \boldsymbol{u}(\boldsymbol{x}') \, d\boldsymbol{x}' \, , \tag{1}$$

in which $\boldsymbol{x} \mapsto \mathbb{1}_{\Omega_j}(\boldsymbol{x}) = 1$ if $\boldsymbol{x}$ is in Ω_j and $= 0$ otherwise, where m_j is the total mass of subdomain Ω_j and where $\rho(\boldsymbol{x})$ is the mass density. This operator carries out an average of the displacements with respect to the mass density in each subdomain (kinematic reduction). We then introduce the $(m \times m)$ matrix $[H^r]$ relative to the finite element discretization of the projection operator h^r defined by Eq. (1). Then, the $(m \times m)$ projected mass matrix $[\mathbb{M}^r]$ is constructed such that $[\mathbb{M}^r] = [H^r]^T[\mathbb{M}][H^r]$.

The basis of the global displacements space is made up of the solutions $\boldsymbol{\varphi}^g$ of the generalized eigenvalue problem

$$[\mathbb{K}]\boldsymbol{\varphi}^g = \lambda^g[\mathbb{M}^r]\boldsymbol{\varphi}^g\,, \tag{2}$$

in which the stiffness matrix is kept exact while the mass matrix is projected. The computation of the eigenvectors is carried out using an adapted subspace iteration algorithm [3]. This algorithm avoid the assembly of matrix $[\mathbb{M}^r]$ which is a full matrix. If needed, a basis of the local displacements space can also be constructed and complete the basis of global displacements (see [2]).

The reduced-order computational model is obtained using the projection of displacement vector $\mathbb{U}(t)$ on the subspace spanned by the family $\{\boldsymbol{\varphi}_1^g, \ldots, \boldsymbol{\varphi}_{n_g}^g\}$ of real vectors associated with the n_g first eigenvalues $0 < \lambda_1^g \leq \ldots \leq \lambda_{n_g}^g$. Then, the n_g-order approximation $\mathbb{U}_{n_g}(t)$ of $\mathbb{U}(t)$ is written as

$$\mathbb{U}_{n_g}(t) = \sum_{j=1}^{n_g} \boldsymbol{\varphi}_j^g {q_j}^g(t) \quad , \tag{3}$$

in which $\boldsymbol{q}^g(t) = (q_1^g(t), \ldots, q_{n_g}^g(t))$. The vector $\boldsymbol{q}^g(t)$ is solution of the following nonlinear reduced matrix equation

$$[M]\ddot{\boldsymbol{q}}^g(t) + [D]\dot{\boldsymbol{q}}^g(t) + [K]\boldsymbol{q}^g(t) + \boldsymbol{f}^{\mathrm{NL}}(\boldsymbol{q}^g(t), \dot{\boldsymbol{q}}^g(t)) = \boldsymbol{f}(t) \quad , t \in]0\,, T]\,, \tag{4}$$

with the initial conditions $\boldsymbol{q}^g(0) = \dot{\boldsymbol{q}}^g(0) = \mathbf{0}$, in which $[M]$, $[D]$ and $[K]$ are the $(n_g \times n_g)$ generalized mass, damping and stiffness matrices, where $\boldsymbol{f}(t)$ is the vector of the generalized forces and where $\boldsymbol{f}^{\mathrm{NL}}(\boldsymbol{q}^g(t), \dot{\boldsymbol{q}}^g(t))$ is the vector of the generalized nonlinear forces. The dynamical systems we are interested in this paper are made up of few eigenvectors of global displacements. Consequently, the size of the nonlinear reduced-order computational model defined by Eq. (3) is very small.

3 APPLICATION

In this Section, we present an industrial application of the methodology which consists in the dynamical analysis of a row of seven fuel assemblies with possibility of collisions between grids and submitted to a seismic loading. A fuel assembly is a slender structure which is made up of 264 flexible fuel rods, 25 stiff guide tubes and 10 stiff grids which hold the tubes in position. The guide tubes are soldered to the grids while the fuel rods are fixed to the grids by springs. The row of assemblies is made up of seven fuel assemblies. The fuel assemblies are linked each to the others by the rigid containment building on which an homogeneous seismic displacement is imposed. The mesh of the finite element model is plotted in Fig. 1. The finite element model has $313,908$ elements and $3,147,060$ DOFs and there are $51,548$ elastic modes in the band $[0\,, 400]$ Hz. The possible contact grid/grid and grid/containment are taken into account by introducing 160 elastic stops. Each grid has a left elastic stop and a right elastic stop, the containment building has 10 elastic stops face to rightmost assembly grids and 10 elastic stops face to leftmost assembly grids.

Each fuel assembly of the row is decomposed into 100 slices yielding 700 subdomains. For the band $[0\,, 400]$ Hz, the reduced-order computational model is constructed with 245 global eigenvectors (instead of $51,548$ elastic modes which would be required with a classical modal analysis). We are interested in the nonlinear transient relative displacement of the row of assemblies.

The nonlinear relative response is calculated in the interval time $[0\,, 19.48]$ s using an explicit Euler integration scheme with an integration time step 10^{-5} s. For an observation point,

Figure 1: Finite element mesh of a row of fuel assemblies: Grids (blue) and guide tubes (red). The fuel rods are not plotted.

the relative transient displacements (for the convergence with respect to the number of global eigenvectors has been analysed) is plotted in Fig. 2.

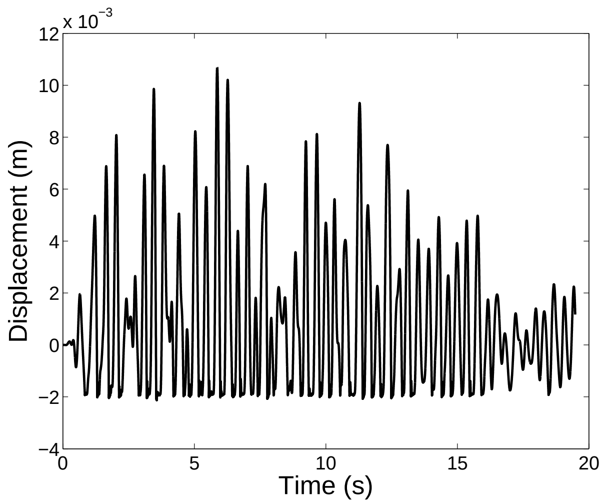

Figure 2. Relative transient displacement.

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