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Reduced-Order Model in Structural Dynamics for High-Modal Density in the LF Range. Applications to Automotive Vehicle and Fuel Assemblies

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Abstract: We are interested in the construction of stochastic reduced-order models in low-frequency range for dynamical structures which are made up of a stiff master structure and several flexible substructures. In the low-frequency range, this type of structure is characterized by the fact that it exhibits not only the classical global elastic modes but also numerous local elastic modes which cannot easily be separated from the global elastic modes. To solve this difficult problem, a new approach has recently been proposed for constructing a reduced-order model adapted to this case. This method consists in introducing two unusual eigenvalue problems. The solutions of this two eigenvalue problems provide a basis of the admissible displacement space which is written as the direct sum of a global displacements space and of a local displacements space. A stochastic reduced-order model is then introduced in order to take into account uncertainties. Two industrial applications are presented. The first one is relative to an automotive vehicle and the second one is relative to a fuel assembly.

Keywords: reduced-order model, ROM, structural dynamics, uncertainties, high modal density

1 INTRODUCTION

This paper is devoted to the construction of a stochastic Reduced-Order Model (ROM) for dynamical structures having a high-modal density in the Low-Frequency (LF) range. We are interested in structures which are made up of stiff parts and flexible components. In the LF range, this type of structures exhibits not only the classical global elastic modes but also numerous local elastic modes. The first objective is to construct a ROM using a basis of the global displacements space. Due to the coupling between the global displacements and the local displacements, the elastic modes cannot strictly and easily be separated into global and local elastic modes.

The objective of this paper is to construct a reduced-order computational model with a very small dimension and which has the capability to predict the dynamical responses of the structure observed on the stiff part with a good accuracy. Since the contributions of the local displacements are negligible in the displacements of the stiff part, we have to construct the reduced-order computational model using a basis adapted to the prediction of the global displacements and therefore, we have to filter the local displacements in the construction of the basis. To achieve this objective, most of previous researches have been based on a spatial filtering of the short wavelengths. Concerning the experimental methods, such a filtering is carried out using regularization techniques (Bucher and Braun, 1997), image-based finite element methods (Hahn and Kikuchi, 2005) or an extraction of eigenvectors of the frequency mobility matrix (Guyader, 2009). Concerning numerical methods, most of the techniques are based on the lumped mass methods. In the Guyan method (Guyan, 1965), the masses are lumped at a few nodes and the inertia forces of the other nodes are neglected. It should be noted that the choice of points in which the masses are concentrated is not obvious to do for complex structures (Bouhaddi and Fillod, 1992), (Ong, 1987), (Li, 2003). The convergence properties of the solution obtained using the lumped mass method have been studied (Chan et al., 1993), (Jensen, 1996), (Belytschko and Mindle 1980). In (Langley and Bremmer, 1999), the authors propose to construct a basis of the global displacements space using a rough finite element model. For slender dynamical structures, another method consists in the construction of an equivalent beam or plate model (Noor et al., 1978), (Planchard, 1995). In (Guyader, 1990), the author circumvent the problem of the high modal density by extrapolating the dynamical response using a few elastic modes. This method is interesting when one has an analytical expression for the modal shape, that is not the case for the structures considered in this paper. In (Ji, 2006), the authors use a free-interface substructuring method in order to extract long wavelength free modes of a master structure. The Proper Orthogonal Decomposition (POD) method (see (Karhunen, 1945), (Loève, 1963), (Holmes et al., 1997)) allows in some cases to extract an accurate small size basis in order to construct a reduced-order computational model of a nonlinear dynamical system (see for instance (Azeez and Vakakis, 2001), (Kunisch and Volkwein, 2001), (Matthies and Keese, 2005), (Sampaio and Soize, 2007)), but this basis has to be constructed *a posteriori*, which means that a sufficiently rich nonlinear response has to be constructed. Moreover, the POD basis is only optimal for a given external load (or imposed displacement). In (Soize, 1998), a similar method to POD is used for linear dynamical systems but the basis which is constructed is independent of any given external load.

Recently, a new method has been proposed to construct a reduced-order computational model in linear structural dynamics for structures having numerous local elastic modes in the low-frequency band (Soize and Batou, 2010). To solve this difficult problem, a new approach has recently been proposed for constructing a reduced-order dynamical model adapted to this type of structures in the LF range (Soize and Batou, 2010). This approach allows a basis of the global displacements and a basis of the local displacements to be constructed by solving two unusual eigenvalue problems. Due to the coupling between the global displacements and the local displacements, a part of the mechanical energy is transferred from the global coordinates to the local coordinates which store this energy and then induced an apparent damping on the global coordinates. The second objective is to take into account uncertainties induced by modelling errors in the computational model and the irreducible errors introduced by neglecting the local displacements. This second objective is achieved using a probabilistic approach. The methodology presented is applied to the case of two complex dynamical systems: an automotive vehicle and the fuel assemblies of a pressurized water reactor.

2 CONSTRUCTION OF THE REDUCED-ORDER MODEL

In this section, we summarize the method introduced in (Soize and Batou, 2010). This method allows a basis of the global displacements and a basis of the local displacements to be constructed by solving two separated eigenvalue problems. It should be noted that these two bases are not made up of the usual elastic modes. The method is based on the construction of a projection operator which reduces the kinetic energy while the elastic energy remains exact.

2.1 Reference reduced model

We are interested in predicting the frequency response functions of a dynamical structure occupying a domain Ω, in the frequency band of analysis $\mathscr{B} = [\omega_{\min}, \omega_{\max}]$ with $0 < \omega_{\min}$. Let $\mathbb{U}(\omega)$ be the complex vector of the m DOF of the computational model constructed by the finite element method. Let $[\mathbb{M}]$ and $[\mathbb{K}]$ be the mass and stiffness matrices which are positive-definite symmetric $(m \times m)$ real matrices. The eigenfrequencies λ and the elastic modes $\boldsymbol{\varphi}$ in $\mathbb{R}^m$ of the conservative part of the dynamical computational model of the structure are the solution of the following eigenvalue problem,

$$[\mathbb{K}]\, \boldsymbol{\varphi} = \lambda\, [\mathbb{M}]\, \boldsymbol{\varphi}\,. \tag{1}$$

Then an approximation $\mathbb{U}_n(\omega)$ at order n of $\mathbb{U}(\omega)$ can be written as

$$\mathbb{U}_n(\omega) = \sum_{\alpha=1}^{n} q_\alpha(\omega)\, \boldsymbol{\varphi}_\alpha = [\Phi]\, \boldsymbol{q}(\omega)\,, \tag{2}$$

in which $\boldsymbol{q}(\omega) = (q_1(\omega), \ldots, q_n(\omega))$ is the complex vector of the n generalized coordinates and where $[\Phi] = [\boldsymbol{\varphi}_1 \ldots \boldsymbol{\varphi}_n]$ is the $(m \times n)$ real matrix of the elastic modes associated with the n first eigenvalues.

2.2 Decomposition in subdomains for kinematic energy reduction.

In this section, we introduce a decomposition of the domain of the structure which allows the kinematic energy to be reduced. We then obtained an associated mass matrix which is adapted to the calculation of the global elastic modes in the low-frequency band of analysis in which there are also a large there are a large number of local elastic modes. The details of the methodology for the the continuous and the discrete cases are presented in (Soize and Batou, 2010).

2.2.1 Decomposition of the domain

The domain Ω is partitioned into n_J subdomains Ω_j such that, for j and k in $\{1, \ldots, n_J\}$,

$$\Omega = \bigcup_{j=1}^{n_J} \Omega_j \quad , \quad \Omega_j \cap \Omega_k = \emptyset\,. \tag{3}$$

The choice of the length of subdomains is related to the smallest "wavelength" of the global elastic modes that we want to extract in presence of numerous local modes.

2.2.2 Projection operator

Let $\boldsymbol{u} \mapsto h^r(\boldsymbol{u})$ be the linear operator defined by

$$\{h^r(\boldsymbol{u})\}(\boldsymbol{x}) = \sum_{j=1}^{n_J} \mathbb{I}_{\Omega_j}(\boldsymbol{x})\, \frac{1}{m_j} \int_{\Omega_j} \rho(\boldsymbol{x}')\, \boldsymbol{u}(\boldsymbol{x}')\, d\boldsymbol{x}'\,, \tag{4}$$

in which $\boldsymbol{x} \mapsto \mathbb{I}_{\Omega_j}(\boldsymbol{x}) = 1$ if $\boldsymbol{x}$ is in Ω_j and equal to 0 otherwise. The local mass m_j is defined, for all j in $\{1,\ldots,n_J\}$, by $m_j = \int_{\Omega_j} \rho(\boldsymbol{x})\,d\boldsymbol{x}$, where $\boldsymbol{x} \mapsto \rho(\boldsymbol{x})$ is the mass density. Let $\boldsymbol{u} \mapsto h^c(\boldsymbol{u})$ be the linear operator defined by

$$h^c(\boldsymbol{u}) = \boldsymbol{u} - h^r(\boldsymbol{u})\,. \tag{5}$$

Function $h^r(\boldsymbol{u})$ will also be denoted by $\boldsymbol{u}^r$ and function $h^c(\boldsymbol{u})$ by $\boldsymbol{u}^c$. We then have $\boldsymbol{u} = h^r(\boldsymbol{u}) + h^c(\boldsymbol{u})$ that is to say, $\boldsymbol{u} = \boldsymbol{u}^r + \boldsymbol{u}^c$. Let $[H^r]$ be the $(m \times m)$ matrix relative to the finite element discretization of the projection operator h^r defined by Eq. (4). Therefore, the finite element discretization $\mathbb{U}$ of $\boldsymbol{u}$ can be written as $\mathbb{U} = \mathbb{U}^r + \mathbb{U}^c$, in which

$$\mathbb{U}^r = [H^r]\,\mathbb{U}$$

and

$$\mathbb{U}^c = [H^c]\,\mathbb{U} = \mathbb{U} - \mathbb{U}^r \quad ,$$

which shows that $[H^c] = [I_m] - [H^r]$. Then, the reduced $(m \times m)$ mass matrix $[\mathbb{M}^r]$ is such that

$$[\mathbb{M}^r] = [H^r]^T [\mathbb{M}][H^r] \quad ,$$

and the complementary $(m \times m)$ mass matrix $[\mathbb{M}^c]$ is such that

$$[\mathbb{M}^c] = [H^c]^T [\mathbb{M}][H^c] \quad .$$

Using the properties of the projection operator defined by Eq. (4), it can be shown [Soize 2010] that

$$[\mathbb{M}^c] = [\mathbb{M}] - [\mathbb{M}^r] \quad .$$

2.3 Global and local displacements bases

There are two methods to calculate the global displacements basis and the local displacements basis. The first one is the direct method that will be used to reduce the matrix equation. In such a method, the basis of the global displacements and the basis of the local displacements are directly calculated using matrix $[\mathbb{M}^r]$. The second one, is the double projection. This method is less intrusive with respect to the commercial software and less time-consuming than the direct method. The global displacements eigenvectors $\boldsymbol{\phi}^g$ in $\mathbb{R}^m$ are solution of the following generalized eigenvalue problem

$$[\mathbb{K}]\boldsymbol{\phi}^g = \lambda^g [\mathbb{M}^r]\boldsymbol{\phi}^g\,. \tag{6}$$

The local displacements eigenvectors $\boldsymbol{\phi}^\ell$ in $\mathbb{R}^m$ are solution of the generalized eigenvalue problem

$$[\mathbb{K}]\boldsymbol{\phi}^\ell = \lambda^\ell [\mathbb{M}^c]\boldsymbol{\phi}^\ell\,. \tag{7}$$

The solutions of the generalized eigenvalue problems defined by Eqs. (6) and (7) are then written, for n sufficiently large, as

$$\boldsymbol{\phi}^g = [\Phi]\,\widetilde{\boldsymbol{\phi}}^g \quad , \quad \boldsymbol{\phi}^\ell = [\Phi]\,\widetilde{\boldsymbol{\phi}}^\ell\,, \tag{8}$$

in which $[\Phi]$, defined in Eq. (2), is the matrix of the elastic modes. The global displacements eigenvectors are the solutions of the generalized eigenvalue problem

$$[\widetilde{K}]\,\widetilde{\boldsymbol{\phi}}^g = \lambda^g\,[\widetilde{M}^r]\,\widetilde{\boldsymbol{\phi}}^g\,, \tag{9}$$

in which $[\widetilde{M}^r] = [\Phi^r]^T\,[\mathbb{M}]\,[\Phi^r]$ and $[\widetilde{K}] = [\Phi]^T\,[\mathbb{K}]\,[\Phi]$, and where the $(m \times n)$ real matrix $[\Phi^r]$ is such that $[\Phi^r] = [H^r]\,[\Phi]$. The local displacements eigenvectors are the solutions of the generalized eigenvalue problem

$$[\widetilde{K}]\,\widetilde{\boldsymbol{\phi}}^\ell = \lambda^\ell [\widetilde{M}^c]\,\widetilde{\boldsymbol{\phi}}^\ell\,, \tag{10}$$

in which $[\widetilde{M}^c] = [\Phi^c]^T\,[\mathbb{M}]\,[\Phi^c]$ and where the $(m \times n)$ real matrix $[\Phi^c]$ is such that $[\Phi^c] = [H^c]\,[\Phi] = [\Phi] - [\Phi^r]$. It is proven in (Soize and Batou, 2010) that the family $\{\boldsymbol{\phi}^g_1,\ldots,\boldsymbol{\phi}^g_{3n_J},\boldsymbol{\phi}^\ell_1,\ldots,\boldsymbol{\phi}^\ell_{m-3n_J}\}$ is a basis of $\mathbb{R}^m$. The mean reduced matrix model is obtained by the projection of $\mathbb{U}(\omega)$ on the family $\{\boldsymbol{\phi}^g_1,\ldots,\boldsymbol{\phi}^g_{n_g},\boldsymbol{\phi}^\ell_1,\ldots,\boldsymbol{\phi}^\ell_{n_\ell}\}$ of real vectors associated with the n_g first global displacements eigenvectors such that $n_g \leq 3n_J \leq m$ and with the n_ℓ first local displacements eigenvectors such that $n_\ell \leq m$. It should be noted that, if the double projection method is used, then we must have $n_g \leq n$, $n_\ell \leq n$ and $n_t \leq n$ in which $n_t = n_g + n_\ell$. Then, the approximation $\mathbb{U}_{n_g,n_\ell}(\omega)$ of $\mathbb{U}(\omega)$ at order (n_g,n_ℓ) is written as

$$\mathbb{U}_{n_g,n_\ell}(\omega) = \sum_{\alpha=1}^{n_g} q^g_\alpha(\omega)\,\boldsymbol{\phi}^g_\alpha + \sum_{\beta=1}^{n_\ell} q^\ell_\beta(\omega)\,\boldsymbol{\phi}^\ell_\beta\,. \tag{11}$$

This decomposition is then used to construct the generalized mass, stiffness and damping matrices which can be written in a block representation as

$$[\mathbb{M}] = \begin{pmatrix} M^{gg} & M^{gl} \\ M^{lg} & M^{ll} \end{pmatrix} , [\mathbb{D}] = \begin{pmatrix} D^{gg} & D^{gl} \\ D^{lg} & D^{ll} \end{pmatrix} , [\mathbb{K}] = \begin{pmatrix} K^{gg} & K^{gl} \\ K^{lg} & K^{ll} \end{pmatrix} . \tag{12}$$

2.4 Reduced-order model for the global displacements

The aim of this work is to construct a reduced-order model adapted to the low-frequency range in which the synthesis of the frequency responses can be obtained using only the global displacements eigenvectors. So the new approximation $\mathbb{U}_{n_g}(\omega)$ of $\mathbb{U}(\omega)$ at order n_g is written as

$$\mathbb{U}_{n_g}(\omega) = \sum_{\alpha=1}^{n_g} q_\alpha^g(\omega)\, \boldsymbol{\phi}_\alpha^g \,. \tag{13}$$

The corresponding reduced-order matrix equation is than written as

$$(-\omega^2[M^{gg}] + i\omega[D^{gg}] + [K^{gg}])\mathbf{g}^g = \mathbf{F}^g \quad . \tag{14}$$

2.5 Probabilistic model of uncertainties

A probabilistic model of uncertainties is introduced in the reduced-order computational model in order to take into account the system-parameter uncertainties and the model uncertainties induced by modeling errors in the reference model from which the reduced-order model has been deduced. We also have to take into account uncertainties induced by the irreducible errors introduced by neglecting the contribution of the local displacements in the constructed reduced-order model. To take into account all these sources of uncertainties, we use the nonparametric probabilistic approach (Soize, 2005) which consists in replacing, in the reduced-order computational model, the deterministic generalized mass, damping and stiffness matrices by random matrices. In this work, the uncertainties are not taken into account on the generalized damping matrix (it has previously been proven that the random frequency responses are not sensitive to the statistical fluctuations of the damping matrix in the framework of the nonparametric probabilistic approach). Therefore the matrices $[M^{gg}]$ and $[K^{gg}]$ are replaced by the random matrices $[\mathbf{M}^{gg}]$ and $[\mathbf{K}^{gg}]$ for which the probability density functions (PDF) and the generator of independent realizations are given in (Soize, 2005). The probability density functions of these two random matrices depend on two dispersion parameters ($\delta_{M^{gg}}$ and $\delta_{K^{gg}}$) which have to be identified using the random frequency response of the stochastic reference model and the maximum likelihood method. Therefore, the random frequency response of the stochastic reduced-order model ($\mathbf{U}^g(\omega;\delta_{M^{gg}};\delta_{K^{gg}})$) is solution of the equation

$$\mathbf{U}^g(\omega;\delta_{M^{gg}};\delta_{K^{gg}}) = \sum_{\alpha=1}^{n_g} \mathbf{Q}^\alpha(\omega;\delta_{M^{gg}};\delta_{K^{gg}})\boldsymbol{\phi}^\alpha \,, \tag{15}$$

$$(-\omega^2[\mathbf{M}^{gg}(\delta_{M^{gg}})] + i\omega[D^{gg}_{mod}] + [\mathbf{K}^{gg}(\delta_{K^{gg}})])\mathbf{Q}^g(\omega;\delta_{M^{gg}};\delta_{K^{gg}}) = F^g \,. \tag{16}$$

This equation is solved using the Monte Carlo simulation method.

3 APPLICATION TO AN AUTOMOTIVE VEHICLE

3.1 Presentation

We are interested in the frequency response of the structural part of an automotive vehicle for which the Finite Element model has 250 000 nodes and contains various types of finite elements such as volume finite elements, surface finite elements and beam elements. The frequency band of analysis is $\mathscr{B} =]0\,,120]\,Hz$. The structure has $1,462,698$ DOF.

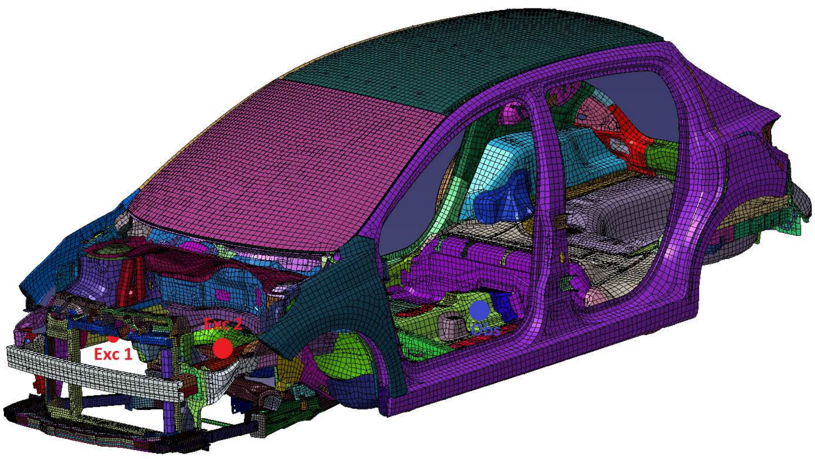

Figure 1 – The Finite Element model of an automotive vehicule

3.2 Decompostion of the domain

The subdomains are generated using the Fast Marching Method [Sethian 1996, Arnoux 2012] which allows fronts to be propagated from a set of starting points. This method is applied to the mesh of the structure of the automotive model. The centers of the subdomains and the subdomains obtained from these centers are represented in Fig. 2.

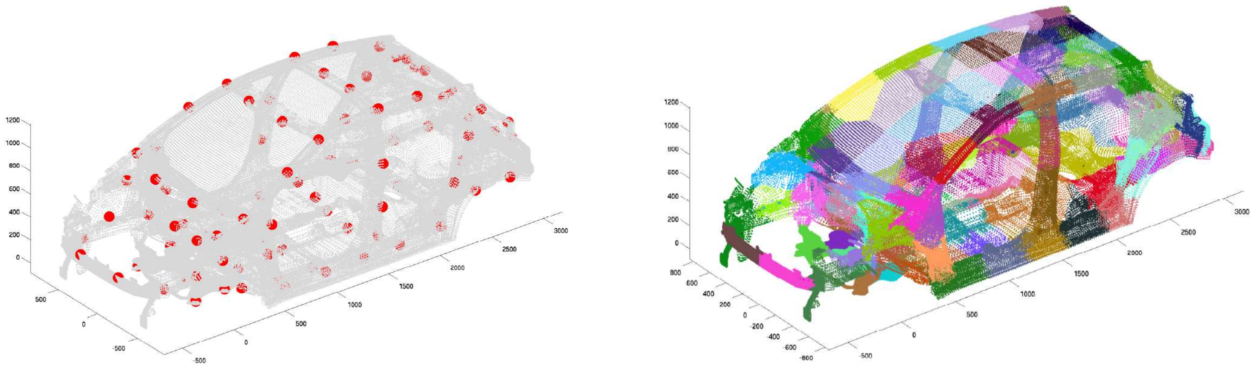

Figure 2 – Centers of the subdomains (left) and subdomains (right)

3.3 Elastic modes, global and local displacements eigenvectors

In a first step, the elastic modes are calculated with the finite element model. There are 160 eigenfrequencies in the frequency band of analysis $\mathscr{B}$. In a second step, the global and local displacements eigenvectors are constructed using the double projection method. In frequency band $]0\,,120]$ Hz, there are $n_g = 36$ global displacements eigenvectors and $n_\ell = 124$ local displacements eigenvectors. To see the good separation obtained between the global displacements eigenvectors and the local displacements eigenvectors, Fig. 3 displays the eleventh elastic mode (right figure) for which there are local displacements and the corresponding fourth global displacements eigenvector (left figure) for which the local displacements have been filtered.

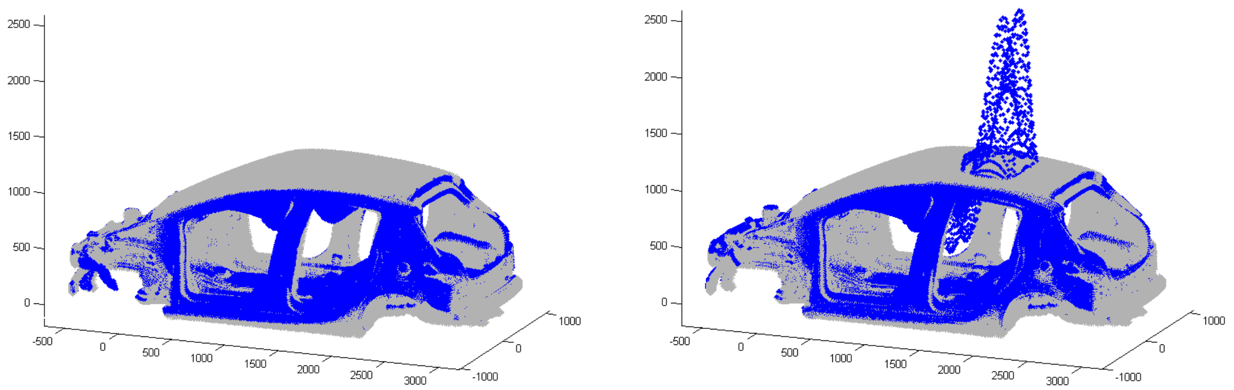

Figure 3 – Fourth global displacements eigenvector (left) and corresponding eleventh elastic mode (right).

3.4 Frequency response functions

For all $\omega \in \mathscr{B}$, the structure is subjected to an external point load equal to 1 N applied to two nodes, Exc1 and Exc2, located in the stiff part of the structure. The frequency response is calculated at one observation point, Obs_1, which is located in the stiff part (see Fig. 1). The modulus, in log scale, of the frequency response function is displayed in Fig. 4. It can be seen that the response calculated with the using the global displacement basis is very close to the reference response calculated with a classical frequency response analysis.

3.5 Random response

The stochastic reference computational model is constructed with the reference nominal computational model and using the non-parametric probabilistic approach of uncertainties as explained in [Durand 2008]. The values of the dispersion parameters δ_M and δ_K are those identified in [Durand 2008].

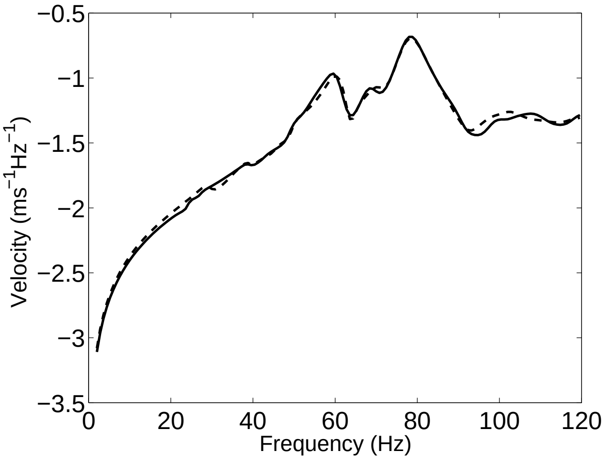

Figure 4 – Modulus, in log scale, of the frequency response function for Obs$_1$: reference (solid line), reduced-order model (dashed line).

All the calculations are carried out with the Monte Carlo simulation method for which 1, 000 independent realisations are used. The confidence regions corresponding to a probability level Pc = 0.95 have been calculated and are plotted in Fig. 5 (dark gray regions).

The random frequency response functions of the stochastic reduced-order model are also calculated with the Monte Carlo simulation method with 1, 000 independent realisations. The first step consists in calculating the optimal values of the dispersion parameters $\delta_{M^{gg}}$ and $\delta_{K^{gg}}$ using the maximum likelihood method. In a second step, for these optimal values of the dispersions parameters, the confidence regions corresponding to a probability level Pc = 0.95 have been calculated and are plotted in Fig. 5 (light gray regions). In the reduced-order model, there is an additional modeling error (with respect to the reference nominal computational model) induced by the projection which is performed only on the global displacements eigenvectors (the local displacements contributions for the prediction of the responses on the stiff part, in the Low-Frequency range, are neglected). Consequently, the level of uncertainties is larger in the reduced-order model than in the reference nominal computational model and therefore, the confidence regions predicted by the stochastic the reduced-order model must be larger than the confidence regions predicted by the stochastic reduced-order model. The validation is obtained if, for each observation, the confidence region computed with the stochastic reduced-order model is included in the confidence region computed with the stochastic reference nominal computational model, for most of the frequencies of band B, that is the case.

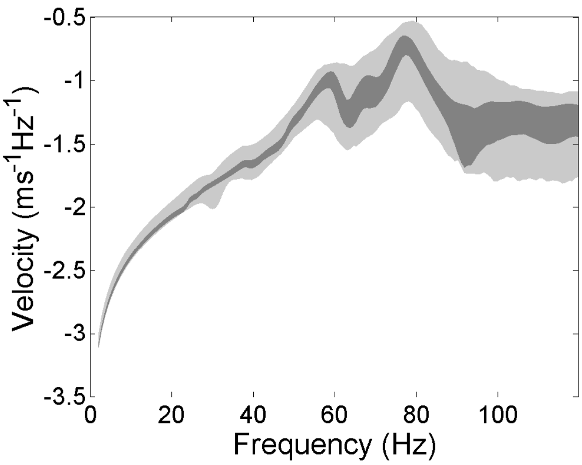

Figure 5 – Modulus, in log scale, of the random frequency response function. Confidence region (dark gray region) computed with the reference computational model. Confidence region (light gray region) computed with the stochatic reduced-order model.

4 APPLICATION TO FUEL ASSEMBLIES

In this Section, we present another industrial application of the methodology which consists in the dynamical analysis of fuel assembly of pressurized water reactor. For this application, only a deterministic ROM is constructed.

4.1 Reference computational model

A fuel assembly is a slender structure which is made up of 264 flexible fuel rods, 25 stiff guide tubes and 10 stiff grids which hold the tubes in position (see the finite element mesh in Fig. 6). The guide tubes are soldered to the grids while the fuel rods are fixed to the grids by springs. The longitudinal (vertical) direction is denoted by $\boldsymbol{z}$. The transverse directions are denoted by $\boldsymbol{x}$ and $\boldsymbol{y}$. The fuel rods and the guide tubes are modeled by Timoshenko beams and the grids are modeled by solid elements. The end of guide tubes are fixed to the containment building. All the displacements following $\boldsymbol{y}$-direction are set to zero. For a single fuel assembly, the finite element model has $44,844$ elements and $449,580$ DOFs. There are $7,364$ elastic modes in the band $[0\,,400]$ Hz. The eight first elastic modes are ensemble modes (all the structure moves in phase), the corresponding eigenfrequencies are 3.09 Hz, 6.31 Hz, 9.78 Hz, 13.5 Hz, 17.6 Hz, 22.2 Hz, 27.3 Hz and 32.7 Hz. Beyond these ensemble modes, there are numerous local elastic modes (only a part of the structure moves) and a few global elastic modes (all the structure moves but not in phase). The 2$^{\text{nd}}$ elastic mode (global) and the 20$^{\text{th}}$ elastic mode (local) are plotted in Fig. 7.

Figure 6 – Finite element mesh of a fuel assembly: Grids (black), fuel rods (blue) and guide tubes (red). Left figure: Complete fuel assembly. Right figure: Grids and guide tubes only.

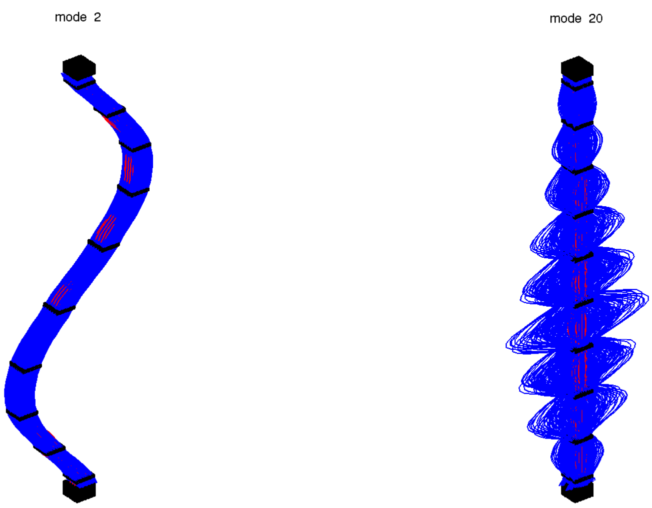

Figure 7 – Left: 2$^{\text{nd}}$ **elastic mode (global). Rigth:** 20$^{\text{th}}$ **elastic mode (local).**

4.2 Construction and validation of the reduced-order computational model

In this section, a single fuel assembly is considered. The first step consists in the construction of the subdomains Ω_j. Since we want to filter the local transverse displacements, the subdomains are chosen as 100 slides of equal thickness. The eigenvectors $\boldsymbol{\varphi}_j^g$ are then computed. In the frequency band $[0\,,400]$ Hz, there are 35 eigenvectors. The 9$^{\text{th}}$ eigenvector is plotted in Fig. 8. In the band $[0,400]$ Hz, the number of eigenvectors (35) is much lower than the number of elastic modes

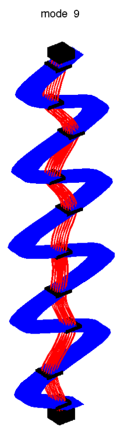

Figure 8 – 9$^{\text{th}}$ eigenvector.

$(7,364)$. A Rayleigh damping model is used and is constructed for the frequencies 3 Hz and 400 Hz with a damping ratio 0.04. A point load is applied to the node P_{exc} which is located at the middle of the 9$^{\text{th}}$ grid (from bottom to top). This load is equal to 1 N in the frequency band $[0\,,400]$ Hz following $\boldsymbol{x}$-direction. The containment building is fixed. The measurement node P_{obs} is located at the middle of the 4$^{\text{th}}$ grid. The frequency response functions at points P_{obs} and P_{exc} are plotted in Figs. 9 and 10. These figures show a very good accuracy of the reduced-order computational model in

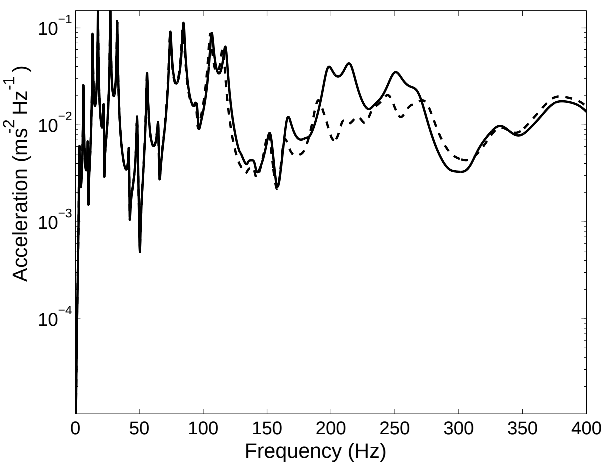

Figure 9 – Modulus of the frequency response function of the acceleration in $\boldsymbol{x}$-direction at point P_{obs}: reduced-order computational model (solid line) and reference computational model (dashed line).

the frequency band $[0\,,100]$ Hz. In the frequency band $[100\,,300]$ Hz, the accuracy of the reduced-order computational model is less. These small deviations are due to the local contributions in the neighborhood of the observation points, which are not taken into account when the basis of the global displacements space is used to construct the reduced-order computational model.

5 CONCLUSIONS

In this work, we have applied a new methodology allowing a reduced-order computational dynamical model to be constructed for the low-frequency domain in which there are simultaneously global and local elastic modes which cannot easily be separated with usual method. An associated stochastic reduced-order model has then been introduced to take

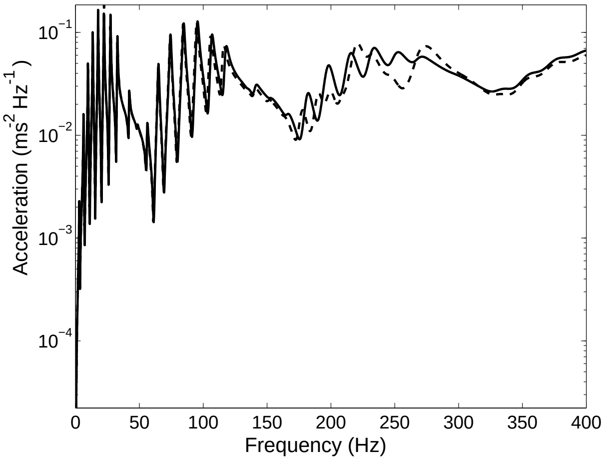

Figure 10 – Modulus of the frequency response function of the acceleration in *x*-direction at point $P_{\rm exc}$: reduced-order computational model (solid line) and reference computational model (dashed line).

into account uncertainties in the adapted reduced-order model. The results obtained are good with respect to the objectives fixed in this work consisting in constructing a reduced-order model with a very low dimension, which has the capability to predict the frequency responses in the low-frequency range.

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