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DYNAMIC AND ACOUSTIC RESPONSE OF COUPLED STRUCTURE/DENSE FLUID AXISYMMETRIC SYSTEMS EXCITED BY A RANDOM WALL PRESSURE FIELD

by

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ABSTRACT

A numerical method for predicting medium frequency random vibrations of coupled fluid/structure axisymmetric systems excited by a random wall pressure field is described. The structure is inhomogeneous, viscoelastic and linear and occupies a bounded domain. The internal inhomogeneous fluid is compressible and occupies a bounded domain. The external homogeneous fluid is compressible and inviscid. The structure and the internal fluid are analyzed by the finite element method and the external fluid by an integral equation method. An application is described, and the numerical predictions are compared with experimental results.

I. – INTRODUCTION

The method uses a numerical model to predict the stationary response in the medium frequency (MF) domain of dynamic axisymmetric coupled fluid/structure systems excited by a random wall pressure field. The structure is inhomogeneous with a linear viscoelastic behavior and occupies a bounded domain of space. It is coupled to an inviscid, compressible, homogeneous external fluid occupying an unbounded domain of space and to a compressible inhomogeneous internal fluid occupying a bounded domain of space. The fluids can have any density (light or heavy). The entire coupled system is axisymmetric. The excitation is a given which is a random wall pressure field, stationary in time, inhomogeneous in space, applied to part of the interface of the structure with the external fluid. It is axisymmetric, *i.e.* the probability law for this field and therefore all its moments are invariant by rotation around the axis of symmetry of revolution. The aim is to calculate the power spectra in the MF domain of the stationary random response of the coupled system, *i.e.* accelerations of the structure, pressures in the internal fluid and, possibly, pressures in the external fluid, *i.e.* noise radiated in the near field and the far field. The system can have any geometry in the generatrix plane. The same is true of the geometry of the internal acoustic cavity and the geometry of the structure/external fluid interface. The structure can have any boundary conditions.

Under these conditions, we use the finite element method to numerically analyze the structure and the internal fluid and a general integral equation method for the external fluid.

As it is the MF domain we are investigating, intermediate between the LF domain for which the modal density of the structure is very small and the HF domain for which it is very large, the number of degrees of freedom (DOF) of the discretized system is generally sufficiently large with respect to a model corresponding to the LF domain.

The state of the three-dimensional ($3D$) coupled system is obtained by $3D$ synthesis of the various states of the circumferential orders n of the Fourier series expansion along the polar angle θ of the cylindrical coordinates. In effect, although the excitation field is axisymmetric, any occurrence at time t is not an axisymmetric pressure field. The excitation field therefore has nonzero components on all circumferential orders n, and not only on $n=0$.

In the first part (Secs. II and III), we give the equations governing the coupled problem and indicate the numerical methods used to construct the frequency response function of the discretized coupled system. In the second part (Sec. IV), we construct the stationary solution. Finally, the third part (Sec. V) concerns an application for which the external and internal fluids are dense (water), the wall pressure field is due to the boundary layer turbulence of an external flow and the structure is a viscoelastic shell. We give the numerical predictions for a wide MF band and comparisons with experimental results.

NOTATIONS

For $x=(x_1, \ldots, x_n)$ and $y=(y_1, \ldots, y_n)$ in $\mathbb{R}^n$, we denote $\langle x, y \rangle = \sum_{j=1}^{n} x_j y_j$ and $\|x\|^2 = \langle x, x \rangle$. Considering $\mathbb{C}$ as the complexification of $\mathbb{R}$, we denote for x and y in $\mathbb{C}$:

$$(x, y)=\sum_{j=1}^{n} x_j \bar{y}_j = \langle x, \bar{y} \rangle \qquad \text{and} \qquad \|x\|^2=(x, x),$$

where $\bar{y}$ is the conjugate of y. Let $\mathbb{K}=\mathbb{R}$ or $\mathbb{C}$, n and m be two positive integers. We identify the vector space of linear mappings of $\mathbb{K}^n$ into $\mathbb{K}^m$ with the matrix $\mathrm{Mat}_{\mathbb{K}}(m, n)$ with dimensions (m, n) and whose elements are in $\mathbb{K}$, and space $\mathbb{K}^n$ with $\mathrm{Mat}_{\mathbb{K}}(n, 1)$. Let $Q \in \mathrm{Mat}_{\mathbb{C}}(n, m)$. We denote the transpose matrix of Q as Q^T, the conjugate matrix as $\bar{Q}$ and the adjoint matrix as $Q^* = \bar{Q}^T$. Let $L^2(\mathbb{R}, \mathbb{C}^m)$ be the space of functions $t \rightarrow U(t)=(U_1(t), \ldots, U_m(t))$ defined dt-almost everywhere on $\mathbb{R}$ with values in $\mathbb{C}^m$, whose square is integrable, equipped with the scalar product:

$$((U, V))=\int_{\mathbb{R}} (U(t), V(t))\, dt,$$

and the associated norm $|||U|||=((U, U))^{1/2}$. For any U in $L^2(\mathbb{R}, \mathbb{C}^m)$, the Fourier transform (FT) of U is the mapping $w \rightarrow (\mathscr{F} U)(w)=\hat{U}(w)$ belonging to $L^2(\mathbb{R}, \mathbb{C}^m)$ such that for almost any w in $\mathbb{R}$:

$$\hat{U}_j(w)=\int_{\mathbb{R}} \exp(-iwt)\, U_j(t)\, dt, \qquad j \in (1, \ldots, m).$$

For almost every t in $\mathbb{R}$, the inverse FT is given by:

$$U_j(t)=(2\pi)^{-1} \int_{\mathbb{R}} \exp(iwt)\, \hat{U}_j(w)\, dw,$$

$$j \in (1, \ldots, m).$$

In the case of a quantity f depending on x and t, the notation $\hat{f}$ always designates the partial FT with respect to t.

II. – FREQUENCY RESPONSE FUNCTIONS FOR 3 D COUPLED SYSTEMS

First we give the formulation for a sufficiently general $3D$ case. The necessary additions for axisymmetric systems are given in Section III.

II,1. – GEOMETRY OF THE COUPLED SYSTEM

The physical space $\mathbb{R}^3$ is referred to a cartesian reference system $Ox_1x_2x_3$ and we denote as $x=(x_1, x_2, x_3)$ the generic point of $\mathbb{R}^3$ (Fig. 1).

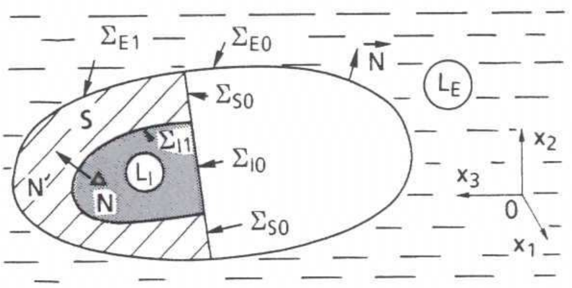


Fig. 1. – Geometry of the coupled system. General case.

Let Ω be a bounded open domain of $\mathbb{R}^3$, simply connected with regular boundary $\sum_E$ (C^0 and C^1 by parts). The structure and the internal fluid are contained in Ω.

(1) The external fluid occupies the domain external to Ω, *i.e.* the open unbounded domain $L_E=\mathbb{R}^3\setminus\bar{\Omega}$ of $\mathbb{R}^3$ with boundary $\sum_E=\sum_{E0}\cup\sum_{E1}$ where $\sum_{E0}\cap\sum_{E1}=\varnothing$.

Below, part $\sum_{E0}$ is undeformable.

(2) The structure is a continuum which occupies the open bounded domain S of $\mathbb{R}^3$ contained in Ω, with regular boundary $\partial S=\sum_{S0}\cup\sum_{S1}$, with $\sum_{S0}\cap\sum_{S1}=\varnothing$, where $\sum_{S0}$ is the part on which the structural displacements are zero. In addition $\sum_{E1}$ is a part of $\sum_{S1}$, the structure/external fluid being coupled through $\sum_{E1}$.

(3) The internal fluid occupies a domain denoted L_I which is an open bounded domain of $\mathbb{R}^3$ contained in $\Omega\setminus S$, with boundary $\sum_I=\sum_{I0}\cup\sum_{I1}$ with $\sum_{I0}\cap\sum_{I1}=\varnothing$, where $\sum_{I0}$ is the part of $\sum_I$ which is undeformable. The structure/internal fluid is coupled through $\sum_{I1}$. In addition, $\sum_I\cap\sum_E=\varnothing$ and $\sum_{I1}$ is a part of $\sum_{S1}$. To simplify the description, we will assume that $\sum_{S1}=\sum_{E1}\cup\sum_{I1}$.

(4) Finally, we denote as $N=(N_1, N_2, N_3)$ the unit normal to $\sum_S$, external to S and to $\sum_E$ external to Ω. We denote as N' the unit normal to $\sum_I$ external to L_I. Therefore, $N'=-N$ on $\sum_{I1}$.

II,2. – EQUATIONS OF THE COUPLED SYSTEM VIBRATIONS

The equations are written in the Fourier domain for variable t and are relative to the linear vibrations of the coupled system around a stable equilibrium state taken as reference state. To simplify the description, we use the $3D$ linear elastodynamic formulation for the structure with a single field. In practice, and for reasons of modeling, we are generally led to consider the structure as a union of $1D$, $2D$ and $3D$ media.

In the present case, the structure is a continuum S whose displacement field is denoted

$$x,\ t\mapsto u(x,t): S\times\mathbb{R}\rightarrow\mathbb{R}^3$$

(1) *External fluid:* the external fluid L_E is inviscid, compressible, homogeneous, with density ρ_E and acoustic velocity a_E (acoustic fluid). Let v_E, φ_E and p_E be the velocity, velocity potential and pressure fields. Herein, we assumed that there is no source term in the external fluid. For $x\in\bar{L}_E=L_E\cup\sum_E$ and $w\in\mathbb{R}$, we have:

$$\hat{v}_E=\operatorname{grad}\hat{\varphi}_E,\qquad \hat{p}_E=-iw\,\rho_E\,\hat{\varphi}_E. \tag{1}$$

Potential $\hat{\varphi}_E$ is the solution of the external Neumann problem:

$$\left.\begin{aligned}&\Delta\hat{\varphi}_E+\frac{w^2}{a_E^2}\hat{\varphi}_E=0\quad\text{in } L_E\\&\frac{\partial\hat{\varphi}_E}{\partial N}=\hat{g}_E(\hat{u})\quad\text{on }\textstyle\sum_E\\&\left|\frac{\partial\hat{\varphi}_E}{\partial r}-i\frac{w}{a_E}\hat{\varphi}_E\right|=\mathcal{O}(r^{-2}),\end{aligned}\right\} \tag{2}$$

$$|\hat{\varphi}_E|=\mathcal{O}(r^{-1}),\qquad r=\|ox\|\rightarrow+\infty$$

where $\hat{g}_E(\hat{u})=0$ on $\sum_{E0}$ and $\hat{g}_E(\hat{u})=iw\langle \hat{u}|_{\Sigma_{E1}}, N\rangle$ on $\sum_{E1}$, where $\hat{u}|_{\Sigma_{E1}}$ is the trace of $\hat{u}$ on $\sum_{E1}$.

(2) *Internal fluid:* the internal fluid L_I is compressible, with constant density ρ_I and acoustic velocity a_I. It is assumed that there is no source term in the internal fluid and we denote as p_I the pressure field on $\bar{L}_I \times \mathbb{R}$ with $\bar{L}_I = L_I \cup \sum_I$. We assume the fluid to be governed by the equations established in [27, 57], where $\hat{\varphi}_I$ is the solution of the internal Neumann problem:

$$\left.\begin{aligned} &\hat{p}_I = -iw\rho_I\hat{\varphi}_I \quad \text{in } \bar{L}_I \\ &\operatorname{div}((1+iw\lambda_I)\operatorname{grad}\hat{\varphi}_I) + \frac{w^2}{a_I^2}\hat{\varphi}_I = 0 \quad \text{in } L_I \\ &(1+iw\lambda_I)\frac{\partial\hat{\varphi}_I}{\partial N'} = \hat{g}_I(\hat{u}) \quad \text{on } \sum_I \end{aligned}\right\} \quad (3)$$

where $\hat{g}_I(\hat{u})=0$ on $\sum_{I0}$ and $\hat{g}_I(\hat{u})=iw\langle \hat{u}|_{\Sigma_{I1}}, N'\rangle$ on $\sum_{I1}$, where $\hat{u}|_{\Sigma_{I1}}$ is the trace of $\hat{u}$ on $\sum_{I1}$. Coefficient λ_I is real and depends on x and w For $\lambda_I \equiv 0$, we have a conventional inviscid compressible fluid (acoustic fluid). For certain models, it is advantageous to have an inhomogeneous dissipation term depending on the frequency to simulate loss phenomena in the neighborhood of walls of the acoustic cavity or in certain parts of the fluid domain L_I.

(3) *Structure:* the structure is an inhomogeneous, anisotropic, linear viscoelastic solid continuum S with density ρ_S. The components of the stress tensor are:

$$\sigma_{jk}(\hat{u}) = a_{jkhi}(x, w)\,\varepsilon_{hi}(\hat{u}), \qquad j, k, h, i \in \{1, 2, 3\},$$

with the Einstein convention for index summing, where $\varepsilon_{jk}(\hat{u}) = (\partial_k \hat{u}_j + \partial_j \hat{u}_k)/2$ is the strain tensor and where $a_{jkhi}(x, w)$ are elasticity constants which verify the usual symmetry properties, are functions of x, depend on w and have values in $\mathbb{C}$ [9, 32, 34, 57, 62]. By hypothesis, there is no volume force applied to S. The structure, coupled with the internal and external fluids, is governed by the following equations for any w in $\mathbb{R}$, $j \in \{1, 2, 3\}$::

$$\left.\begin{aligned} &-w^2\rho_S\hat{u}_j - \partial_k\sigma_{jk}(\hat{u}) = 0 \quad \text{in } S \\ &\sigma_{jk}N_k = -(\hat{p}_{E1}+\hat{p}_{I1})N_j + \hat{f} \quad \text{on } \sum_{S1} \\ &\hat{u}_j = 0 \quad \text{on } \sum_{S0} \end{aligned}\right\} \quad (4)$$

where $x, t \mapsto f(x, t): \sum_{S1} \times \mathbb{R} \to \mathbb{R}$ is a given external force field applied to the structure and $x, w \mapsto \hat{p}_{I1}(x, w)$, (resp. $x, w \mapsto \hat{p}_{E1}(x, w)$) is the function of $\sum_{S1} \times \mathbb{R}$ in $\mathbb{C}$, with support $\sum_{I1}$ (resp. $\sum_{E1}$), and such that for any w in $\mathbb{R}$, $\hat{p}_{I1}$ (resp. $\hat{p}_{E1}$) is equal to the trace of $\hat{p}_I$ on $\sum_{I1}$ (resp. $\hat{p}_E$ on $\sum_{E1}$).

II,3. – EQUATIONS FOR THE VIBRATIONS OF THE DISCRETIZED COUPLED SYSTEM

The state of the discretized coupled system is represented by the three fields $\hat{u}$, $\hat{\varphi}_E$ and $\hat{\varphi}_I$, which are solutions of the coupled problem (2)-(3)-(4). For any w in $\mathbb{R}$, problem (2) has a unique solution [45, 49, 55, 57] which can be written $\hat{\varphi}_E = iw\mathscr{B}_w(\hat{g}_E(\hat{u}))$ where $\mathscr{B}_w$ is a linear operator which can be characterized by an integral equation formulation on surface $\sum_E$ [1, 37]. $\hat{p}_{E1}$ can then be expressed as a function of $\hat{u}$ in (4). This yields a symmetric (but not Hermitian) variational formulation of the problem (3)-(4) where the only unknown fields are $\hat{u}$ and $\hat{\varphi}_I$ and which involves the operator $\mathscr{B}_w$ [27, 49, 55, 57]. This problem in $\hat{u}$, $\hat{\varphi}_I$ (and therefore the problem (2)-(3)-(4) in $\hat{u}$, $\hat{\varphi}_I$, $\hat{\varphi}_E$) has, for any w in $\mathbb{R}$, a unique solution which can be analyzed.

A finite dimension approximation can then be constructed. As the internal fluid is inhomogeneous, it and the structure are analyzed by the finite element method [13, 33, 64]. To construct an approximation of $\mathscr{B}_w$ with a finite rank, surface $\sum_E$ is meshed with finite elements which are compatible on $\sum_{E1}$ with the finite elements of structure S. Let m_S and m_I be the number of DOFs of the models of the structure and of the internal fluid L_I respectively. We denote as $U(t) \in \mathbb{R}^{m_S}$, $F(t) \in \mathbb{R}^{m_S}$ and $\Phi_I(t) \in \mathbb{R}^{m_I}$, the nodal unknowns of the structure, the external nodal forces applied to the structure and the nodal unknowns of the internal fluid due to the finite approximation in space of fields u, f and φ_I. Then, for any w in $\mathbb{R}$, the equations of the discretized coupled system are the matrix equation on $\mathbb{C}^m$, $m = m_S + m_I$ which is written by blocks:

$$\left\{-w^2\begin{bmatrix} M_S + M_E(w) & 0 \\ 0 & -M_I \end{bmatrix} + iw\begin{bmatrix} C_S(w) + C_E(w) & G \\ G^T & -C_I(w) \end{bmatrix} + \begin{bmatrix} K_S(w) & 0 \\ 0 & -K_I \end{bmatrix}\right\}\begin{bmatrix} \hat{U} \\ \hat{\Phi}_I \end{bmatrix} = \begin{vmatrix} \hat{F} \\ 0 \end{vmatrix} \quad (5)$$

where:

(1) For the structure: M_S, $C_S(w)$ and $K_S(w) \in \mathrm{Mat}_{\mathbb{R}}(m_S, m_S)$ are the mass, damping and stiffness matrices which are positive-definite symmetric [6, 7, 14, 48, 67]. The dependency on w of C_S

and K_S is due to the presence of viscoelastic materials [9, 57, 68].

(2) For the internal fluid:

$$M_I,\ C_I(w),\ K_I \in \mathrm{Mat}_{\mathbb{R}}(m_I, m_I)$$

are the mass, internal dissipation (term in λ_I) and stiffness matrices which are symmetric, M_I and K_I being positive-definite and $C_I(w)$ being nonnegative. Matrix C_I depends on w via λ_I and is not a priori positive-definite, since λ_I can be zero in certain parts of domain L_I. Matrix $G \in \mathrm{Mat}_{\mathbb{R}}(m_S, m_I)$ is the coupling matrix with the structure [27, 57, 68]. It can be noted that all the signs of the second equation (5) (that in $\hat{\Phi}_I$) have been changed. This makes it possible to have a symmetrical formulation for the coupled problem. In addition, the formulation chosen for the coupled internal fluid introduces only one DOF per node.

(3) For the external fluid: $M_E(w)$ and $C_E(w) \in \mathrm{Mat}_{\mathbb{R}}(m_S, m_S)$ are the added mass matrix and the matrix of dissipation by radiation to infinity. They are symmetric and nonnegative. Numerical methods and general programs [25, 26] were developed to construct $M_E(w)$ and $C_E(w)$ as well as radiation in the fluid L_E.

(*a*) For a surface of revolution $\sum_E$, or any 3D surface, an integral equation formulation is used [1, 37]. This formulation was validated for the LF domain [20, 25, 37, 39], and for the MF domain [21, 22, 23, 25, 40, 55, 56, 57].

(*b*) Where $\sum_E$ is a tapered surface of revolution, an asymptotic method was developed. It allows a decrease in the numerical costs. The theory is developed and validated in [17, 50] for the LF domain and in [11, 52] for the MF domain.

II,4. – FREQUENCY RESPONSE FUNCTION OF THE DISCRETIZED SYSTEM

We set $V(t)=\{U(t), \Phi_I(t)\} \in \mathbb{R}^m$ $\mathbb{F}(t)=\{F(t), 0\} \in \mathbb{R}^m$, $m=m_s+m_I$. Equation (5) is written $Z(w)\hat{V}(w)$ $\mathcal{O} \times \mathbb{R}^d$, where $Z(w) \in \mathrm{Mat}_{\mathbb{C}}(m, m)$ is the impedance of the discrete coupled system. In the usual framework of elastodynamics of solid media and subject to hypotheses on x, $w \mapsto \lambda_I(x, w)$ which are not restrictive for the applications, it is determined from the variational formulation that for any w in $\mathbb{R}$, matrix $Z(w)$ is symmetric, non-Hermitian, inversible, and that the associated frequency response function $w \mapsto H(w)=Z(w)^{-1}$ is a bounded function of $\mathbb{R}$ in $\mathrm{Mat}_{\mathbb{C}}(m, m)$, $H(w)$ being symmetric and non-Hermitian. Under these conditions, for any $\mathbb{F} \in L^2(\mathbb{R}, \mathbb{R}^m)$, the solution of (5) is a function $V \in L^2(\mathbb{R}, \mathbb{R}^m)$ such that:

$$\hat{V}(w)=H(w)\hat{\mathbb{F}}(w), \qquad \forall w \in \mathbb{R}. \tag{6}$$

A combined time-frequency numerical method with two scales was developed to construct function $w \mapsto H(w)$ over a wide medium frequency band for a reasonable numerical cost, knowing that m is very large in the MF models [57]. This method is implemented in hydroelastoacoustic software systems [25, 26, 68].

III. – FREQUENCY RESPONSE FUNCTION FOR AN AXISYMMETRIC COUPLED SYSTEM EXCITED BY A WALL PRESSURE FIELD

III,1. – GEOMETRY, SURFACE Σ AND AXIS SYSTEMS

(1) *Geometry:* We consider the coupled system described in Section II,1 but whose geometry and mechanical properties are axisymmetric with an axis of revolution Ox_3.

(2) *Cylindrical coordinates*: With the cartesian coordinates of point $x=(x_1, x_2, x_3)$ are associated cylindrical coordinates $x=(\theta, r, z)$ such that $x_1=-r\sin\theta$, $x_2=r\cos\theta$, $x_3=z$. The origin and orientation of polar angle θ are indicated in Figure 2.

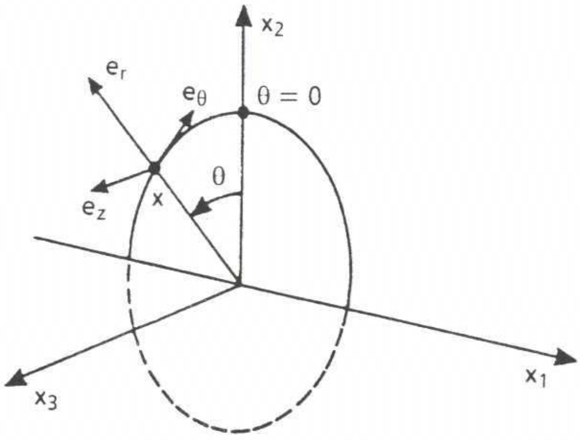


Fig. 2. – Cylindrical coordinates and local cylindrical axis system.

The generatrix plane $\theta=0$ is therefore Ox_2x_3. The local cylindrical reference system is the direct orthonormal reference system (e_θ, e_r, e_z) attached to point $x=(\theta, r, z)$.

(3) *Surface* Σ: the wall pressure field p (excitation of the system) is applied to the surface $\Sigma \subset \sum_{E1}$ defined by:

$$\Sigma=\{x \mid \theta \in]0, 2\pi[,\ r=R(z),\ z \in]z_0, z_1[\} \tag{7}$$

where $R \in C^1(]z_0, z_1[, \mathbb{R}^+)$ We denote as s the curvilinear abscissa of the generatrix of Σ whose origin is the point $(0, R(z_0), z_0)$, and which points positively in the direction of increasing z. The curvilinear measure ds on the generatrix of Σ and the surface measure $d\Sigma$ of Σ are expressed as a function of the parameterizing:

$$ds = (1 + R'(z)^2)^{1/2}\, dz; \qquad d\Sigma = R(z)\, d\theta\, ds(z), \quad (8)$$

where $R'(z) = dR(z)/dz$.

(4) *Local physical reference system on* Σ: in any point x of Σ, we define a local physical direct orthonormal reference system (b_1, b_2, b_3) attached to point x such that $b_1 = e_\theta$, $b_2 = N$, $b_3 = e_\theta \wedge N$ where N always denotes the unit normal to $\sum_E$, and therefore to Σ, external to Ω (Sec. II,1). The orthogonal (3×3) matrix used for transition from the local physical reference system to the local cylindrical reference system depends only on z and is written:

$$\left.\begin{aligned} \mathscr{R}(z) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha(z) & \beta(z) \\ 0 & -\beta(z) & \alpha(z) \end{bmatrix}; \\ \alpha(z) &= (1 + R'(z)^2)^{-1/2}; \qquad \beta(z) = R'(z)\,\alpha(z). \end{aligned}\right\} \quad (9)$$

III,2. – FOURIER SERIES EXPANSION OF THE FIELDS

Problem (2)-(3)-(4) is expressed in cylindrical coordinates and the components of the structure displacement field u are expressed in the local cylindrical reference system. We denote as $\mathscr{U}(\theta, r, z, t) \in \mathrm{Mat}_{\mathbb{R}}(3, 1)$ the column matrix of components u_θ, u_r and u_z of u on (e_θ, e_r, e_z). For any fixed r, z and t, fields $\theta \mapsto u(\theta)$, $\varphi_E(\theta)$, $\varphi_I(\theta)$, $p_I(\theta)$ and $p_E(\theta)$ are periodic, with period 2π. The Fourier series expansions of these fields with respect to θ are expressed:

– For the internal and external fluids, if q denotes any one of fields φ_E, φ_I, p_E or p_I with scalar values, we have:

$$q(\theta, r, z, t) = \sum_{n \geqq 0} \{q_S^{(n)}(r, z, t) \cos n\theta + q_{AS}^{(n)}(r, z, t) \sin n\theta\}. \quad (10)$$

– for the structure, we have:

$$\mathscr{U}(\theta, r, z, t) = \sum_{n \geqq 0} \{Q_S(n\theta)\, \mathscr{U}_S^{(n)}(r, z, t) + Q_{AS}(n\theta)\, \mathscr{U}_{AS}^{(n)}(r, z, t)\} \quad (11)$$

where:

$$Q_S(n\theta) = \begin{bmatrix} \sin n\theta & 0 & 0 \\ 0 & \cos n\theta & 0 \\ 0 & 0 & \cos n\theta \end{bmatrix};$$

$$Q_{AS}(n\theta) = \begin{bmatrix} \cos n\theta & 0 & 0 \\ 0 & -\sin n\theta & 0 \\ 0 & 0 & -\sin n\theta \end{bmatrix} \quad (12)$$

where

$$\{q_S^{(n)}(r, z, t) \in \mathbb{R},\ \mathscr{U}_S^{(n)}(r, z, t) \in \mathrm{Mat}_{\mathbb{R}}(3, 1)\}$$

and $\{q_{AS}^{(n)}(r, z, t) \in \mathbb{R},\ \mathscr{U}_{AS}^{(n)}(r, z, t) \in \mathrm{Mat}_{\mathbb{R}}(3, 1)\}$ are the symmetric and antisymmetric parts respectively for the circumferential index $n \in \mathbb{N}$.

Considering the particular choice of the physical local reference system on Σ, we first verify that for any x on Σ, representation (11) is invariant if the components of field u are expressed in the local physical reference system, since we have:

$$\begin{aligned} &\mathscr{R}(z)^T Q_S(n\theta)\, \mathscr{R}(z) = Q_S(n\theta); \\ &\mathscr{R}(z)^T Q_{AS}(n\theta)\, \mathscr{R}(z) = Q_{AS}(n\theta). \end{aligned} \quad (13)$$

III,3. – FREQUENCY RESPONSE FUNCTION OF THE DISCRETIZED COUPLED SYSTEM

The methodology is the same as that described in Sections II,3 and II,4. The 1D and 2D finite element meshes concern only the generatrix plane of the coupled external fluid/internal fluid/structure system. Let $m' = m'_S + m'_I$ be the total number of axisymmetric DOFs introduced in the model of the structure and the internal fluid. The nodes of the structure mesh on the generatrix of Σ are equipped with the local physical reference system. The force field applied to the structure is written:

$$x, t \mapsto f(x, t) = -p(x, t)\, \mathbf{1}_\Sigma(x)\, N(x) \quad (14)$$

and therefore has a nonzero component only along b_2 of the local physical reference system.

Let $N < m'$ be the number of DOFs of the structure relative to the generatrix of Σ. For any $n \in \mathbb{N}$, equation (6) gives:

$$\begin{aligned} &\hat{V}_S^{(n)}(w) = H^{(n)}(w)\, \hat{F}_S^{(n)}(w); \\ &\hat{V}_{AS}^{(n)}(w) = H^{(n)}(w)\, \hat{F}_{AS}^{(n)}(w) \end{aligned} \quad (15)$$

where $V_S^{(n)}(t)$ and $V_{AS}^{(n)}(t)$ in $\mathbb{R}^{m'}$ (or $F_S^{(n)}(t)$ and $F_{AS}^{(n)}(t)$ in $\mathbb{R}^N$) are the nodal unknowns of the structure and the internal fluid (or the external applied nodal forces) due to the finite approximation of the symmetric part and the antisymmetric part of fields $\{u, \varphi_I\}$ (or of field f) for the circumferential index n. The frequency response matrix $H^{(n)}(w) \in \mathrm{Mat}_{\mathbb{C}}(m', N)$ is constructed for each n with the general programs mentioned in Secs. II,3 and II,4.

Let $W(t)=(W_1(t), \ldots, W_L(t))$, be the L DOFs observed of the 3D system. We have:

$$\hat{W}(w)=\sum_{n \geq 0}\{T_S^{(n)} \hat{V}_S^{(n)}(w)+T_{AS}^{(n)} \hat{V}_{AS}^{(n)}(w)\} \quad (16)$$

where $T_S^{(n)}$ and $T_{AS}^{(n)} \in \mathrm{Mat}_{\mathbb{R}}(L, m')$ are real constant matrices constructed very simply from equations (10) and (11). For the numerical computations, we can only preserve a finite subset $\mathcal{N}$ of $\mathbb{N}$ in sum (16). The contents of this set $\mathcal{N}$ are related to each problem analyzed and the convergence with respect to n must systematically be checked numerically. Finally, using (15) and (16) yields the desired frequency response function of the 3D coupled system:

$$\hat{W}(w)=\sum_{n \in \mathcal{N}}\{\mathbb{H}_S^{(n)}(w) \hat{F}_S^{(n)}(w)+\mathbb{H}_{AS}^{(n)}(w) \hat{F}_{AS}^{(n)}(w)\} \quad (17)$$

where $\mathbb{H}_S^{(n)}(w)$ and $\mathbb{H}_{AS}^{(n)}(w)$ are the matrices of $\mathrm{Mat}_{\mathbb{C}}(L, N)$ such that:

$$\mathbb{H}_S^{(n)}(w)=T_S^{(n)} H^{(n)}(w), \quad \mathbb{H}_{AS}^{(n)}(w)=T_{AS}^{(n)} H^{(n)}(w). \quad (18)$$

As in Section II,4, functions $\mathbb{H}_S^{(n)}$ and $\mathbb{H}_{AS}^{(n)}$ are bounded functions of $\mathbb{R}$ in $\mathrm{Mat}_{\mathbb{C}}(L, N)$

III,4. – EXPRESSION OF THE NODAL FORCES DUE TO THE WALL PRESSURE FIELD

Considering the use of the local physical reference system on Σ, the work of the force field f defined by (14) is expressed:

$$\mathscr{T}(t)=\int_\Sigma \langle f(x, t), u(x, t)\rangle d\Sigma(x)$$

$$=-\int_{z_0}^{z_1}\int_0^{2\pi} p(\theta, z, t)\, u_2(\theta, z, t)\, R(z)\, d\theta\, ds(z) \quad (19)$$

where, to simplify the expression, we set $p(\theta, z, t)=p(\theta, R(z), z, t)$ and

$$u_2(\theta, z, t)=u_2(\theta, R(z), z, t)=\langle u(x, t), b_2(x)\rangle.$$

Considering (11) and (13), we have:

$$u_2(\theta, z, t)=\sum_{n \in \mathcal{N}}\{u_{S,2}^{(n)}(z, t)\cos n\theta - u_{AS,2}^{(n)}(z, t)\sin n\theta\}. \quad (20)$$

Substituting (20) in (19) yields:

$$\left.\begin{aligned}
&\mathscr{T}(t)=\sum_{n \in \mathcal{N}}\{\mathscr{T}_S^{(n)}(t)+\mathscr{T}_{AS}^{(n)}(t)\}\\
&\mathscr{T}_S^{(n)}(t)=-\int_{z_0}^{z_1}\int_0^{2\pi} p(\theta, z, t)\, u_{S,2}^{(n)}(z, t) \times \cos n\theta\, R(z)\, d\theta\, ds(z)\\
&\mathscr{T}_{AS}^{(n)}(t)=\int_{z_0}^{z_1}\int_0^{2\pi} p(\theta, z, t)\, u_{AS,2}^{(n)}(z, t) \times \sin n\theta\, R(z)\, d\theta\, ds(z).
\end{aligned}\right\} \quad (21)$$

But in addition, for $I \in \{S, AS\}$, we have:

$$\mathscr{T}_I^{(n)}(t)=\int_{z_0}^{z_1}\langle f_I^{(n)}(z, t), u_I^{(n)}(z, t)\rangle ds(z)$$

$$=\int_{z_0}^{z_1} f_{I,2}^{(n)}(z, t)\, u_{I,2}^{(n)}(z, t)\, ds(z). \quad (22)$$

Identification of (21) and (22) yields, for any n in $\mathcal{N}$ and z in $]z_0, z_1[$:

$$\left.\begin{aligned}
&f_{S,2}^{(n)}(z, t)=-R(z)\int_0^{2\pi} p(\theta, z, t)\cos n\theta\, d\theta\\
&f_{AS,2}^{(n)}(z, t)=R(z)\int_0^{2\pi} p(\theta, z, t)\sin n\theta\, d\theta.
\end{aligned}\right\} \quad (23)$$

Let $\varphi_1, \ldots, \varphi_N$ be a basis of interpolation relative to the finite element mesh of the generatrix of surface Σ. Functions $z \mapsto \varphi_j(z)$ are defined dz-almost everywhere on $]z_0, z_1[$ with values in $\mathbb{R}$, integrable on $]z_0, z_1[$ with respect to $ds(z)$. Then, the components of $F_S^{(n)}(t)$ and $F_{AS}^{(n)}(t)$, which are defined by:

$$F_{I,j}^{(n)}(t)=\int_{z_0}^{z_1} f_{I,2}^{(n)}(z, t)\, \varphi_j(z)\, ds(z), \quad I \in \{S, AS\}, \quad (24)$$

are written, for $j \in \{1, \ldots, N\}$:

$$\left.\begin{aligned}
&F_{S,j}^{(n)}(t)=-\int_{z_0}^{z_1}\int_0^{2\pi} p(\theta, z, t)\cos n\theta\, d\theta\, d\mu_j(z)\\
&F_{AS,j}^{(n)}(t)=\int_{z_0}^{z_1}\int_0^{2\pi} p(\theta, z, t)\sin n\theta\, d\theta\, d\mu_j(z)
\end{aligned}\right\} \quad (25)$$

where:

$$d\mu_j(z)=\varphi_j(z)\, R(z)\, ds(z). \quad (26)$$

IV. – STATIONARY RANDOM VIBRATIONS OF THE AXISYMMETRIC COUPLED SYSTEM SUBJECTED TO AN AXISYMMETRIC FIELD

The (external fluid/internal fluid/structure) axisymmetric coupled system considered in Section III is subjected to a random wall pressure field on Σ, stationary in time. It is attempted to construct the second-order characteristics of the stationary response, *i.e.* the spectra on a wide MF band [53, 57].

IV,1. – MODELING OF THE EXCITATION

Let $x=(\theta, R(z), z)$ be a point of Σ. The wall pressure field applied to Σ, $p(x,t)$, $x \in \Sigma t \in \mathbb{R}$ is a stochastic field defined on a probabilistic space indexed on $\Sigma \times \mathbb{R}$ with values in $\mathbb{R}$ centered, second-order, stationary in quadratic mean for variable t [18, 28, 35, 41, 51]. We denote the mathematical expectation as E. The mean function

$$x, t \mapsto E(p(x, t))$$

of this field is therefore zero and its cross autocorrelation function defined on $\Sigma \times \Sigma \times \mathbb{R}$ is written:

$$R_p(x, x', \tau) = E(p(x, t+\tau)\, p(x', t)). \tag{27}$$

As the field is second order, we have:

$$E(p(x, t)^2) = R_p(x, x, 0) < +\infty, \quad \forall t \in \mathbb{R}, \quad \forall x \in \Sigma. \tag{28}$$

Since $p(x, t)$ has values in $\mathbb{R}$, R_p verifies the following property:

$$R_p(x, x', -\tau) = R_p(x', x, \tau), \quad \forall x, x' \in \Sigma, \quad \forall \tau \in \mathbb{R}. \tag{29}$$

It is assumed that field $p(x, t)$ is continuous in quadratic mean and therefore $x, x', \tau \mapsto R(x, x', \tau)$ is continuous on $\Sigma \times \Sigma \times \mathbb{R}$ and, for any x and x' in Σ, its cross spectral measure has a spectral density $x, x', w \mapsto S_p(x, x', w) : \Sigma \times \Sigma \times \mathbb{R} \to \mathbb{C}$:

$$R_p(x, x', \tau) = \int_{\mathbb{R}} e^{iw\tau} S_p(x, x', w)\, dw. \tag{30}$$

Considering equation (29), we have

$$S_p(x, x', -w) = \overline{S_p(x, x', w)}; \quad S_p(x, x', w) = \overline{S_p(x', x, w)}. \tag{31}$$

The spectral power density of the stationary process $(p(x, t), t \in \mathbb{R})$ is denoted $\Phi_p(x, w) = S_p(x, x, w)$ for any fixed x.

Conventionally, we have:

$$\forall x \in \Sigma, \quad E(p(x, t)^2) = R_p(x, x, 0) = \int_{\mathbb{R}} \Phi_p(x, w)\, dw < +\infty. \tag{32}$$

Below, for x and $x' \in \Sigma$, we will also denote $S_p(x, x', w) \equiv S_p(\theta, z, \theta', z', w)$.

Field p is assumed to be axisymmetric in quadratic mean, *i.e.* its cross spectral density verifies the following properties.

(AX1) For any z and z' in $]z_0, z_1[$ and for any w in $\mathbb{R}$, we have, for any θ_0:

$$S_p(\theta_0+\theta, z, \theta_0+\theta', z', w) = S_p(\theta, z, \theta', z', w). \tag{33}$$

Function S_p depends only on $\theta-\theta'$ and we will now denote it $S_p(\theta-\theta', z, z', w)$.

(AX2) For any z and z' in $]z_0, z_1[$ and for w in $\mathbb{R}$, function $\gamma \mapsto S_p(\gamma, z, z', w)$ of $[-2\pi, 2\pi]$ in $\mathbb{C}$ verifies the properties:

$$\left.\begin{aligned} &\forall \gamma \in [-2\pi, 0], \\ &S_p(\gamma+2\pi, z, z', w) = S_p(\gamma, z, z', w) \\ &\forall \gamma \in [0, 2\pi], \\ &S_p(\gamma-2\pi, z, z', w) = S_p(\gamma, z, z', w) \\ &\forall \gamma \in [-2\pi, 2\pi], \\ &S_p(-\gamma, z, z', w) = S_p(\gamma, z, z', w) \end{aligned}\right\} \tag{34}$$

IV,2. – SECOND-ORDER CHARACTERISTICS OF THE NODAL FORCES

Considering the hypotheses, it is verified that processes $F_I^{(n)}(t)$,, $n \in \mathbb{N}$, $I \in \{S, AS\}$ defined by (25) indexed on $\mathbb{R}$ with values in $\mathbb{R}^N$ are second-order, centered, stationary, continuous in quadratic mean, dependent and that their spectral and interspectral matrix measures have densities $S_{I,J}^{n,n'}(w) \in \mathrm{Mat}_{\mathbb{C}}(N, N)$ for any n and n' in $\mathbb{N}$ and for any I and J in $\{S, AS\}$, such that:

$$R_{I,J}^{n,n'}(\tau) = E(F_I^{(n)}(t+\tau)\, F_J^{(n')}(t)^T) = \int_{\mathbb{R}} \exp(iw\tau)\, S_{I,J}^{n,n'}(w)\, dw \tag{35}$$

which, for any j and k in $\{1, \ldots, N\}$ and w in $\mathbb{R}$, are expressed:

$$[S_{I,J}^{n,n'}(w)]_{jk} = \varepsilon_{I,J} \int_{z_0}^{z_1}\int_{z_0}^{z_1} d\mu_j(z)\, d\mu_k(z') \times \int_0^{2\pi}\int_0^{2\pi} g_I(n\theta)\, g_J(n'\theta')\, S_p(\theta-\theta', z, z', w)\, d\theta\, d\theta' \tag{36}$$

where

$$K_S(w) \in \mathrm{Mat}_{\mathbb{R}}(m_S, m_S), \qquad \varepsilon_{S,AS} = \varepsilon_{AS,S} = -1,$$

$g_S(y) = \cos y$, $g_{AS}(y) = \sin y$. We then demonstrate the following results (as the proof is somewhat long, we cannot give it herein [54]):

Let $c_0 = 2\pi$, $c_{n \geq 1} = \pi$. Then, considering hypotheses (33) and (34), for any j and k in $\{1, \ldots, N\}$ and any $w \in \mathbb{R}$, we have:

$$\left.\begin{aligned} &\forall n \geqq 0, \quad \forall n' \geqq 0, \qquad S_{S,AS}^{n,n'}(w) = S_{AS,S}^{n,n'}(w) = 0 \\ &\forall n \geqq 0, \quad \forall n' \geqq 0, \qquad [S_{S,S}^{n,n'}(w)]_{jk} = 2\, c_n\, \delta_{nn'} \\ &\times \int_{z_0}^{z_1}\int_{z_0}^{z_1} d\mu_j(z)\, d\mu_k(z') \int_0^{2\pi} \cos n\gamma\, S_p(\gamma, z, z', w)\, d\gamma \\ &\forall n \geqq 1, \quad \forall n' \geqq 1, \qquad S_{AS,AS}^{n,n'}(w) = S_{S,S}^{n,n'}(w) \\ &\forall n \geqq 0, \quad \forall n' \geqq 0, \qquad S_{AS,AS}^{n,0}(w) = S_{AS,AS}^{0,n'}(w) = 0 \end{aligned}\right\} \tag{37}$$

where

$\delta_{nn'} = 1$ if $n = n'$ and $\delta_{nn'} = 0$ if $n \neq n'$.

IV,3. – SECOND-ORDER CHARACTERISTICS OF THE RESPONSE

Let n_c be the cardinal of $\mathcal{N}$. We consider the linear convolution filter $\{F_S^{(n)}(t),\ F_{AS}^{(n)}(t),\ n\in\mathcal{N}\}\to W(t)$, whose frequency response function defined by (17) is a bounded function of $\mathbb{R}$ and $\mathrm{Mat}_{\mathbb{C}}(L, 2\,N\,n_c)$. Under these conditions [19, 28, 35, 38, 41, 51, 58], the process $\{W(t),\ t\in\mathbb{R}\}$ indexed on $\mathbb{R}$ with values in $\mathbb{R}^L$ is second-order, centered, stationary, continuous in quadratic mean and its spectral matrix measure has a density $S_W(w)\in\mathrm{Mat}_{\mathbb{C}}(L, L)$ which, considering result (37) is expressed as follows for any w in $\mathbb{R}$:

$$S_W(w)=\sum_{n\in\mathcal{N}}\{\mathbb{H}_S^{(n)}(w)\,S_{S,S}^{n;n}(w)\,\mathbb{H}_S^{(n)}(w)^* + \mathbb{H}_{AS}^{(n)}(w)\,S_{AS,AS}^{n;n}(w)\,\mathbb{H}_{AS}^{(n)}(w)^*\} \quad (38)$$

For any n in $\mathcal{N}$, we set:

$$S_V^{(n)}(w) = H^{(n)}(w)\,S_{S,S}^{n;n}(w)\,H^{(n)}(w)^*\in\mathrm{Mat}_{\mathbb{C}}(m', m'). \quad (39)$$

Then, considering (18), equation (38) can also be written:

$$S_W(w)=\sum_{n\in\mathcal{N}}\{T_S^{(n)}\,S_V^{(n)}(w)\,T_S^{(n)\,T} + \varepsilon_n\,T_{AS}^{(n)}\,S_V^{(n)}(w)\,T_{AS}^{(n)\,T}\} \quad (40)$$

where $\varepsilon_0=0$ and $\varepsilon_{n\geq 1}=1$. We note that for an observation DOF $W_j(t)$ with form (10) or (11), *i.e.*:

$$W_j(t)=\sum_{n\in\mathcal{N}}\{[V_S^{(n)}(w)]_j\cos n\,\theta_0 \pm [V_{AS}^{(n)}(w)]_j\sin n\,\theta_0\} \quad (41)$$

with any θ_0, we have:

$$[S_W(w)]_{jj}=\sum_{n\in\mathcal{N}}[S_V^{(n)}(w)]_{jj}. \quad (42)$$

V. – APPLICATION: HYDROELASTOACOUSTIC RESPONSE IN DENSE FLUIDS TO AN AXISYMMETRIC TURBULENT BOUNDARY LAYER

V,1. – MECHANICAL SYSTEM STUDIED

The geometry of the coupled system studied is schematically illustrated in Figure 3. It is a 3D system of revolution with axis Ox_3. We use the notations of Section II,1.

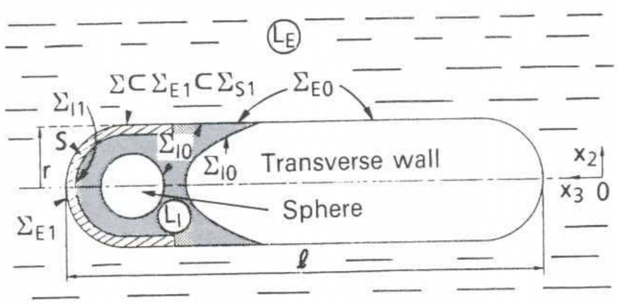


Fig. 3. – Geometry of the generatrix plane of the coupled system.

The two characteristic geometric dimensions are the length l and the radius r (Fig. 3). Surface $\sum_E$ is a slender surface: $r/l \ll l$. The structure S (solid part considered deformable) is an elastic shell made of composite materials. The internal acoustic cavity L_I is bounded (1) by the internal surface of elastic shell S; (2) by an undeformable sphere centered on the axis of revolution Ox_3; (3) by a transverse wall; and (4) by an undeformable part of $\sum_E$ (*see* Fig. 3). The external fluid L_E and the internal fluid L_I are dense (water). We have

$$a_I=a_E=1{,}500\ \mathrm{m/s}, \qquad \rho_E=\rho_I=1\,000\ \mathrm{kg/m^3}.$$

Below, with the angular frequency w (rad/s), we associate the reduced frequency:

$$kr=wr/a, \qquad a=a_E=a_I. \quad (43)$$

The system is placed in a low velocity external flow with constant infinite upstream velocity V_∞, with direction $-Ox_3$. On surface $\sum_E$ develops an axisymmetric boundary layer which is attached and turbulent on part Σ of $\sum_{E1}$, and therefore of $\sum_{S1}$ (Fig. 3). Velocity V_∞ is sufficiently small so that the hydrodynamics of the external flow is decoupled from the elastoacoustic problem. Structure S is therefore excited by the turbulent wall pressure field applied to $\Sigma\subset\sum_{S1}$, and it is the second-order characteristics of the coupled system's response which are sought, *i.e.* the spectra, on a wide MF band: $kr\in B=(3, 25)$, of the accelerations of structure S, the pressures radiated in L_E and mainly the pressures in the internal fluid L_I. The analysis and the results given below correspond to a velocity V_∞ for which the spectral measures of the turbulent wall pressure excitation on Σ and the system response were made [46]. For this case, the flow, the mechanical system and the wall pressure field on Σ are axisymmetric.

V,2. – MODEL OF THE TRANSVERSE SPECTRUM OF THE TURBULENT WALL PRESSURE FIELD

For the flat wall turbulent boundary layers, theoretical and experimental data are available on the wall pressure spectra in the homogeneous case (for instance, *see* [5, 8, 36, 44, 59, 61, 65, 66]). As regards models, one of the first was proposed by **Corcos** [15, 16], then subsequent improvements were proposed, for instance, by **Chase** [12] and **Ffowcs Williams** [30, 31].

In the present application, wall Σ is not flat, there is a variation in the curve radius of the generatrix of Σ, and the pressure field is not homogeneous. For this case, from the standpoint of numerical predictions, the first step consists of determining the external potential flow around the body of $\sum_E$, for V_∞ fixed. This allows us to estimate the external velocity $U_E(z)$ for computation of the boundary layer with weak coupling between inviscid fluid and viscous fluid. The second step is computation of the boundary layer, which allows the laminar, transition and turbulent regions to be estimated, yielding z_0 and therefore Σ, and the boundary layer parameters (critical Reynolds number, displacement thickness, friction coefficient, etc.) [2, 3].

Finally, a model of the cross spectrum is chosen for the wall pressure. This spectrum depends on the external velocity U_E and the boundary layer parameters [46, 54]. It should be noted that for the case at hand, much less data are available on the spectrum models [4, 43] than for flat walls.

(1) *General properties:* the turbulent wall pressure field $p(x,t)$, $x \in \Sigma$, $t \in \mathbb{R}$, verifies the properties discussed in Section IV,1. Its cross spectral density can always be written:

$$S_p(\gamma, z, z', w) = (\Phi_p(z, w)\,\Phi_p(z', w))^{1/2}\,\boldsymbol{\Gamma}(\gamma, z, z', w) \quad (44)$$

where $\Phi_p(z, w) = S_p(0, z, z, w) \geqq 0$ is the autospectrum. The complex coherence function $\boldsymbol{\Gamma}$: $[-2\pi, 2\pi] \times]z_0, z_1[\times]z_0, z_1[\times \mathbb{R} \to \mathbb{C}$ must have the following properties:

$$\left.\begin{array}{l} \boldsymbol{\Gamma}(-\gamma, z', z, w) = \overline{\boldsymbol{\Gamma}(\gamma, z, z', w)} \\ \boldsymbol{\Gamma}(\gamma, z, z', -w) = \overline{\boldsymbol{\Gamma}(\gamma, z, z', w)} \\ |\boldsymbol{\Gamma}(\gamma, z, z', w)| \leqq 1; \qquad \boldsymbol{\Gamma}(0, z, z, w) = 1. \end{array}\right\} \quad (45)$$

(2) *Wall Σ and wall pressure autospectrum:* we did not use a model to determine Σ and the autospectrum $\Phi_p(z, w)$, since we had data on the transition region and the experimental wall spectra [46] from measurements made on the wall.

(3) *Model for complex coherence:* as we had no experimental data for this function, we constructed a model based on the flat wall complex coherence $\Gamma(\xi, \eta, w)$, where $|\xi|$ and $|\eta|$ are the longitudinal distance (in the direction of the flow) and the lateral distance (perpendicular to the direction of the flow) respectively of two points in the plane of the flat wall. This function $\Gamma : \mathbb{R}^3 \to \mathbb{C}$ should verify properties similar to (45):

$$\left.\begin{array}{l} \Gamma(-\xi, -\eta, w) = \overline{\Gamma(\xi, \eta, w)} \\ \Gamma(\xi, \eta, -w) = \overline{\Gamma(\xi, \eta, w)} \\ |\Gamma(\xi, \eta, w)| \leqq 1; \qquad \Gamma(0, 0, w) = 1 \end{array}\right\} \quad (46)$$

and the property related to symmetry of the flow on a flat wall:

$$\Gamma(\xi, -\eta, w) = \Gamma(\xi, \eta, w). \quad (47)$$

The longitudinal and lateral correlation scales, $L_1(w)$ and $L_2(w)$, at frequency w are given in this case by:

$$\left.\begin{array}{l} L_1(w) = \displaystyle\int_0^{+\infty} |\Gamma(\xi, 0, \omega)|\,d\xi; \\ L_2(w) = \displaystyle\int_0^{+\infty} |\Gamma(0, \eta, \omega)|\,d\eta. \end{array}\right\} \quad (48)$$

We used the following model for $\mathbb{L}$ [54]:

$$\left.\begin{array}{l} \boldsymbol{\Gamma}(\gamma, z, z', w) = \Gamma(\xi(z, z'), \eta(\gamma, z, z'), w) \\ \xi(z, z') = s(z) - s(z') \\ \eta(\gamma, z, z') = R_m(z, z')\,g(\gamma) \end{array}\right\} \quad (49)$$

where:

- $z \mapsto s(z) : [z_0, z_1] \to \mathbb{R}^+$ is the curvilinear abscissa function of the generatrix of Σ, introduced in Section III,1;
- $\gamma \mapsto g(\gamma) : [-2\pi, 2\pi] \to \mathbb{R}$ is a function such that $g(-2\pi) = g(0) = g(2\pi) = 0$ and:

$$\forall \gamma \in [-2\pi, 0], \quad g(\gamma + 2\pi) = g(\gamma)$$
$$\forall \gamma \in [0, 2\pi], \quad g(\gamma - 2\pi) = g(\gamma)$$
$$\forall \gamma \in [-2\pi, 2\pi], \quad g(-\gamma) = g(\gamma).$$

- $z, z' \mapsto R_m(z, z') : [z_0, z_1] \times [z_0, z_1] \to \mathbb{R}^+$ is a function related to the generatrix $z \mapsto R(z)$ and verifying $R_m(z, z') = R_m(z', z)$.

It should be noted that the structure of model (49) is necessary for equations (45) to be satisfied, considering the fact that we have (46), and for equations (34) expressing the axisymmetric character of the turbulent wall pressure field to be satisfied. For instance, model $\eta(\theta - \theta', z, z') = \theta R(z') - \theta' R(z)$ is not invariant around Ox_3 and would not be correct.

For the configuration analyzed, we chose:

$$R_m(z, z') = 0.5\,(R(z) + R(z')) \qquad (50)$$

$$g(\gamma) = \begin{cases} \gamma & \text{if} \quad -\pi \leqq \gamma \leqq \pi \\ \gamma - 2\pi & \text{if} \quad \pi < \gamma \leqq 2\pi \\ \gamma + 2\pi & \text{if} \quad -2\pi \leqq \gamma < -\pi \end{cases} \qquad (51)$$

and for Γ we chose Corcos's model [15, 16]:

$$\left.\begin{aligned} &\Gamma(\xi, \eta, w) \\ &= \exp\{i\xi w \underline{U}_c^{-1} - |\xi|\, L_1(w)^{-1} - |\eta|\, L_2(w)^{-1}\} \\ &L_1(w) = \underline{U}_c (0.115\,|w|)^{-1}; \quad L_2(w) = \underline{U}_c (0.7\,|w|)^{-1} \\ &\underline{U}_c = 0.65\, \underline{U}_E \end{aligned}\right\} \qquad (52)$$

where $\underline{U}_E$ is an average on $z \in]z_0, z_1[$ of the external velocity $U_E(z)$ and $\underline{U}_c$ is the associated average convection velocity. For our application, in the frequency band aB/r considered, scales $L_1(w)$ and $L_2(w)$ are very small compared with radius r and the response is relatively insensitive to the scale models used. For instance, using **Chase**'s model [12] for Γ gives scales which do not significantly differ from model (52) for the parametric domain considered [24].

V,3. – FINITE ELEMENT MODEL OF THE COUPLED SYSTEM

We use the method described in Sections II and III. Figure 4 shows part of the finite element mesh of the generatrix plane containing elastic shell S and part of the internal acoustic cavity L_I. Structure S is not meshed with mean plane shell elements but with viscoelastic solid 2D elements, axisymmetric n by n. The generatrix of $\sum_E$ is also meshed to construct the coupling matrices with the external fluid. On the part $\sum_E \cap (\sum_{S1} \cup \sum_{I0})$, the mesh consists of the trace of the mesh of the structure and the internal fluid on $\sum_E$.

Fig. 4. – Mesh of part of the generatrix plane: elastic shell and part of the internal fluid domain.

The characteristic dimensions of the finite elements of S and L_I and of $\sum_E$ are small compared with the wavelengths involved in S, L_I and L_E for any w in aB/r.

V,4. – EXPRESSION OF THE SPECTRAL DENSITIES OF THE NODAL EXCITATION FORCES

For any $n \in \mathscr{N}$, the only nonzero terms of (37) are expressed as follows, considering (44), (49)-(52) and according to the computations:

$$[S^{n,n}_{S,S}(w)]_{jk} = 2\,c_n \int_{z_0}^{z_1}\int_{z_0}^{z_1} A(\xi(z, z'), w) \times \mathscr{E}_n(z, z', w)\, d\beta_j(z, w)\, d\beta_k(z', w) \qquad (53)$$

where

$$\left.\begin{aligned} d\beta_j(z, w) &= (\Phi_p(z, w))^{1/2}\, d\mu_j(z) \\ A(\xi, w) &= \exp(i\xi w \underline{U}_C^{-1} - |\xi|\, L_1(w)^{-1}) \\ \mathscr{E}_n(z, z', w) &= \frac{L_2(w)\, R_m(z, z')}{R_m^2(z, z') + n^2 L_2^2(w)} [1 - (\cos n\pi) \\ &\quad \times \exp(-\pi R_m(z, z')\, L_2(w)^{-1})]. \end{aligned}\right\} \qquad (54)$$

For $w \in aB/r$, the oscillations of function Σ may be rapid on a mesh. It is therefore necessary to accurately make the quadratures of (53).

For this reason, we were led to consider a submesh of the generatrix of Σ to compute (53).

Let $Z_1 < Z_2 < \ldots < Z_N$ be the abscissas along Ox_3 of the N nodes of the mesh of S located on the generatrix of Σ. Let $z_0 = X_1 < X_2 < \ldots < X_N < X_{N+1} = z_1$, such that $Z_j = 0{,}5\,(X_j + X_{j+1})$. We consider the submesh defined by the $(N+1)$ nodes of the generatrix of Σ, with coordinates X_j and $Y_j = R(X_j)$ in plane $Ox_2 x_3$. Since the mesh of the structure is very fine, it is legitimate to use a linear interpolation for the geometric quantities. The j-th submesh is therefore a linear 1D element with the interval $\mathscr{I}_j = [X_j, X_{j+1}]$ as parametric domain in z, with which is associated the parametrizing $\alpha \in [-0.5; 0.5]$, and we have:

$$\left.\begin{aligned} z = \psi_j(\alpha) &= (X_j + X_{j+1})/2 + \alpha\,(X_{j+1} - X_j) \in \mathscr{I}_j \\ R(z) &= (Y_j + Y_{j+1})/2 + \alpha\,(Y_{j+1} - Y_j) \\ ds(z) &= \lambda_j\, d\alpha, \\ \lambda_j &= [(X_{j+1} - X_j)^2 + (Y_{j+1} - Y_j)^2]^{1/2}. \end{aligned}\right\} \qquad (55)$$

By the construction of the structure mesh, valid for any w in aB/r, we can use a constant approximation per submesh of the normal structural displacement field to compute the nodal forces. This amounts to taking in (26):

$$\varphi_j(z) = \mathbf{1}_{\mathscr{I}_j}(z), \qquad \forall z \in [z_0, z_1]. \qquad (56)$$

Finally, as an initial approximation, and considering (56), it is legitimate to write:

$$\left.\begin{array}{l}\forall z \in \mathscr{J}_j,\ d\beta_j(z, w) \simeq R_j \Phi_j(w)^{1/2}\, ds(z) \\ \forall z \in \mathscr{J}_j,\quad z' \in \mathscr{J}_k,\qquad \mathscr{E}_n(z, z', w) \simeq \mathscr{E}_{n,jk}\end{array}\right\} \quad (57)$$

where we set:

$$\left.\begin{array}{r}R_j = R(Z_j) \simeq 0.5\,(Y_j + Y_{j+1}); \\ \Phi_j(w) = \Phi_p(Z_j, w); \\ \mathscr{E}_{n,jk}(w) = \mathscr{E}_n(Z_j, Z_k, w)\end{array}\right\} \quad (58)$$

Under these conditions, equation (53) is written:

$$[S_{S,S}^{n,n}(w)]_{jk} = 2\,c_n \lambda_j \lambda_k R_j R_k (\Phi_j(w)\,\Phi_k(w))^{1/2}\, \mathscr{E}_{n,jk}(w)\, A_{jk}(w) \quad (59)$$

where we set:

$$A_{jk}(w) = \int_{-1/2}^{1/2}\int_{-1/2}^{1/2} A(\xi(\psi_j(\alpha), \psi_k(\alpha')), w)\, d\alpha\, d\alpha'. \quad (60)$$

Setting [54]:

$$\left.\begin{array}{l}b_w = L_1(w)^{-1} - iw\, \underline{U}_c^{-1} \in \mathbb{C}; \\ s_j = s(X_j);\qquad s_{j+1} = s_j + \lambda_j,\qquad s_1 = 0\end{array}\right\} \quad (61)$$

the explicit computation of (60) gives:

(*a*) For $k < j$:

$$A_{jk}(w) = -(\lambda_j \lambda_k b_w^2)^{-1}(1 - \exp(-\lambda_j b_w)) \times (1 - \exp(\lambda_k b_w)) \exp(-b_w(s_j - s_k)). \quad (62)$$

(*b*) For $k = j$:

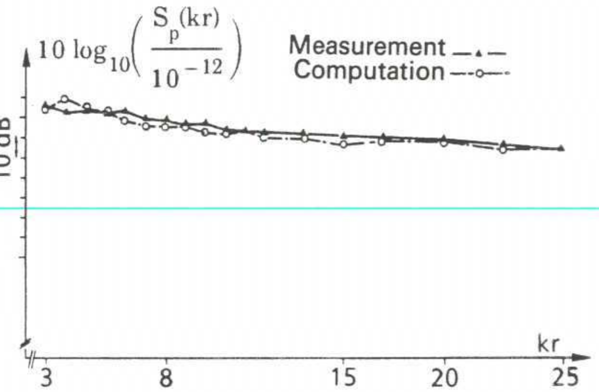
$$A_{jj}(w) = \lambda_j^{-1}(\bar{b}_w^{-1} + b_w^{-1}) - \lambda_j^{-2}(\bar{b}_w^{-2} + b_w^{-2}) + \lambda_j^{-2}(\bar{b}_w^{-2}\exp(-\lambda_j \bar{b}_w) + b_w^{-2}\exp(-\lambda_j b_w)). \quad (63)$$

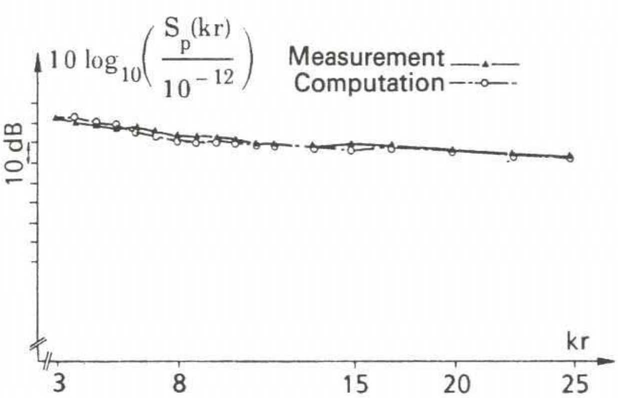
As matrix $S_{S,S}^{n,n}(w)$ is Hermitian, we therefore have its construction for $k > j$.

V,5. – ANALYSIS BY THE NUMERICAL MODEL AND COMPARISONS WITH EXPERIMENT

The analysis was made on a Cray 2 using a hydroelastoacoustic program based on the above developments [25, 26, 68]. A specific analysis of convergence with respect to the circumferential orders $n\,(\mathcal{N} \to \mathbb{N})$ was conducted. The experimental results used for the comparisons were taken from study [46]. Figures 5 to 8 illustrate the quality of the prediction made by the numerical model.

In these figures, the reduced frequency kr is on the abscissa and $10\log(10^{12} S_p(kr))$ is on the ordinate, where $S_p(kr)$ is the spectral pressure power density in a point of the internal fluid. Figures 5 and 6 are relative to comparison between computation and measurement in two representative points of the internal fluid, located in the regions near the internal surface of the elastic shell, and Figures 7 and 8 show two representative points located on the surface of the sphere



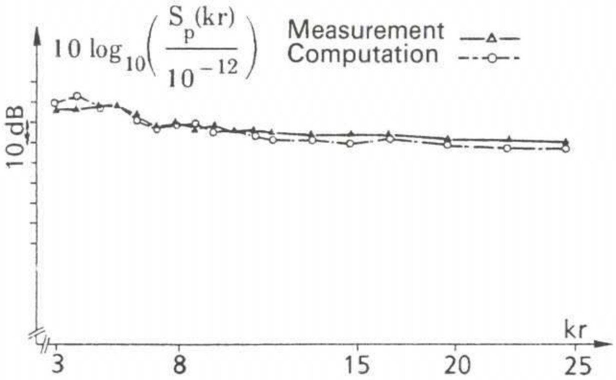


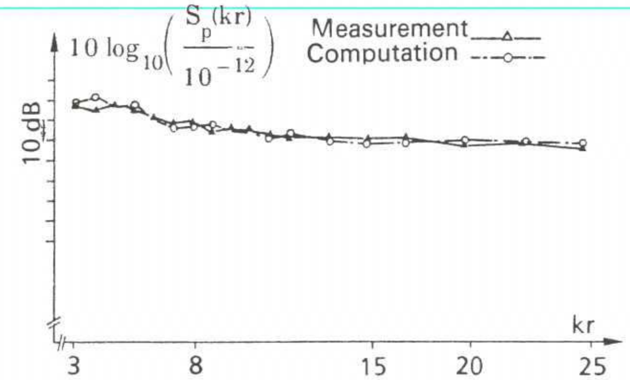
Figs. 5 and 6. – Comparison between computation and measurement of the pressure spectrum in the internal fluid in two points near the internal surface of the elastic shell.

VI. – CONCLUSIONS

We described a numerical method which allows predictive computations to be made of the stationary dynamic responses in the medium frequency domain for coupled bounded structure/bounded compressible internal fluid/unbounded compressible external fluid axisymmetric systems with a linear behavior and excited by an axisymmetric random wall pressure field. The hydroelastoacoustic program developed is very general. It allows any geometries and boundary conditions to be taken into account in a generatrix plane, the structure and internal fluid being inhomogeneous.

The application described, relative to a composite elastic shell coupled with dense external and internal fluids, shows that the numerical model gives a good prediction by comparison with experimental results, for a wide medium frequency band and for which, as





Figs. 7 and 8. – Comparison between computation and measurement of the pressure spectrum in the internal fluid in two points of the surface of the sphere.

the coupled system response is not of the LF type, difficulties arise with numerical methods using modal representation. The program developed also allows analyses to be made for purely 3D cases, *i.e.* not axisymmetric.

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