

# Uncertainty quantification for post-buckling analysis of cylindrical shells with experimental comparisons

Evangéline Capiez-Lernout, Christian Soize, M. P. Mignolet

► **To cite this version:**

Evangéline Capiez-Lernout, Christian Soize, M. P. Mignolet. Uncertainty quantification for post-buckling analysis of cylindrical shells with experimental comparisons. Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2012), Vienna University of Technology, Sep 2012, Vienna, Austria. pp.1-2. hal-00734167

**HAL Id: hal-00734167**

**<https://hal-upec-upem.archives-ouvertes.fr/hal-00734167>**

Submitted on 21 Sep 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Uncertainty quantification for post-buckling analysis of cylindrical shells with experimental comparisons (ECCOMAS 2012)

E. Capiez-Lernout<sup>†</sup>, C. Soize<sup>†\*</sup>, M.-P. Mignolet<sup>‡</sup>

<sup>†</sup>Université Paris-Est, Modélisation et Simulation Multi Échelle, MSME UMR 8208 CNRS  
5, Boulevard Descartes F-77455 Marne-la-Vallée  
evangeline.capiez-lernout@univ-paris-est.fr, christian.soize@univ-paris-est.fr

<sup>‡</sup> SEMTE, Faculties of Mechanical and Aerospace Engineering  
Arizona State University, Tempe, Arizona 85287-6106, USA  
marc.mignolet@asu.edu

**Keywords:** post-buckling analysis, uncertain nonlinear computational model, experimental identification, model uncertainties.

## 1 Introduction

The present work concerns the experimental identification of an uncertain nonlinear computational model in the context of the post-buckling analysis of a cylindrical shell. It proposes an alternative approach to existing methodologies for which only system parameter uncertainties are modeled [5]. This methodology is adapted to the analysis of large static deformations of geometrically nonlinear structural systems in the presence of both system parameters uncertainties and model uncertainties. The available experimental data is made up of the nonlinear static deflection of a cylindrical shell. First, the deterministic nonlinear computational model is constructed using the finite element method and the corresponding nonlinear response is used as a reference deterministic solution for which a reduced-order basis is deduced using the POD (Proper Orthogonal Decomposition) analysis. The mean reduced-order nonlinear computational model is then explicitly constructed in the context of three-dimensional solid finite elements. Moreover, a positive-definite operator related to the nonlinear stiffness of the structure is defined, allowing the use of the nonparametric probabilistic methodology for constructing the uncertain nonlinear reduced-order computational model. Finally, the experimental identification of the uncertain nonlinear computational model is carried out in order to validate the proposed methodology.

## 2 Methodology of analysis

The structure under consideration is a cylindrical shell for which an experimental characterization of its post-buckling response has been conducted [2]. First, the experimental description of the cylindrical shell allows a deterministic computational finite element model to be constructed with the finite element method and the post-buckling deterministic nonlinear response, solution of

$$[K^{(1)}] \mathbf{u} + \mathbf{f}^{NL}(\mathbf{u}) = \mathbf{f} \quad , \quad (1)$$

is calculated as a reference solution. The projection basis  $\varphi^\alpha$ ,  $\alpha = \{1, \dots, N\}$ , required for the construction of the mean reduced-order nonlinear computational model is deduced from this reference

calculation using the Proper Orthogonal Decomposition (POD) method which is known to be particularly efficient in nonlinear static cases [4]. Decomposing the reference solution on this POD basis yields the following set of nonlinear equations

$$\mathcal{K}_{\alpha\beta}^{(1)} q_\beta + \frac{1}{2} \left( \hat{\mathcal{K}}_{\alpha\beta\gamma}^{(2)} + \hat{\mathcal{K}}_{\beta\gamma\alpha}^{(2)} + \hat{\mathcal{K}}_{\gamma\alpha\beta}^{(2)} \right) q_\beta q_\gamma + \mathcal{K}_{\alpha\beta\gamma\delta}^{(3)} q_\beta q_\gamma q_\delta = \mathcal{F}_\alpha \quad , \quad (2)$$

in which the expressions of  $\mathcal{K}_{\alpha\beta}^{(1)}$ ,  $\hat{\mathcal{K}}_{\alpha\beta\gamma}^{(2)}$  and  $\mathcal{K}_{\alpha\beta\gamma\delta}^{(3)}$  and its symmetry properties can be found in [3]. Using the finite element method, the elementary contributions of the linear, quadratic and cubic internal forces projected on this POD basis are calculated for each finite element before assemblage. The mean reduced-order nonlinear computational model described by Eq.(2) is then explicitly obtained as shown in [1]. From the knowledge of these terms and using a reshaping operation, a positive-definite stiffness operator is constructed [3], allowing the nonparametric probabilistic model of uncertainties to be used for modeling both system parameter uncertainties and model uncertainties. The random response is then obtained by solving the random equation

$$\mathcal{K}_{\alpha\beta}^{(1)} \mathbf{Q}_\beta + \frac{1}{2} \left( \hat{\mathcal{K}}_{\alpha\beta\gamma}^{(2)} + \hat{\mathcal{K}}_{\beta\gamma\alpha}^{(2)} + \hat{\mathcal{K}}_{\gamma\alpha\beta}^{(2)} \right) \mathbf{Q}_\beta \mathbf{Q}_\gamma + \mathcal{K}_{\alpha\beta\gamma\delta}^{(3)} \mathbf{Q}_\beta \mathbf{Q}_\gamma \mathbf{Q}_\delta = \mathcal{F}_\alpha \quad , \quad \text{with } \mathbf{U} = [\Phi] \mathbf{Q} \quad . \quad (3)$$

### 3 Experimental validation

The experimental validation consists in formulating the identification of the uncertain nonlinear computational model from the experimental post-buckling nonlinear response. The random post-buckling response is considered as a function of one scalar dispersion parameter  $\delta$  controlling the uncertainty level in the random stiffness operator. The experimental identification requires to solve an optimization problem. The cost function to be minimized contains penalty terms only in the areas for which the experimental data is not within the confidence region described by the uncertain nonlinear computational model. Despite of the sensitivity of the post-buckling behaviour of the cylindrical shell, a good agreement between the experimental response and between the nonlinear uncertain model is observed.

### References

- [1] E. Capiiez-Lernout, C. Soize, M.-P. Mignolet: Computational stochastic statics of an uncertain curved structure with geometrical nonlinearity in three-dimensional elasticity. *Computational Mechanics* , to appear in 2012, doi 10.1007/s00466-011-0629-y.
- [2] G. Michel, A. Combescure, J.-F. Jullien: Finite element simulation of dynamic buckling of cylinders subjected to periodic shear. *Thin-Walled Structures*, 36 (2000), 111–135.
- [3] M.-P. Mignolet, C. Soize: Stochastic reduced order models for uncertain geometrically nonlinear dynamical systems. *Computer Methods in Applied Mechanics and Engineering*, 197 (2008), 3951–3963.
- [4] R. Sampaio, C. Soize: Remarks on the efficiency of POD for model reduction in non-linear dynamics of continuous elastic systems. *International Journal for Numerical Methods in Engineering*, 72(1) (2007), 22–45.
- [5] C.A. Schenk, G.I. Schuëller: Buckling analysis of cylindrical shells with random geometric imperfections. *International Journal of Nonlinear Mechanics*, 38 (2003), 1119–1132.