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Three-dimensional idealized unit-cell based method for computing acoustic properties of low-density reticulated foams

Camille Perrot[a], Minh-Tan Hoang, Guy Bonnet
Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME FRE3160 CNRS, 5 bd Descartes, 77454 Marne-la-Vallée, France

Fabien Chevillotte
INSA de Lyon, Laboratoire Vibrations Acoustique, bâtiment St. Exupéry, 25 bis avenue Jean Capelle
69621 Villeurbanne Cedex, France

François-Xavier Bécot, Luc Jaouen
Matelys - Acoustique & Vibrations, 20/24 rue Robert Desnos, F-69120 Vaulx-en-Velin, France

Arnaud Duval, Jean-François Rondeau
Faurecia Acoustics and Soft Trim Division, R&D Center, Route de Villemontry, Z.I. BP13,
08210 Mouzon, France

Laurent Gautron, Rossana Combes
Université Paris-Est, Laboratoire Géomatériaux et Géologie de l'Ingénieur,
G2I EA 4119 CNRS, 5 bd Descartes, 77454 Marne-la-Vallée, France

ABSTRACT

This paper presents recent developments in the field of micro-acoustics of porous media. The fundamental idea of the proposed approach is that it is possible to assess porous microstructure and acoustical properties from the three-dimensional implementation of micro-acoustics based scaling relations on porous materials. We illustrate this approach through results from the application of this technique to real samples of predominately open-cell polymeric foams.

1. INTRODUCTION

The determination of the dynamic viscous permeability of porous materials is a long-standing problem of great interest, for instance, for the oil industry[1]. Various methods have been proposed to solve it on a rigorous basis.

The first problem consists of the idealization at the local scale of real media. Open cell foams can be idealized as regular arrays of polyhedrons, for example. A presentation of various idealized shapes is given by Gibson and Ashby[2] for cellular solids, and more specifically by Weaire and Hutzler[3] for foams.

The second problem consists in the determination of the macroscopic transport properties, such as the dynamic viscous permeability. The number of media which can be analytically addressed is deceptively small[4-5], and many techniques have been developed in the literature, such as the self consistent method providing bound estimates (see, for instance, the recent work of Boutin and Geindreau and references therein[6]).

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[a] Email camille.perrot@univ-paris-est.fr

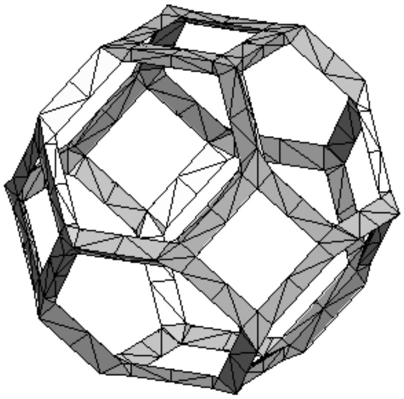


Figure 1: The tetrakaidecahedron is proposed as an example of idealized periodic unit-cell for identification of open-cell foams average characteristic lengths

The purpose of this paper is to present a technique based on first-principles calculations of dynamic permeability[7] in reconstructed porous media[8] which can be applied to model real open-cell foams; and to compare its predictions to multi-scale experimental data. The main difficulty in modeling the frequency-dependant flow through open-cell foams lies in accurately determining micro-structural characteristics, and how they collectively dictate the dynamic permeability.

The paper is divided into three sections. The method for linking idealized three-dimensional micro-structures and macro-acoustic properties of open-cell foams is dealt with in Section 2. A generation of three-dimensional idealized periodic unit cells mimicking the microstructure of real open-cell foam samples is presented, together with a computational technique in reconstructed unit-cells of the macroscopic properties of sound absorbing materials. In Section 3, results at micro- and macro- scales are successfully compared with experimental data obtained from (optical and scanning electron) microscopy and standing wave tube measurements and discussed. The fourth Section is devoted to some concluding remarks which summarize the progress recently made.

2. MICRO-STRUCTURE AND MACRO-ACOUSTIC PROPERTIES

Only packings of identical isotropic polyhedrons were investigated. Tetrakaidecahedron with ligaments of equilateral triangle cross section shapes were considered. They are usually defined from the basis of truncated octahedrons, with ligament lengths L and thicknesses l. A tetrakaidecahedron is a 14-sided polyhedron, with six squares faces and eight hexagonal faces. The average number of edges per face, another polyhedron shape indicator, is equal to (6×4 + 8×6) / 14 ≈ 5.14. The cells have diameter D equal to $(2\sqrt{2})L$, between two parallel square faces, of one unit. All subsequent calculations are performed in a cubic sample of volume D^3. The ligaments length L is generally taken much larger than the ligaments thickness l. An example of tetrakaidecahedron for such packings is given in Figure 1.

The simplest macroscopic parameter characterizing the packing geometry is its porosity, or air volume fraction Φ. The porosities of these packed polyhedron samples might be expressed as a function of the inverse of the ligaments elongation L/l,

$$\Phi \cong 1 - C_1 \left(l/L + 1/2 \right)^2 \ . \tag{1a}$$

When l denotes the basis of the ligaments triangular cross-section shapes, $C_1 = 3\sqrt{3} / 8\sqrt{2}$. The porosity increases with the square of the ligaments elongation L/l.

The second parameter which is widely used to characterize the macroscopic geometry of porous media, and thus polyhedron packings, is the specific surface area S, defined as the total solid surface area per unit volume. The hydraulic radius is defined as twice the ratio of the total pore volume to its surface area. This quantity is generally referred as the thermal characteristic length Λ' in the context of sound absorbing materials[9] such that $\Lambda' = 2\Phi/S$. As for the porosity, the thermal characteristic length might be expressed in terms of the microstructural parameters,

$$\Lambda' \cong C_2 \frac{\left(L + l/2\right)^2}{l} - C_3 l \ , \tag{1b}$$

where $C_2 = 4\sqrt{2} / 9$ and $C_3 = 1 / 2\sqrt{3}$ for triangular cross-section shapes of basis l. Above equations are linking micro- to macro- geometric parameters assuming a tetrakaidecahedron unit-cell with triangular cross-section shapes.

When laboratory measurements of porosity and specific surface area (or thermal characteristic length) are available, average ligaments lengths and thicknesses can be identified by inversion, such that :

$$l \cong \Lambda'\left(1-\Phi\right)\big/C_3\Phi \ , \tag{2a}$$

and

$$L \cong \Lambda'\left[\sqrt{C_1\left(1-\Phi\right)} - \left(1-\Phi\right)/2\right]\Big/C_3\Phi \ . \tag{2b}$$

Note that a mixed approach combining micro- and macro- geometric parameters is also possible by taking advantage of available experimental means (measurements of Φ and l, for instance). In this case, detailed expressions are as follows,

$$L \cong \left[\sqrt{C_1\left(1-\Phi\right)} - 1/2\right] l \ , \tag{3a}$$

and

$$\Lambda' \cong l C_3 \Phi / \left(1-\Phi\right) \ . \tag{3b}$$

It was then shown how the long-wavelengths acoustic properties of rigid porous media can be numerically determined by solving the local equations governing the asymptotic frequency-dependent dissipation phenomena with the adequate boundary conditions. A detailed presentation of this type of derivation is given by Perrot *et al.*[10]. Since the media considered in this section are macroscopically homogeneous, they are considered as infinite periodic media, made of three-dimensional identical unit cells of size D^3. It is assumed that $\lambda >> D$, where λ is the wavelength of an incident acoustic plane wave. This means that for characteristic lengths on the order of $D \sim 0.5$ mm, this assumption is valid for frequencies reaching up to 6000 - 7000 Hz. The most frequently studied asymptotic macroscopic properties of sound absorbing materials are computed from the numerical solution of the incompressible fluid equations (the static viscous permeability k_0) and from Laplace equation (the high-frequency tortuosity α_∞ and the surface length's parameter Λ). The dynamic viscous permeability $\Pi_v(\omega)$ is then derived from the model of Johnson *et al.*[1] and compared with standing wave tube measurements using an adapted experimental setup[11].

Table 1: *Multi-scale geometric and transport properties of low-density open cell foams*

PUC	Φ (-)	Λ' (μm)	L (μm)	l (μm)	k_0 (m^2)	α_0 (-)	Λ (μm)	α_∞ (-)
Foam 1	0.98	440	133	31	2.70×10^{-9}	1.27	256	1.06
Foam 2	0.97	418	153	45	3.00×10^{-9}	1.33	245	1.08
Foam 3	0.98	594	180	42	4.90×10^{-9}	1.27	344	1.06

3. MULTI-SCALE RESULTS FOR RECONSTRUCTED FOAMS AND DISCUSSION

In order to evaluate our multi-scale methodology, experimental data from microscopy and standing wave tube measurements were used. Also, the porosity of three real samples of open-cell foams was non-destructively measured from the perfect gas law properties as described by Beranek[12]. The thermal characteristic length determination[13] was based on the measurement of the dynamic bulk modulus of the materials and analytical inverse solutions derived from Lafarge *et al.* model[14]. Note that Λ' determination relying on direct characterization principles, such as the standard BET method[15] also exist. Starting from Φ and Λ' measurements, average ligaments lengths L and thicknesses l where estimated from macro-micro equations (2). Results of these multi-scale geometric properties are given in Table 1. Numerical computations of k_0, α_∞ and Λ are performed in reconstructed unit cells and also given in Table 1 from adequate asymptotic velocity fields averaging which are shown in Figure 2. $\Pi_v(\omega)$ is then analytically derived as described in Section 2. Based on the impedance tube technique by Utsono *et al.*[11] the complex and frequency-dependant characteristic impedance $Z_c(\omega)$ and propagation constant $q(\omega)$ of each material were measured, and the equivalent dynamic viscous permeability $\Pi_v(\omega)$ deduced - see for example Lafarge et al.[14] for the detailed relationships. There is an excellent agreement between computed (present microstructural method), measured (impedance tube measurements), and characterized[16] dynamic viscous permeabilities $\Pi_v(\omega)$, see Figure 3. This actually proves that a three-dimensional implementation the periodic unit-cell based method for computing the dynamic viscous permeability of low-density reticulated foams corrects the static viscous permeability overestimation provided by similar two-dimensional micro-cellular models[17]. The computed and characterized viscous transition frequencies between viscous and inertial regimes, $f_v = \nu\Phi / 2\pi k_0\alpha_\infty$ (ν is the air cinematic viscosity), are also in good accordance. Finally, in order to check the relevance of the micro-cellular model, ligaments lengths and thicknesses were respectively measured on optical and scanning electron micrographies on one of the foam samples, see Figure 4. For Foam 1, results of measured local characteristic lengths to be compared with Table 1 are as follows: L = 169 ± 62 µm and l = 27.4 ± 4.6 µm. Considering measurements standard deviations and the two-dimensional nature of the micrographs, this comparison is acceptable but must be further checked on more pictures and foams samples.

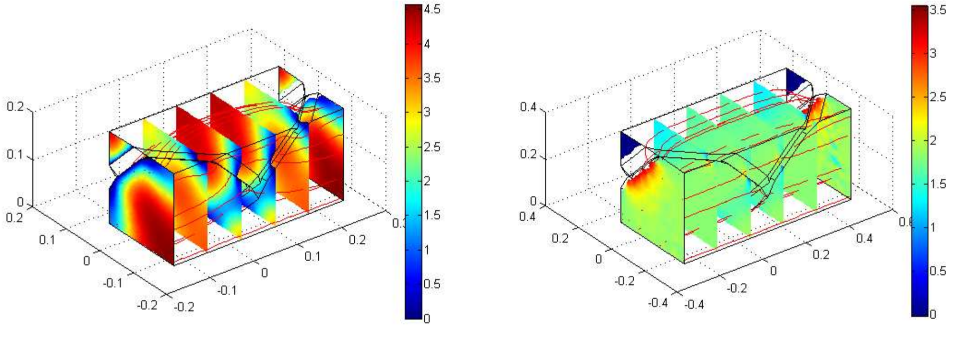
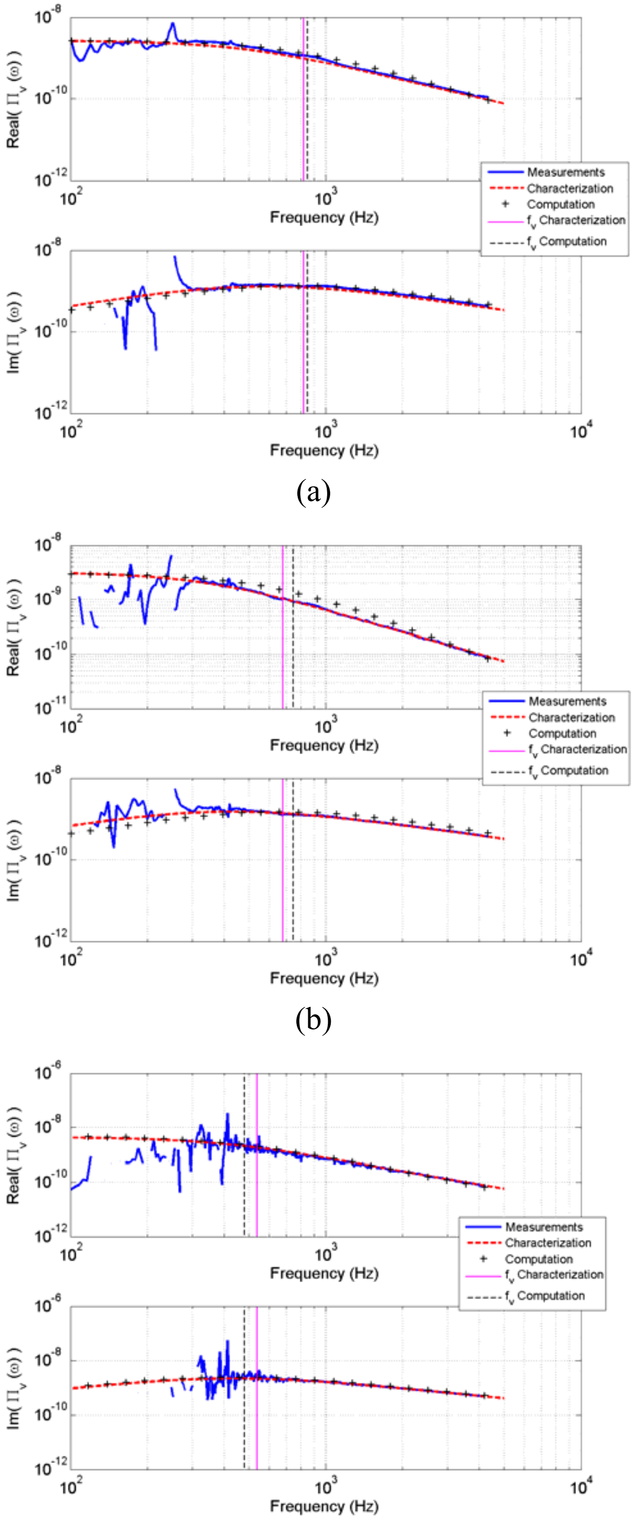


Figure 2: Permeability fields in asymptotic low (left - [× 10^{-9} m^2]) and high (right - [× 10^{-11} m^2]) for Foam sample 1.



(a)

(b)

(c)

Figure 3: Dynamic viscous permeabilities of foam sampes: (a) Foam 1, (b) Foam 2, (c) Foam 3.

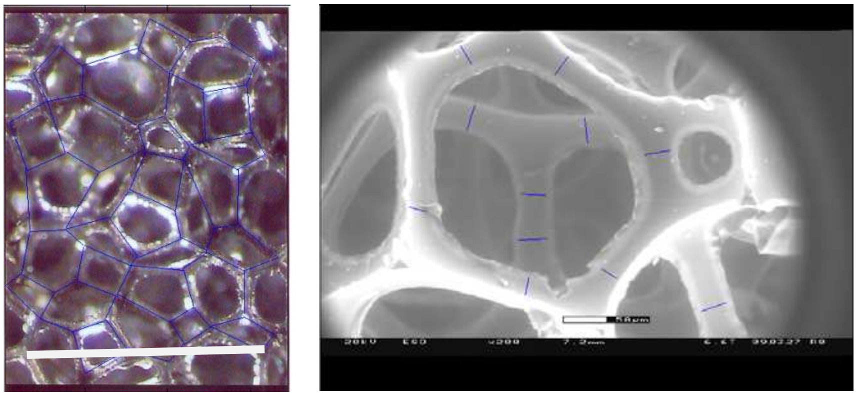


Figure 4: Measurements of ligaments lengths (left) and thicknesses (right) of Foam 1, respectively on optical and scanning electron micrographies. At left, the white rule represent 1000 micrometers, and measurements are realized on 29 cell faces, with an average of 4.93 ± 0.75 edges/face.

4. CONCLUSIONS

A three-dimensional idealized periodic unit-cell (PUC) based method to obtain the acoustic properties of real open cell foam samples was described. The first step was to provide the local characteristic lengths of the representative unit cell. For isotropic open cell foams, two input parameters were required in principle. This was found to be (i) the average ligaments length and thicknesses (bottom-up approach), (ii) the porosity and specific surface area (inverse model), or (iii) a local characteristic length combined with a macroscopic parameter (mixed approach). Long wavelengths acoustic properties were derived from the three-dimensional reconstructed PUC by solving the boundary value problems governing the micro-scale propagation and visco-thermal dissipation phenomena with adequate periodic boundary conditions, and further field phase averaging. The computed acoustic properties of the foams were found to be in good agreement with experimental data. The overall picture that emerges from that work is that the acoustical response of these materials is governed by their three-dimensional micro-cellular morphology for which an idealized unit-cell based method is a formidable framework of multi-scale analysis.

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