Recent advances in prior probabilistic representations for non-Gaussian mesoscale random fields of apparent properties

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Abstract :

In this presentation, we propose and review the most recent advances in the construction and random generation of prior algebraic probabilistic representations for random fields of mesoscale properties. The approach is illustrated on elasticity tensor random fields exhibiting material symmetry properties.

Keywords : **Apparent properties; Random microstructure; Probabilistic model**

1 Introduction

Computational homogenization typically involves, prior to the upscaling procedure, the definition of a parametrized representation for the underlying random microstructure. Such an issue has been extensively discussed within the framework of morphological approaches [7], in which some (e.g. erosion) operators presumably separate the phase geometries and allow random sets (or even random points) models to be used. While such techniques have proven their efficiency in many applications (such as those dealing with matrix-inclusion-type microstructures), they may fail in a few cases of primary importance, such as the modeling of more complex multiphase materials (e.g. biological tissues) for which the random geometry does not appear as well-separated at the microscale. Furthermore, such topology-based strategies cannot accommodate nonhomogeneous properties of some constitutive phases and may yield very high probabilistic dimensions. Instead of pursuing such parametric approaches, one may subsequently consider describing the random material at a slightly coarsest mesoscopic scale, hence considering (random) fields of apparent properties. Interestingly, the latter exhibit less statistical fluctuations than their microscopic counterparts and may be identified from limited experimental data, provided that suitable stochastic representations are available. It is worth noting that such models are also relevant when no scale separation can be invoked, so that a mesoscopic modeling is naturally involved. In this presentation, we propose and review the most recent advances in the construction and random generation of such prior representations for random fields of mesoscale elastic properties.

2 Prior algebraic representation and random generation algorithms for random fields

Here, we denote as $\mathbf{x} \mapsto [\mathbf{C}(\mathbf{x})]$ the random field, defined on a given probability space and indexed by a bounded domain Ω, for which a probabilistic model is sought. The methodology for defining a class of random fields essentially consists in first introducing a family of independent second-order centered homogeneous Gaussian random fields and in writing then the non-Gaussian non-homogeneous tensor-valued random field $\mathbf{x} \mapsto [\mathbf{C}(\mathbf{x})]$ as the memoryless non-linear transformation of these Gaussian random fields. Such a strategy basically amounts to prescribe the system of first-order marginal distributions and has been pioneered for constructing a class of tensor-valued non-Gaussian random fields associated with anisotropic elasticity in [6]. A cornerstone of the approach is the definition of the random matrix ensemble $\mathcal{RM}$ to which $[\mathbf{C}(\mathbf{x})]$ is assumed to belong for $\mathbf{x}$ fixed in Ω. In this work, the definition of ensemble $\mathcal{RM}$ is carried out within the framework of information theory and more specifically, by having recourse to the maximum entropy principle [5]. This approach allows the most objective probabilistic model to be constructed, under some constraints defining the available information for the modeled quantity. Recently, several models have been proposed for both random matrices and random fields, including information about boundedness [1] and/or symmetry properties [2] [3] [4]. In this talk, we first review the aforementioned approaches and discuss the adequacy between some mathematical properties exhibited by random matrices (belonging to usual random matrix ensembles) and mechanical issues related to anisotropy modeling. We then address the issue of random generation and propose new algorithms which are based on solving Itô stochastic differential equations. We finally exemplify the theoretical concepts and algorithms by considering the modeling of non-Gaussian elasticity tensor random fields exhibiting transverse isotropy.

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