Stochastic reduced-order model for dynamical structures with high modal density in the low-frequency range

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STOCHASTIC REDUCED-ORDER MODEL FOR DYNAMICAL STRUCTURES WITH HIGH MODAL DENSITY IN THE LOW-FREQUENCY RANGE

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Abstract. *The problem considered here concerns the construction of a stochastic reduced-order model for dynamical structures having a high modal density in the low frequency range. The classical methods used for the low-frequency range to construct a reduced-order model are not adapted in this case. We then use a recently proposed method which consists in constructing a basis of the global displacements and a basis of the local displacements by solving two separate eigenvalue problems. We then construct a stochastic reduced-order model using the basis of the global displacements and the contribution of the local displacements is taken into account using a probabilistic approach. The theory is presented and is applied to tube bundles structures which is are quasi-periodic structures for which the dynamical response is characterized by ensemble (global) displacements and more local displacements.*

Keywords. *Reduced-order model, structural dynamics, global and local displacements*

1 INTRODUCTION

This paper is devoted to the construction of a stochastic reduced-order model for dynamical structures having a high modal density in the low frequency range. In general, dynamical structures exhibit well separated resonances in the low-frequency range. This low modal density allows classical methods (such as modal analysis) to be completed. This low-frequency range can clearly be separated from the medium-frequency range (for which the modal density is larger but not uniform in frequency). In some cases, in the low-frequency, a dynamical structure can exhibit both the global elastic modes (which characterize this low-frequency range) and numerous local elastic modes. This situation appears for complex heterogeneous structures presenting stiff parts which support global displacements and flexible parts which support local displacements. The presence of these flexible parts induces numerous local reasonance in the low frequency range. Furthermore, in this case, the elastic modes cannot be separated into global elastic modes and local elastic modes. Indeed, due to the coupling between global elastic modes and local elastic modes, the deformation of some global elastic modes have local contributions and the deformation of some local elastic modes have global contributions. Then there are no efficient sorting method which could be used to select the elastic modes as global elastic modes or as local elastic modes. In addition, although the reduced-order model must be constructed with respect to the global elastic modes, this reduced-order model must have the capability to predict the amplitudes of the responses of the structure in this low-frequency range. Since there are local elastic modes in the frequency band, a part of the mechanical energy is transferred from the global elastic modes to the local elastic modes which store this energy and then induces an apparent damping at the resonances associated with the global elastic modes.

The objective of this paper is double: (1) The first one is to contruct a basis of global displacements and a basis of local displacements by solving two generalized eigenvalue problems. The elements of theses two bases will not be classical elastic modes (2) The second one is to construct a reduced-order model with the basis of the global displacements but in taking into account the effects of the local displacements, in order to correctly predict the frequency response functions in the low-frequency range.

These two objectives are achieved using the method developed in [1]. This method is based on a kinematic reduction of the kinetic energy. Then, this reduced kinetic energy is used to construct a new global eigenvalue problem for which the solutions form a basis of global displacements. This basis can be completed with a basis of local displacements which is obtained by introducing a complementary kinetic energy, and then a local eigenvalue problem. A classical method to construct a reduced-order model for quasi-periodic beam structures consists in modeling these structures by simplified models using homogenisation methods ([2],[3] or equivalent beams [4]). Such a simplified model provides quite a good approximation of the global contributions of the displacements but can clearly not take into account the local contributions. Furthermore, the construction of an acurate simplified model cannot be carried out automatically and a procedure of validation of the simplified model is always needed.

In this paper we first develop the details of the methodology presented in [1]. Then, we present the application of this methodology to the dynamical structures we are interested in.

2 REFERENCE REDUCED MATRIX MODEL

We are interested in predicting the frequency response functions of a three dimensional linear damped structure, occupying a bounded domain Ω, in the frequency band of analysis $\mathscr{B} = [\omega_{\min}, \omega_{\max}]$ with $0 < \omega_{\min}$. The complex vector $\mathbb{U}(\omega)$ of the m DOF of the computational model constructed by the finite element method is solution of the following complex matrix equation,

$$(-\omega^2[\mathbb{M}] + i\omega[\mathbb{D}] + [\mathbb{K}])\mathbb{U}(\omega) = \mathbb{F}(\omega) \quad , \tag{1}$$

in which $[\mathbb{M}]$, $[\mathbb{D}]$ and $[\mathbb{K}]$ are respectively the $(m\times m)$ positive-definite symmetric real mass, damping and stiffness matrices and where $\mathbb{F}(\omega)$ is relative to the discretization of the external forces. The eigenfrequencies and the elastic modes of the associated conservative dynamical system consists in finding λ and $\boldsymbol{\varphi}$ in $\mathbb{R}^m$ such that

$$[\mathbb{K}]\,\boldsymbol{\varphi} = \lambda\,[\mathbb{M}]\,\boldsymbol{\varphi}\,. \tag{2}$$

Using the modal method, the approximation $\mathbb{U}_n(\omega)$ at order n of $\mathbb{U}(\omega)$ is written as

$$\mathbb{U}_n(\omega) = \sum_{\alpha=1}^{n} q_\alpha(\omega)\,\boldsymbol{\varphi}_\alpha = [\Phi]\,\boldsymbol{q}\,, \tag{3}$$

in which $\boldsymbol{q} = (q_1,\ldots,q_n)$ is the complex vector of the n generalized coordinates and where $[\Phi] = [\boldsymbol{\varphi}_1 \ldots \boldsymbol{\varphi}_n]$ is the $(m\times n)$ real matrix of the elastic modes associated with the n first eigenvalues.

3 DECOMPOSITION OF THE MASS MATRIX

The details of the methodology for the discrete and the continuous cases are presented in [1]. The domain Ω is partitioned into n_J subdomains Ω_j such that, for j and k in $\{1,\ldots,n_J\}$,

$$\Omega = \bigcup_{j=1}^{n_J}\Omega_j \quad , \quad \Omega_j\cap\Omega_k = \emptyset\,. \tag{4}$$

Let $\boldsymbol{u}\mapsto h^r(\boldsymbol{u})$ be the linear operator defined by

$$\{h^r(\boldsymbol{u})\}(\boldsymbol{x}) = \sum_{j=1}^{n_J} \mathbb{1}_{\Omega_j}(\boldsymbol{x})\,\frac{1}{m_j}\int_{\Omega_j}\rho(\boldsymbol{x})\,\boldsymbol{u}(\boldsymbol{x})\,d\boldsymbol{x}\,, \tag{5}$$

in which $\boldsymbol{x}\mapsto \mathbb{1}_{\Omega_j}(\boldsymbol{x}) = 1$ if $\boldsymbol{x}$ is in Ω_j and $= 0$ otherwise. The local mass m_j is defined, for all j in $\{1,\ldots,n_J\}$, by $m_j = \int_{\Omega_j}\rho(\boldsymbol{x})\,d\boldsymbol{x}$, where $\boldsymbol{x}\mapsto\rho(\boldsymbol{x})$ is the mass density. Let $\boldsymbol{u}\mapsto h^c(\boldsymbol{u})$ be the linear operator defined by

$$h^c(\boldsymbol{u}) = \boldsymbol{u} - h^r(\boldsymbol{u})\,. \tag{6}$$

Function $h^r(\boldsymbol{u})$ will also be denoted by $\boldsymbol{u}^r$ and function $h^c(\boldsymbol{u})$ by $\boldsymbol{u}^c$. We then have $\boldsymbol{u} = h^r(\boldsymbol{u}) + h^c(\boldsymbol{u})$ that is to say, $\boldsymbol{u} = \boldsymbol{u}^r + \boldsymbol{u}^c$. Let $[H^r]$ be the $(m\times m)$ matrix relative to the finite element discretization of the projection operator h^r defined by Eq. (5). Therefore, the finite element discretization $\mathbb{U}$ of $\boldsymbol{u}$ can be written as $\mathbb{U} = \mathbb{U}^r + \mathbb{U}^c$, in which $\mathbb{U}^r = [H^r]\,\mathbb{U}$ and $\mathbb{U}^c = [H^c]\,\mathbb{U} = \mathbb{U} - \mathbb{U}^r$ which shows that $[H^c] = [I_m] - [H^r]$. Then, the $(m\times m)$ reduced mass matrix $[\mathbb{M}^r]$ is constructed such that $[\mathbb{M}^r] = [\mathbb{M}][H^r] = [H^r]^T[\mathbb{M}] = [H^r]^T[\mathbb{M}][H^r]$ and where the $(m\times m)$ complementary mass matrix $[\mathbb{M}^c]$ is constructed such that $[\mathbb{M}^c] = [\mathbb{M}] - [\mathbb{M}^r]$.

4 BASIS OF GLOBAL DISPLACEMENTS AND BASIS OF LOCAL DISPLACEMENTS

The basis of global displacements and the basis of local displacements are calculated using the decomposition of the mass matrix $[\mathbb{M}]$. The basis of global displacements is made up of the solutions $\boldsymbol{\phi}^g$ in $\mathbb{R}^m$ of the generalized eigenvalue problem

$$[\mathbb{K}]\boldsymbol{\phi}^g = \lambda^g[\mathbb{M}^r]\boldsymbol{\phi}^g\,. \tag{7}$$

This generalized eigenvalue problem admits an increasing sequence of $3n_J$ positive global eigenvalues $0 < \lambda_1^g \leq \ldots \leq \lambda_{3n_J}^g$, associated with the finite family of algebraically independent global eigenvectors $\{\boldsymbol{\phi}_1^g,\ldots,\boldsymbol{\phi}_{3n_J}^g\}$. The family $\{\boldsymbol{\phi}_1^g,\ldots,\boldsymbol{\phi}_{3n_J}^g\}$ is defined as the basis of the global displacements. In general, this family is not made up elastic modes. The basis of the local displacements is made up of the solutions $\boldsymbol{\phi}^\ell$ in $\mathbb{R}^m$ of the generalized eigenvalue problem

$$[\mathbb{K}]\boldsymbol{\phi}^\ell = \lambda^\ell[\mathbb{M}^c]\boldsymbol{\phi}^\ell\,. \tag{8}$$

This generalized eigenvalue problem admits an increasing sequence of positive local eigenvalues $0 < \lambda_1^\ell \leq \ldots \leq \lambda_{m-3n_J}^\ell$, associated with the infinite family of local eigenvectors $\{\boldsymbol{\phi}_1^\ell, \ldots, \boldsymbol{\phi}_{m-3n_J}^\ell\}$. The family $\{\boldsymbol{\phi}_1^\ell, \ldots, \boldsymbol{\phi}_{m-3n_J}^\ell\}$ is defined as the basis of local displacements. In general, this family is not made up of elastic modes.

In practice, the basis of global displacements and the basis of local displacements are calculated by using a double projection method which is less intrusive with respect to the commercial software and less time-consuming than the direct method. The solutions of the generalized eigenvalue problems defined by Eqs. (7) and (8) are then written, for n sufficiently large, as

$$\boldsymbol{\phi}^g = [\Phi]\,\widetilde{\boldsymbol{\phi}}^g \quad , \quad \boldsymbol{\phi}^\ell = [\Phi]\,\widetilde{\boldsymbol{\phi}}^\ell . \tag{9}$$

The global eigenvectors are the solutions of the generalized eigenvalue problem

$$[\widetilde{K}]\,\widetilde{\boldsymbol{\phi}}^g = \lambda^g\,[\widetilde{M}^r]\,\widetilde{\boldsymbol{\phi}}^g , \tag{10}$$

in which $[\widetilde{M}^r] = [\Phi^r]^T\,[\mathbb{M}]\,[\Phi^r]$ and $[\widetilde{K}] = [\Phi]^T\,[\mathbb{K}]\,[\Phi]$, and where the $(m \times n)$ real matrix $[\Phi^r]$ is such that $[\Phi^r] = [H^r]\,[\Phi]$. The local eigenvectors are the solutions of the generalized eigenvalue problem

$$[\widetilde{K}]\,\widetilde{\boldsymbol{\phi}}^\ell = \lambda^\ell[\widetilde{M}^c]\,\widetilde{\boldsymbol{\phi}}^\ell , \tag{11}$$

in which $[\widetilde{M}^c] = [\Phi^c]^T\,[\mathbb{M}]\,[\Phi^c]$ and where the $(m \times n)$ real matrix $[\Phi^c]$ is such that $[\Phi^c] = [H^c]\,[\Phi] = [\Phi] - [\Phi^r]$.

5 MEAN REDUCED MATRIX MODEL

It is proven in [1] that the family $\{\boldsymbol{\phi}_1^g, \ldots, \boldsymbol{\phi}_{3n_J}^g, \boldsymbol{\phi}_1^\ell, \ldots, \boldsymbol{\phi}_{m-3n_J}^\ell\}$ is a basis of $\mathbb{R}^m$. The mean reduced matrix model is obtained using the projection of $\mathbb{U}(\omega)$ on the subspace of $\mathbb{C}^m$ spanned by the family $\{\boldsymbol{\phi}_1^g, \ldots, \boldsymbol{\phi}_{n_g}^g, \boldsymbol{\phi}_1^\ell, \ldots, \boldsymbol{\phi}_{n_\ell}^\ell\}$ of real vectors associated with the n_g first global elastic modes such that $n_g \leq 3n_J \leq m$ and with the n_ℓ first local elastic modes such that $n_\ell \leq m$. Then, the approximation $\mathbb{U}_{n_g,n_\ell}(\omega)$ of $\mathbb{U}(\omega)$ at order (n_g, n_ℓ) is written as

$$\mathbb{U}_{n_g,n_\ell}(\omega) = \sum_{\alpha=1}^{n_g} q_\alpha^g(\omega)\,\boldsymbol{\phi}_\alpha^g + \sum_{\beta=1}^{n_\ell} q_\beta^\ell(\omega)\,\boldsymbol{\phi}_\beta^\ell . \tag{12}$$

Let $\boldsymbol{q}(\omega) = (\boldsymbol{q}^g(\omega), \boldsymbol{q}^\ell(\omega))$ be the vector in $\mathbb{C}^{n_t}$ of all the generalized coordinates such that $\boldsymbol{q}^g(\omega) = (q_1^g(\omega), \ldots, q_{n_g}^g(\omega))$ and $\boldsymbol{q}^\ell(\omega) = (q_1^\ell(\omega), \ldots, q_{n_\ell}^\ell(\omega))$. Consequently, vector $\boldsymbol{q}(\omega)$ is solution of the following mean reduced matrix equation such that

$$(-\omega^2[M] + i\omega[D] + [K])\,\boldsymbol{q}(\omega) = \mathscr{F}(\omega) , \tag{13}$$

where $[M]$, $[D]$ and $[K]$ are the $(n_t \times n_t)$ mean generalized mass, damping and stiffness matrices defined by blocks as

$$[M] = \begin{bmatrix} M^{gg} & M^{g\ell} \\ (M^{g\ell})^T & M^{\ell\ell} \end{bmatrix} , [D] = \begin{bmatrix} D^{gg} & D^{g\ell} \\ (D^{g\ell})^T & D^{\ell\ell} \end{bmatrix} , [K] = \begin{bmatrix} K^{gg} & K^{g\ell} \\ (K^{g\ell})^T & K^{\ell\ell} \end{bmatrix} . \tag{14}$$

Let A (or $\mathbb{A}$) be denoting M, D or K (or $\mathbb{M}$, $\mathbb{D}$ or $\mathbb{K}$). Therefore, the block matrices are defined by

$$[A]_{\alpha\beta}^{gg} = (\boldsymbol{\phi}_\alpha^g)^T[\mathbb{A}]\boldsymbol{\phi}_\beta^g , [A]_{\alpha\beta}^{g\ell} = (\boldsymbol{\phi}_\alpha^g)^T[\mathbb{A}]\boldsymbol{\phi}_\beta^\ell , [A]_{\alpha\beta}^{\ell\ell} = (\boldsymbol{\phi}_\alpha^\ell)^T[\mathbb{A}]\boldsymbol{\phi}_\beta^\ell , \tag{15}$$

which can be rewritten, using Eq. (9),

$$[A]_{\alpha\beta}^{gg} = (\widetilde{\boldsymbol{\phi}}_\alpha^g)^T[\widetilde{A}]\widetilde{\boldsymbol{\phi}}_\beta^g , [A]_{\alpha\beta}^{g\ell} = (\widetilde{\boldsymbol{\phi}}_\alpha^g)^T[\widetilde{A}]\widetilde{\boldsymbol{\phi}}_\beta^\ell , [A]_{\alpha\beta}^{\ell\ell} = (\widetilde{\boldsymbol{\phi}}_\alpha^\ell)^T[\widetilde{A}]\widetilde{\boldsymbol{\phi}}_\beta^\ell , \tag{16}$$

in which the $[\widetilde{A}]$ is the $(n \times n)$ matrix defined by $[\widetilde{A}] = [\Phi]^T[\mathbb{A}][\Phi]$. The matrices $[K]^{gg}$ and $[K]^{\ell\ell}$ are diagonal. The generalized force is a vector in $\mathbb{C}^{n_t}$ which is written as $\mathscr{F}(\omega) = (\mathscr{F}^g(\omega), \mathscr{F}^\ell(\omega))$ in which $\mathscr{F}_\alpha^g(\omega) = (\boldsymbol{\phi}_\alpha^g)^T\,\mathbb{F}(\omega)$ and $\mathscr{F}_\alpha^\ell(\omega) = (\boldsymbol{\phi}_\alpha^\ell)^T\,\mathbb{F}(\omega)$. Then, for all ω fixed in $\mathscr{B}$, the generalized coordinates are calculated by inverting Eq. (13) and the response $\mathbb{U}_{n_g,n_\ell}(\omega)$ is calculated using Eq. (12).

6 PROBABILISTIC MODEL FOR THE LOCAL CONTRIBUTIONS

In the low-frequency range, the global displacements are not really sensitive to uncertainties introduced in the computational model. Nevertheless, we have assumed that the structure under consideration had also local contributions in the same low-frequency band. It is well known that the modal density of such local modes increases rapidly with the frequency and that, in addition, the local modes are sensitive both to the system parameters uncertainties and to the model errors which induce model uncertainties. In order to improve the predictability of the computational model, the nonparametric probabilistic approach (see [5]) is used to take into account uncertainties for the local contributions.

6.1 Random reduced matrix model

The nonparametric probabilistic approach consists in replacing the matrices of the reduced mean matrix model by random matrices for which the probability distributions are constructed by using the maximum entropy principle with the constraints defined by the available information. We introduce the random matrices $[\widetilde{\boldsymbol{M}}]$, $[\widetilde{\boldsymbol{D}}]$ and $[\widetilde{\boldsymbol{K}}]$ with values in the set of all the positive-definite symmetric $(n \times n)$ real matrices, for which their mean values are such that $E\{[\widetilde{\boldsymbol{M}}]\} = [\widetilde{M}]$, $E\{[\widetilde{\boldsymbol{D}}]\} = [\widetilde{D}]$ and $E\{[\widetilde{\boldsymbol{K}}]\} = [\widetilde{K}]$, and finally, verify the following inequalities $E\{\|[\widetilde{\boldsymbol{M}}]^{-1}\|_F^2\} < +\infty$, $E\{\|[\widetilde{\boldsymbol{D}}]^{-1}\|_F^2\} < +\infty$ and $E\{\|[\widetilde{\boldsymbol{K}}]^{-1}\|_F^2\} < +\infty$ which assure that there exists a second-order random solution to the stochastic reduced-order equation. The probability distribution of each random matrix $[\boldsymbol{M}]$, $[\boldsymbol{D}]$ or $[\boldsymbol{K}]$ depend on the mean value $[\widetilde{M}]$, $[\widetilde{D}]$ or $[\widetilde{K}]$ and on a dispersion parameter δ_M, δ_D or δ_K defined by

$$\delta_A^2 = \frac{E\{\|[\widetilde{\boldsymbol{A}}] - [\widetilde{A}]\|_F^2\}}{\|[A^{\ell\ell}]\|_F^2}\,, \tag{17}$$

in which $\widetilde{A}$ (or $\widetilde{\boldsymbol{A}}$) is $\widetilde{M}$, $\widetilde{D}$ or $\widetilde{K}$ (or, $\widetilde{\boldsymbol{M}}$, $\widetilde{\boldsymbol{D}}$ or $\widetilde{\boldsymbol{K}}$). The dispersion parameters allow the level of uncertainties to be controlled. For $\widetilde{A}$ (or $\widetilde{\boldsymbol{A}}$) being $\widetilde{M}$, $\widetilde{D}$ or $\widetilde{K}$ (or, $\widetilde{\boldsymbol{M}}$, $\widetilde{\boldsymbol{D}}$ or $\widetilde{\boldsymbol{K}}$), we introduce the Cholesky factorization $[\widetilde{\boldsymbol{A}}] = [\boldsymbol{L}_{\tilde{\boldsymbol{A}}}]^T[\boldsymbol{L}_{\tilde{\boldsymbol{A}}}]$ and $[\widetilde{A}] = [L_{\tilde{A}}]^T[L_{\tilde{A}}]$. Then, the random generalized mass, damping and stiffness matrices are written as

$$[\boldsymbol{M}] = \begin{bmatrix} M^{gg} & \boldsymbol{M}^{g\ell} \\ (\boldsymbol{M}^{g\ell})^T & \boldsymbol{M}^{\ell\ell} \end{bmatrix}, [\boldsymbol{D}] = \begin{bmatrix} D^{gg} & \boldsymbol{D}^{g\ell} \\ (\boldsymbol{D}^{g\ell})^T & \boldsymbol{D}^{\ell\ell} \end{bmatrix}, [\boldsymbol{K}] = \begin{bmatrix} K^{gg} & \boldsymbol{K}^{g\ell} \\ (\boldsymbol{K}^{g\ell})^T & \boldsymbol{K}^{\ell\ell} \end{bmatrix}, \tag{18}$$

in which the block matrices are defined for $\widetilde{A}$ (or $\widetilde{\boldsymbol{A}}$) being $\widetilde{M}$, $\widetilde{D}$ or $\widetilde{K}$ (or, $\widetilde{\boldsymbol{M}}$, $\widetilde{\boldsymbol{D}}$ or $\widetilde{\boldsymbol{K}}$) by

$$[A]^{gg}_{\alpha\beta} = (\widetilde{\boldsymbol{\phi}}^g_\alpha)^T[\widetilde{A}]\widetilde{\boldsymbol{\phi}}^g_\beta\,, [\boldsymbol{A}]^{g\ell}_{\alpha\beta} = (\widetilde{\boldsymbol{\phi}}^g_\alpha)^T[L_{\tilde{A}}]^T[\boldsymbol{L}_{\tilde{\boldsymbol{A}}}]\widetilde{\boldsymbol{\phi}}^\ell_\beta\,, [\boldsymbol{A}]^{\ell\ell}_{\alpha\beta} = (\widetilde{\boldsymbol{\phi}}^\ell_\alpha)^T[\widetilde{\boldsymbol{A}}]\widetilde{\boldsymbol{\phi}}^\ell_\beta\,. \tag{19}$$

6.2 Random frequency responses

The random response $\boldsymbol{U}_{n_g,n_l}(\omega)$ is then written as

$$\boldsymbol{U}_{n_g,n_\ell}(\omega) = \sum_{\alpha=1}^{n_g} Q^g_\alpha(\omega)\,\boldsymbol{\phi}^g_\alpha + \sum_{\beta=1}^{n_\ell} Q^\ell_\beta(\omega)\,\boldsymbol{\phi}^\ell_\beta\,, \tag{20}$$

in which the random vector $\boldsymbol{Q}(\omega) = (\boldsymbol{Q}^g(\omega), \boldsymbol{Q}^\ell(\omega))$ with values in $\mathbb{C}^{n_t}$ of all the generalized coordinates is such that $\boldsymbol{Q}^g(\omega) = (Q^g_1(\omega), \ldots, Q^g_{n_g}(\omega))$ and $\boldsymbol{Q}^\ell(\omega) = (Q^\ell_1(\omega), \ldots, Q^\ell_{n_\ell}(\omega))$. Consequently, vector $\boldsymbol{Q}(\omega)$ is solution of the following stochastic reduced matrix equation such that

$$(-\omega^2[\boldsymbol{M}] + i\omega[\boldsymbol{D}] + [\boldsymbol{K}])\,\boldsymbol{Q}(\omega) = \mathscr{F}(\omega)\,. \tag{21}$$

This equation is solved using the Monte Carlo simulation method.

7 APPLICATION TO A TUBE BUNDLE STRUCTURE

7.1 Mean Finite Element Model

The dynamical system is made up of 49 tubes linked each to the others by four grids. There are two types of tubes:

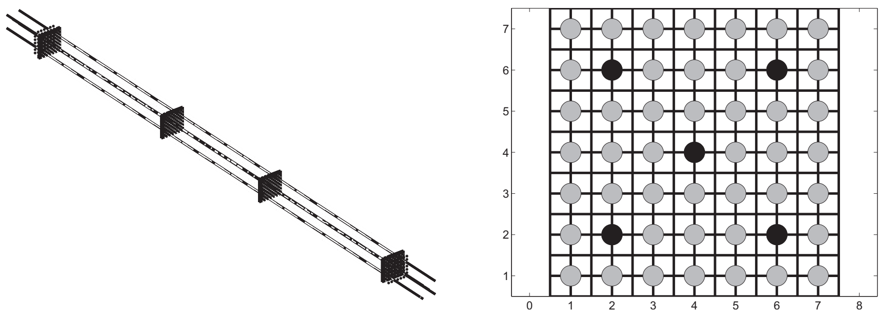

Figure 1: Geometry of the dynamical system. Right figure: 3D-Mesh. Left figure: Grid-view.

(1) the guid-tubes (Black tubes on Fig. 1) which are clamped at their ends and welded to the grids and (2) the plain-tubes (Grey tubes on Fig. 1) which are free at their ends and linked to the grids by linear springs . Guid-tubes are circular, homogeneous, isotropic beam with constant outer radius 6.0×10^{-3} m, thickness 4.0×10^{-4} m, length 2.25 m,

mass density $6,526\ kg/m^3$, Poisson ratio 0.3, Young modulus $9.84\times 10^{10}\ N/m^2$. Plain-tubes are circular, homogeneous, isotropic beam with constant outer radius $4.75\times 10^{-3}\ m$, thickness $5.7\times 10^{-4}\ m$, length $1.91\ m$, mass density $3.79\times 10^4\ kg/m^3$ (equivalent mass density), Poisson ratio 0.3, Young modulus $9.84\times 10^{10}\ N/m^2$. The four grids are assemblages of rectangular, homogeneous, isotropic beams with constant heigth $2.7\times 10^{-2}\ m$, thickness $4.8\times 10^{-4}\ m$, mass density $1.2\times 10^4\ kg/m^3$, Poisson ratio 0.3, Young modulus $9.84\times 10^{10}\ N/m^2$. The guid-tube/grid springs have stiffness equal to $5.0\times 10^9\ N/m$ for the three translations and $5.0\times 10^6\ N/m$ for the three rotations. The plain-tube/grid springs have stiffness equal to $1.8\times 10^5\ N/m$ for the three translations and $92\ N/m$ for the three rotations.

The frequency band of analysis is $\mathscr{B} =]0\,,120]\ Hz$. The finite element model is made up of Timoshenko beam elements and linear spring elements. The structure has $m = 12,750$ DOF.

7.2 Modal analysis, global and local elastic modes

In a first step, the elastic modes are calculated with the finite element model defined by Eq. (2). There are 447 eigenfrequencies in the frequency band of analysis $\mathscr{B}$ and $n = 500$ eigenfrequencies in the band $]0\,,147.3]\ Hz$. The 4$^{\text{th}}$ elastic mode $\boldsymbol{\phi}_1$ and the 10$^{\text{th}}$ elastic mode $\boldsymbol{\phi}_2$ are displayed in Fig. 2. We can see that $\boldsymbol{\phi}_4$ is a global elastic mode while $\boldsymbol{\phi}_{10}$ is a local elastic mode. In a second step, the global eigenvectors and the local eigenvectors are constructed. In order

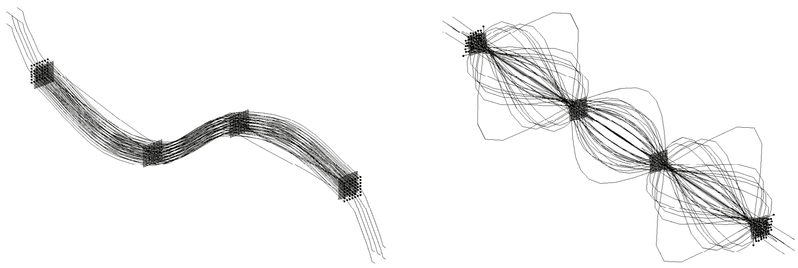

Figure 2: 4$^{\text{th}}$ elastic (left) and 10$^{\text{th}}$ elastic mode (right).

to construct the matrix $[H^r]$, the domain is splitted into 49 subdomains which correspond to 49 longitudinal slices of the structure. In the band $]0\,,147.3]\ Hz$, there are $n_g = 23$ global eigenvectors and $n_\ell = 477$ local eigenvectors. Fig. 3 displays the distribution of the number of eigenfrequencies for the global eigenvectors and for the local eigenvectors. It can be seen that there are numerous local eigenfrequencies intertwined with the global eigenfrequencies.

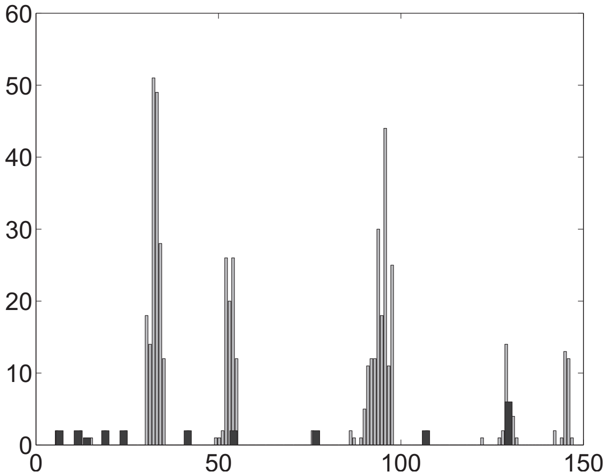

Figure 3: Distribution of the number of eigenfrequencies for the global eigenfrequencies (black histogram) and for the local eigenfrequencies (grey histogram) as a function of the frequency in Hz.

7.3 Frequency responses calculated with the mean model

For all $\omega \in \mathscr{B}$, the structure is subjected to two external point loads equal to $1\ N$ applied to a node which belongs to the lowest grid (stiff part) and a node belonging to the plain-tube 3-3 (see Fig. 1) (flexible part) located between the two lowest grids. The mean damping matrix is constructed using a modal damping corresponding to a damping rate $\xi = 0.01$. The response is calculated at two observation points, the point Pobs_1 located in the highest grid (stiff part) and the point Pobs_2 belonging to the plain-tube 3-3 (flexible part) located between the two highest grids. The response is calculated for different projections associated with the different bases: for the initial elastic modes with Eq. (3) ($n = 500$), for global eigenvectors with Eq. (12) ($n_g = 23$ and $n_\ell = 0$), for local eigenvectors with Eq. (12) ($n_g = 0$ and $n_\ell = 477$) and finally, for global and local eigenvectors with Eq. (12) ($n_g = 23$ and $n_\ell = 477$). The modulus in log scale of the responses are displayed in Fig. 4. It can be seen that the responses calculated using global and local eigenvectors are exactly the same

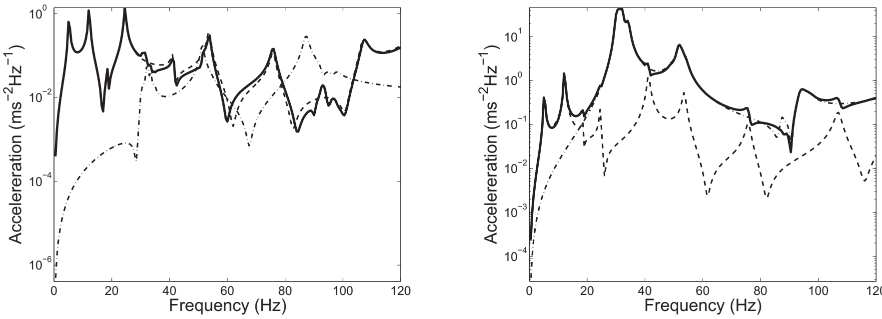

Figure 4: FRF for Pobs_1 (left) and Pobs_2 (right). Comparisons between different projection bases: initial elastic modes (solid thick line), global eigenvectors (dashed line), local eigenvectors only (mixed line), global and local eigenvectors (solid thin line superimposed to the solid thick line).

that the response calculated using the initial elastic modes. For point Pobs_1 in the stiff part, the contribution of the global eigenvectors is preponderant. For point Pobs_2 in the flexible part, the contribution of the local eigenvectors is important except for the two first resonances corresponding to the first global eigenvectors.

7.4 Random frequency responses calculated with the stochastic model

The random frequency responses is calculated as explained in Section 6. The dispersion parameters are chosen as $\delta_M = 0.1$, $\delta_D = 0.0$ and $\delta_K = 0.1$. The Monte Carlo simulation method is carried out with 400 simulations. The confidence regions corresponding to a probability level $P_c = 0.98$ are presented in Fig. 5. It can be seen that for observation points

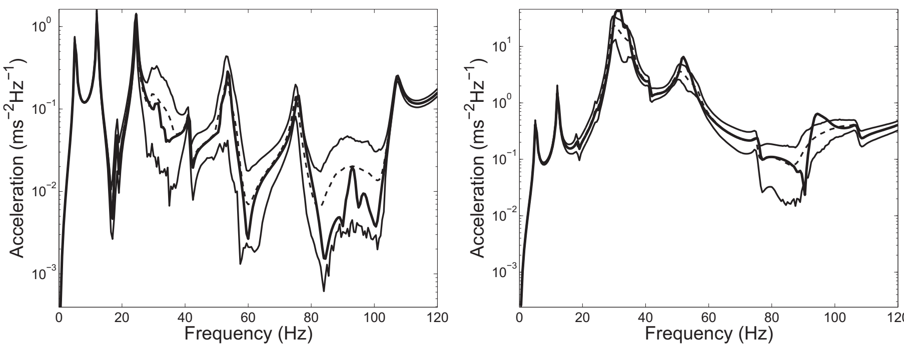

Figure 5: Random FRF for Pobs_1 (left) and Pobs_1 (right). Confidence region (lower and upper lines), mean response (solid middle line), deterministic response calculated with the initial elastic modes (dashed line).

Pobs_1 and Pobs_2 the sensitivity of the resonances relative to the global eigenvectors with respect to uncertainties is low. This variability increases at the frequencies for which the local contributions are not negligeable.

8 CONCLUSIONS

A general method has been developed and validated to construct a stochastic reduced-order model in low-frequency dynamics in presence of numerous local elastic modes. The projection basis is made up of two families of vector bases: the global eigenvectors and the local eigenvectors which are separately computed. This separation allows a probabilistic model of uncertainties to be implemented only for the local eigenvectors wich are not robust with respect to uncertainties..

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