# Construction and experimental identification of an uncertain model in computational dynamics using a generalized probabilistic approach of uncertainties

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## Abstract

We are interested in constructing an uncertain model of a nominal motor CSS of pressurized water reactor using a generalized probabilistic approach of uncertainties and in identifying this model using experimental measurements of the first eigenfrequencies. This generalized probabilistic approach of uncertainties allows both model-parameter uncertainties and model uncertainties to be taken into account and identified separately in the context of the experimental modal anaysis. Finally, the identified uncertain model allows statistics on quantities of interest to be estimated.

## 1 Introduction

The problem presented here concerns the development and the identification of a unique uncertain computational dynamical model of a nominal motor CSS of pressurized water reactor, having the capability to predict the responses of motors belonging to a given set of motors for which experimental measurements are available. We are interested in the prediction in a statistical sense of the first eigenfrequencies of the nominal motor using a computational dynamical models for which there are two types of uncertainties. The first one is related to the uncertainties of the computational model parameters (model-parameter uncertainties). The second one is due to the modeling errors which induce model uncertainties and which are due to the approximation introduced by the modeling process. Recently, a new generalized approach of uncertainties has been proposed (see [2]) to construct a prior probabilistic model of uncertainties. This method allows both model-parameter uncertainties and model uncertainties to be taken into account. In this method, unlike the classical output-prediction-error method (see [1]), the model errors are taken into account using the nonparametric probabilistic approach. The prior probabilistic models of the two types of uncertainties are constructed and identified in a separate way. We apply this method to construct a stochastic computational dynamical model of the motor CSS. Then, the identification of this model is performed using a set of experimental measurements corresponding to the motors belonging to the given set (and associated with several sites). Section 2 is devoted to the description of the motor CSS and to the experimental measurements. Sections 3 and 4 respectively present the mean computational model and the stochastic computational model. Section 5 is devoted to the identification of the dispersion parameters of the stochastic computational model.

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In the last Section, statistics on the three first eigenfrequencies are constructed using the identified stochastic computational model.

## 2 CSS motors and experimental measurements

CSS motors are components of Emergency Core Cooling Systems of Pressurized Water Reactors. We are interested in the low-frequency dynamical behaviour of such structures and more especially in the range of variation of the three first eigenfrequencies. The rotation frequency of this motor is fixed. In this paper, for reasons of confidentiality, all experimental and numerical frequencies have been divided by this rotation frequency and are therefore dimensionless. Therefore, the dimensionless rotation frequency is equal to 1. The first dimensionless eigenfrequencies have to be out of the range $[0.92, 1.08]$ in order to avoid a reasonance induced by the rotation of the motor. It should be noted that the results presented in this paper concern a nake motor without any added mass or added stiffness which would allow the first eigenfrequencies to be out this range with sufficient probability (Some CSS motors are already equipped of such added mass and added stiffness). The motor is represented on Fig. 1. The motor is fixed on a rectangular base plate which is fixed to the rectangular metallic plate. The Metallic plate is fixed on the concrete slab. The gimbal housing is fixed on the base plate. The motor is very rigid while the gimbal housing is flexible. An experimental modal analysis is carried out for 11 motors. The idenfied first dimensionless eigenfrequency for the 11 motors are $0.77$, $0.96$, $0.85$, $0.95$, $0.97$, $0.85$, $0.75$, $0.91$, $0.78$, $0.77$ and $0.81$ . The idenfied second dimensionless eigenfrequency for the 11 motors are $0.84$, $1.04$, $0.88$, $0.98$, $0.98$, $0.9$, $0.92$, $0.92$, $0.86$, $0.79$ and $0.85$. The third dimensionless is identified for only 11 motors. The idenfied third dimensionless eigenfrequencies are $1.01$, $1.0$, $1.13$, $0.85$ and $1.02$. These eigenfrequencies will be used for the identification of the dispersion parameters.

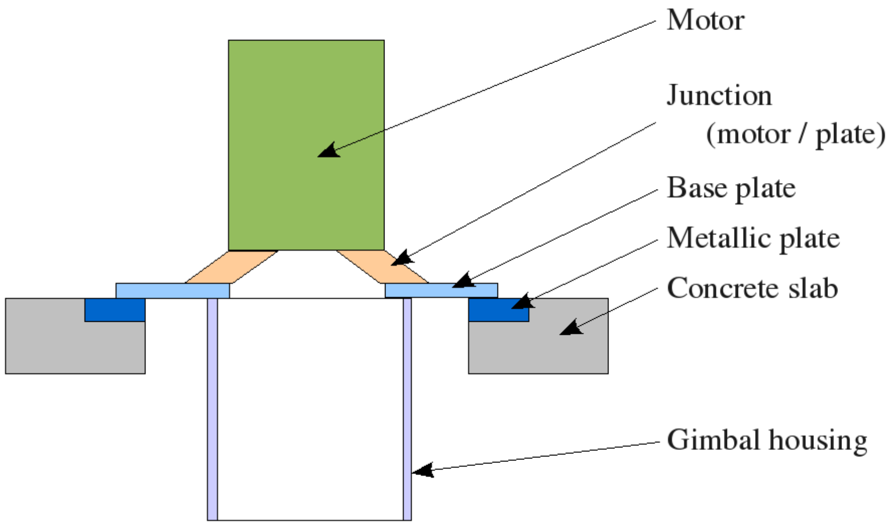


Figure 1: CSS motor

## 3 Mean computational model

The mesh of the finite element model of the mean computational model is represented on Fig. 2. The concrete slab, the base plate and the metallic plate are modeled by volume finite elements. The gimbal housing is modeled by a Timoshenko beam for which the area of the section is $A = 5.4 \times 10^{-3}\ m^2$ and the flexion inertia coefficients are $I_y = I_z = 3.4 \times 10^{-6}\ m^4$. The motor is modeled by a rigid body for which the total mass, the position of the center of and the tensor of inertia reduced at the center of mass are defined. The junction between the motor and the base plate is modeled by a rotation spring for which the rotation stiffness with respect to the two direction of flexion of the structure are both equal to $k_r = 8.4 \times 10^7 \; Nm/rad$. The caracteristics of the materials are summarized in Tab. 1. The four lateral sides of the concrete slab are fixed. The finite element model has $m = 40866$ DOFs. Then the eigenfrequencies $f_1, \dots, f_m$ are calculated by

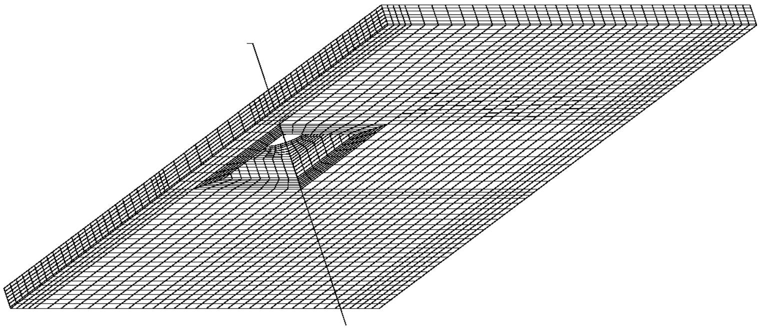

Figure 2: FE model

| | Mass density | Young modulus | Poisson modulus |
|---|---|---|---|
| Concrete slab | $2.6 \times 10^3 \; kg/m^3$ | $1.33 \times 10^{10} \; N/mm^2$ | 0.15 |
| Base plate | $7.8 \times 10^3 \; kg/m^3$ | $2.1 \times 10^{11} \; N/mm^2$ | 0.3 |
| Metallic plate | $7.1 \times 10^3 \; kg/m^3$ | $2.4 \times 10^{11} \; N/mm^2$ | 0.31 |
| Gimbal housing | $7.8 \times 10^3 \; kg/m^3$ | $2.1 \times 10^{11} \; N/mm^2$ | 0.3 |

Table 1: Caracteristics of the materials.

finding the solutions $\omega = 2\pi f$ and $\boldsymbol{\varphi}$ of the generalized eigenvalue problem

$$[K]\,\boldsymbol{\varphi} = \omega^2\,[M]\,\boldsymbol{\varphi}\,, \tag{1}$$

in which $[M]$ and $[K]$ are respectively the positive-definite symmetric real mass and stiffness matrices. Let $[\phi] = [\varphi_1, ..., \varphi_n]$ be the matrix the $n$ first eigenmodes. Then the generalized mass and stiffness matrices are defined by $[\tilde{M}] = [\phi]^T[M][\phi]$ and $[\tilde{K}] = [\phi]^T[K][\phi]$. The two first eigenfrequencies for the mean computational model are equal to the mean value of the two first eigenfrequencies which have been identifed experimentally, that is $f_1 = 0.86$ and $f_2 = 0.91$ (dimensionless values). The three first eigenmodes are shown on Fig. 3.

## 4 Stochastic computational model

The mean computational model presented in Section 3 ownes two type of uncertainties : model-parameter uncertainties and model uncertainties induced by modeling errors. The model-parameter uncertainites are induced by the variability of eight parameters $x_1, \dots, x_8$ of the mean computational model : the mass density, the Young modulus and the Poisson modulus of the concrete slab, the mass density, the Young modulus and the Poisson modulus of the metallic plate, the flexion inertia coefficient of the gimbal housing and the rotation stiffness of the spring. These two types of uncertainties are taken into account using the generalized probabilistic approach of uncertainties introduced in [2] and which allows an independent modeling of both model-parameter uncertainties and modeling errors to be performed.

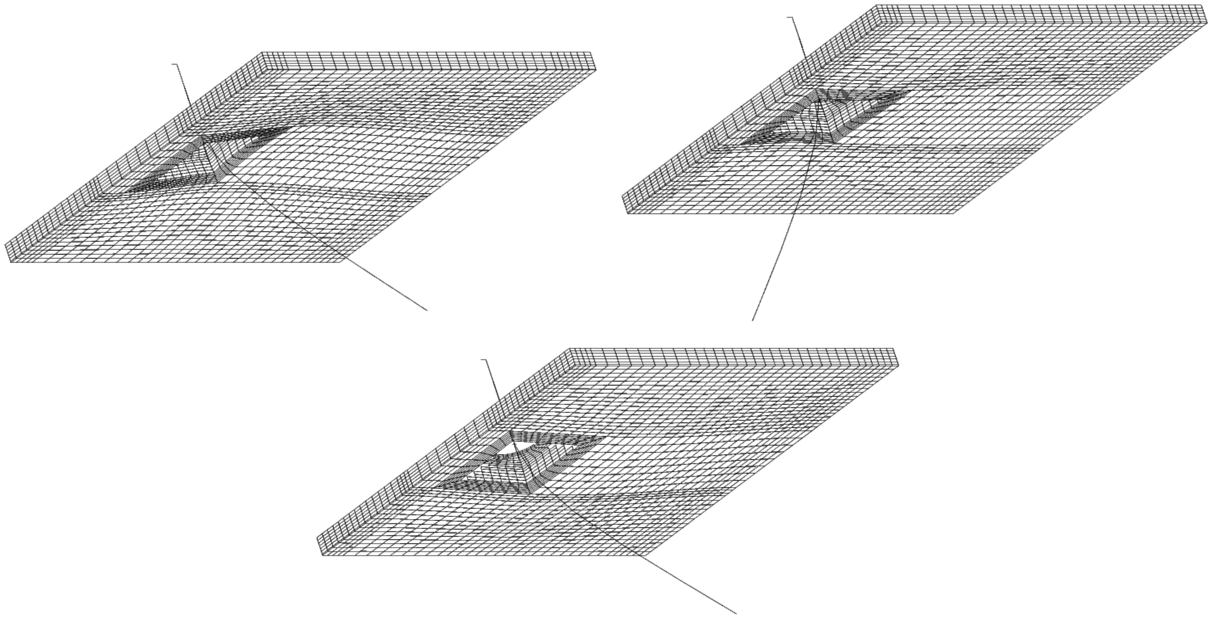

Figure 3: First, second and third eigenmodes

### 4.1 Construction of the probability model of the uncertain parameters

Each of the eight uncertain parameters $x_i$ is replaced by a random variable $X_i$ for which the probability density function is constructed using the maximum entropy principle under the available information defined by (1) the random variable $X_i$ is strictly positive (2) the mean value of the random variable is equal to the value of the nominal value $\underline{x}_i$ of the mean computational model (3) the inverse of the random variable $X_i$ is a second-order random variable (so that the random eigenvalues are also second order-random variables). Then, the probability density function $p_{X_i}(x_i)$ of the random variable $X_i$ is defined by

$$p_{X_i}(x_i;\delta_{x_i}) = \mathbb{1}_{]0,+\infty[}(x_i)\frac{1}{\underline{x}_i}\left(\frac{1}{\delta_{x_i}^2}\right)^{\frac{1}{\delta_{x_i}^2}}\frac{1}{\Gamma(1\delta_{x_i}^2)}\left(\frac{x_i}{\underline{x}_i}\right)^{\frac{1}{\delta_{x_i}^2}-1}\text{exp}\left(-\frac{x_i}{\delta_{x_i}^2\underline{x}_i}\right) \quad , \tag{2}$$

where $\Gamma(\alpha) = \int_0^{+\infty} t^{\alpha-1}e^{-t}\,dt$ is the Gamma function and where $\mathbb{1}_{]0,+\infty[}(x)$ is the indicator function. The parameter $\delta_{x_i} = \sigma_{X_i}/\underline{x}_i$ where $\sigma_{X_i}$ is the standard deviation of the random variable $X_i$, such that $0 \leq \delta_{x_i} < 1/\sqrt{2}$. Therefore, the probability density function of the random variable $X_i$ is completely defined by the mean value $\underline{x}_i$ which is given by the nominal model and by the dispersion parameter $\delta_{x_i}$ which has to be identified using the experimental measurements of the eigenfrequencies. Let $\boldsymbol{X} = (X_1,\ldots,X_8)$. Then, the random eigenfrequencies $F_1,\ldots,F_m$ are calculated by finding the solutions $\Omega = 2\pi F$ and $\boldsymbol{\varphi}(\boldsymbol{X})$ of the random generalized eigenvalue problem

$$[K(\boldsymbol{X})]\,\boldsymbol{\varphi}(\boldsymbol{X}) = \Omega(\boldsymbol{X})^2\,[M(\boldsymbol{X})]\,\boldsymbol{\varphi}(\boldsymbol{X})\,. \tag{3}$$

Then the random generalized mass and stiffness matrices are defined by $[\tilde{M}(\boldsymbol{X})] = [\phi(\boldsymbol{X})]^T[M(\boldsymbol{X})][\phi(\boldsymbol{X})]$ and $[\tilde{K}(\boldsymbol{X})] = [\phi(\boldsymbol{X})]^T[K(\boldsymbol{X})][\phi(\boldsymbol{X})]$.

### 4.2 Construction of the probability model of model uncertainties

The construction of the probability model of model uncertainties is based on the nonparametric probabilistic approach of uncertainties (see [2], [3], [4]). Therefore, dependent random matrices $[\tilde{M}(\boldsymbol{X})]$ and $[\tilde{K}(\boldsymbol{X})$ are replaced by the dependent random matrices $[\tilde{\boldsymbol{M}}(\boldsymbol{X})]$ and $[\tilde{\boldsymbol{K}}(\boldsymbol{X})]$. For each realisation $\boldsymbol{x}_i$ of the random variable $\boldsymbol{X}$, the probabilty density function of the random matrices $[\tilde{\boldsymbol{M}}(\boldsymbol{x}_i)]$ and $[\tilde{\boldsymbol{K}}(\boldsymbol{x}_i)]$ are constructed using the maximum entropy principle under the available information defined by (1) the random matrices $[\tilde{\boldsymbol{M}}(\boldsymbol{x}_i)]$ and $[\tilde{\boldsymbol{K}}(\boldsymbol{x}_i)]$ are positive definite (2) the mean value of the random matrices $[\tilde{\boldsymbol{M}}(\boldsymbol{x}_i)]$ and $[\tilde{\boldsymbol{K}}(\boldsymbol{x}_i)]$ are respectively equal to $[\tilde{M}(\boldsymbol{x}_i)]$ and $[\tilde{K}(\boldsymbol{x}_i)]$ (3) the frobenius norm of the random matrices $[\tilde{\boldsymbol{M}}(\boldsymbol{x}_i)]^{-1}$ and $[\tilde{\boldsymbol{K}}(\boldsymbol{x}_i)]^{-1}$ are second-order random variables (so that the random eigenvalues are also second order-random variables). The probability density functions of these random matrices depend on the dispersion parameters $\delta_M$ and $\delta_K$ which have to be identified using the experimental values of the eigenfrequencies. Then, the random eigenfrequencies $F_1, \ldots, F_m$ are calculated by finding the solutions $\Omega = 2\pi F$ and $\boldsymbol{\eta}$ of the random reduced generalized eigenvalue problem

$$[\tilde{\boldsymbol{K}}(\boldsymbol{X})]\,\boldsymbol{\eta} = \Omega^2\,[\tilde{\boldsymbol{M}}(\boldsymbol{X})]\,\boldsymbol{\eta}\,. \tag{4}$$

# 5 Estimation of the dispersion parameters

This section is devoted to the experimental identification of the dispersion parameters of the stochastic computational model constructed in the previous section.

## 5.1 Estimation of the dispersion parameter of the probability model of the uncertain parameters

In this step, the dispersion parameters $\delta_{x_1}, \ldots, \delta_{x_8}$ of the eight random variables $X_1, \ldots, X_8$ have to be identified using an observation which is weakly sensitive to model uncertainties. In general, the fundamental eigenfrequency is correctly represented by the Finite Element model of a considered structure. Therefore, the identification of the dispersion parameter of the probability model of the uncertain parameters is performed using only the experimental observation of the first eigenfrequency. The optimal value of the dispersion parameters is estimated using the maximum likelihood method. Such an estimation for eight dispersion parameters is difficult for two reasons: (1) The dimension of the admissible space of the dispersion parameters is relatively high (2) There are only 11 expermimental observations of the first eigenfrequency. We then have to make additional assumptions to reduce the number of dispersion parameters which have to be identified: (1) the dispersion parameters of the random mass density, of the random Young modulus and of the random Poisson modulus of the concrete slab are identical and denoted by $\delta_{\text{sl}}$, (2) the dispersion parameters of the random mass density, of the random Young modulus and of the random Poisson modulus of the metallic plate are identical and denoted by $\delta_{\text{pl}}$ and (3) the dispersion parameters of the random flexion inertia coeficient of the gimbal housing and of the random rotation stiffness of the spring are identical and denoted by $\delta_{\text{fl}}$. We then have to identify three dispersion parameters $\delta_{\text{sl}}$, $\delta_{\text{pl}}$ and $\delta_{\text{fl}}$. Let $\boldsymbol{\delta}_X = (\delta_{\text{sl}}, \delta_{\text{pl}}, \delta_{\text{fl}})$ and $\mathcal{C}_X$ denote the admissible space of $\delta_X$. The optimal value of $\boldsymbol{\delta}_X$ denoted by $\boldsymbol{\delta}_X^{opt}$ is solution of the following optimization problem

$$\boldsymbol{\delta}_X^{opt} = \arg\max_{\boldsymbol{\delta}_X \in \mathcal{C}_X} \sum_{i=1}^{11} \log(p_{F_1}(f_1^{\text{exp, i}}; \boldsymbol{\delta}_X)) \quad , \tag{5}$$

in which $p_{F_1}(f_1)$ is the probability density function of the random first eigenfrequency $F_1$ calculated using the stochastic computational model (with $\delta_M = \delta_K = 0$) and where $f_1^{\text{exp, 1}}, \ldots, f_1^{\text{exp, 11}}$ are the 11 experimental values of the first eigenfrequency. This optimization problem is solved using the trial method. The optimal value is $\boldsymbol{\delta}_X^{opt} = (0.28, 0.2, 0.32)$. The probability density function of the first and the third eigenfrequencies estimated using the stochastic computational model with $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and $\boldsymbol{\delta}_{MK} = (0, 0)$ are plotted on Fig. 4.

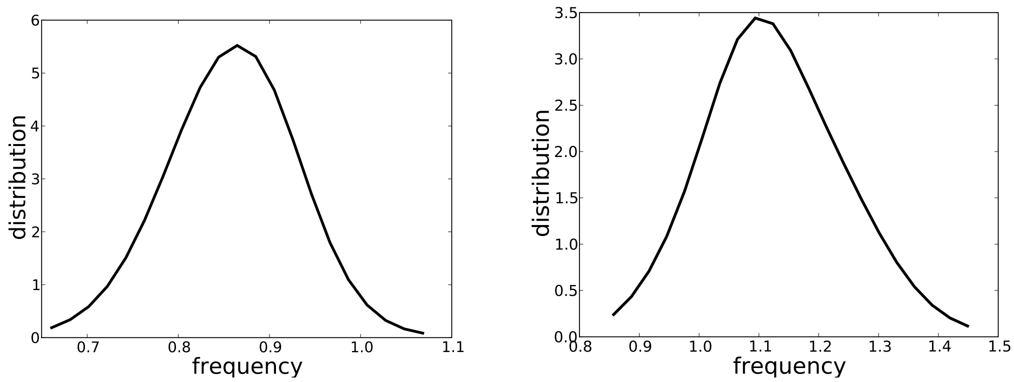


Figure 4: Probability density function of the first (left figure) and the third (right figure) dimensionless eigenfrequencies.

### 5.2 Estimation of the dispersion parameters of the probability model of model uncertainties

In this step, the dispersion parameters $\delta_M$ and $\delta_K$ have to be identified using only the experimental observation of the third eigenfrequency (which is sensitive to both model parameter uncertainties and model uncertainties). Let $\boldsymbol{\delta}_{MK} = (\delta_M, \delta_K)$ and $\mathcal{C}_{MK}$ be the admissible space of $\boldsymbol{\delta}_{MK}$. The optimal value of $\delta_{MK}$ denoted by $\delta_{MK}$ is estimated using the maximum likelihood method. Then, $\boldsymbol{\delta}_{MK}^{opt}$ is solution of the following optimization problem

$$\boldsymbol{\delta}_{MK}^{opt} = \arg \max_{\boldsymbol{\delta}_{MK} \in \mathcal{C}_{MK}} \sum_{i=1}^{5} \log(p_{F_1,F_2,F_3}(f_1^{\text{exp, i}}, f_2^{\text{exp, i}}, f_3^{\text{exp, i}}; \boldsymbol{\delta}_{MK})) \quad , \tag{6}$$

in which $p_{F_1,F_2,F_3}(f_1, f_2, f_3)$ is the joint probability density function of the three first eigenfrequencies $F_1$, $F_2$ and $F_3$ calculated using the stochastic computational model with $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and where $f_1^{\text{exp, 1}}, f_2^{\text{exp, 1}}, f_3^{\text{exp, 1}}$, $\ldots, f_1^{\text{exp, 5}}, f_2^{\text{exp, 5}}, f_3^{\text{exp, 5}}$ are the 5 experimental values of the first, second and third eigenfrequencies measured on five motors. The estimation of the likelihood is performed using the Monte Carlo simultation method (see [5]) with $n_s = 350$ simulations. The graph of function $(\delta_M, \delta_K) \mapsto \sum_{i=1}^{5} \log(\hat{p}_{F_1,F_2,F_3}(f_1^{\text{exp, i}}$, $f_2^{\text{exp, i}}, f_3^{\text{exp, i}}; (\delta_M, \delta_K)))$ , where $\hat{p}_{F_1,F_2,F_3}(f_1, f_2, f_3; \boldsymbol{\delta}_{MK}^{opt}, n_s)$ is the estimated probability density function of the three first eigenfrequencies, is plotted on Fig. 5. The maximum is reached for $\boldsymbol{\delta}_{MK}^{opt} = (0.42, 0)$. Figure 6 shows the convergence function $n_s \mapsto \sum_{i=1}^{5} \log(\hat{p}_{F_3}(f_3^{\text{exp, i}}; \boldsymbol{\delta}_{MK}^{opt}, n_s))$. The probability density function of the first and the third eigenfrequencies estimated using the stochastic computational model with $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and $\boldsymbol{\delta}_{MK} = \boldsymbol{\delta}_{MK}^{opt}$ are plotted on Fig. 7.

## 6 Statistics for the three first eigenfrequencies

In this section, we estimate statistics for the three first eigenfrequencies using the stochastic computational model constructed in Section 4 for which the dispersion parameters have been identified in Section 5. We are interested to the probability for each eigenfrequency to belong to the range $[0.92, 1.08]$. The statistics are estimated using the Monte carlo simuation method for two cases: (1) without taking into account model uncertainties, i.e. $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and $\boldsymbol{\delta}_{MK} = (0, 0)$ and (2) taking into account model uncertainties, i.e. $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and $\boldsymbol{\delta}_{MK} = \boldsymbol{\delta}_{MK}^{opt}$. The results for this two cases are respectively reported on Table 2 and Table 3. We can remark that model uncertainties increase the variability of the first eigenfrequencies and modify the probability of being in the range $[0.92, 1.08]$. These model uncertainties can be reduced by improving the mean computational model.

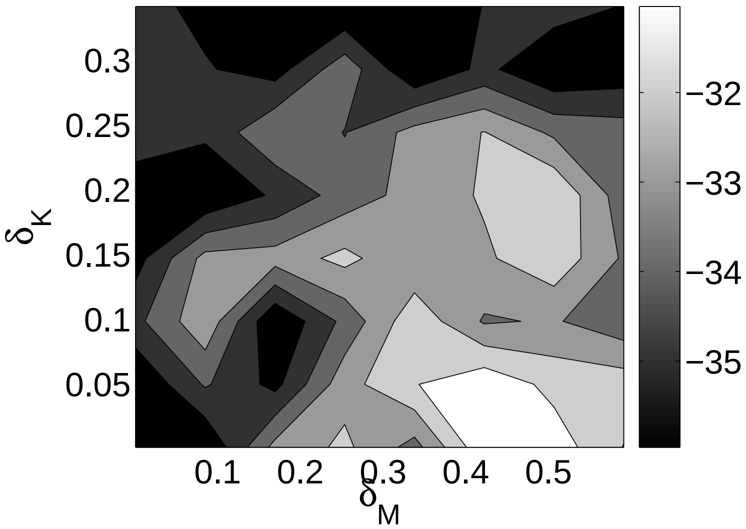


Figure 5: Graph of the function $(\delta_M, \delta_K) \mapsto \sum_{i=1}^{5} \log(p_{F_1,F_2,F_3}(f_1^{\mathrm{exp,\,i}}, f_2^{\mathrm{exp,\,i}}, f_3^{\mathrm{exp,\,i}}; (\delta_M, \delta_K)))$

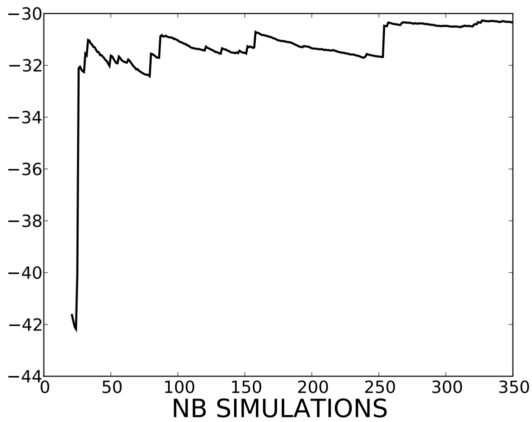


Figure 6: Graph of the function $n_s \mapsto \sum_{i=1}^{5} \log(\hat{p}_{F_1,F_2,F_3}(f_1^{\mathrm{exp,\,i}}, f_2^{\mathrm{exp,\,i}}, f_3^{\mathrm{exp,\,i}}; \boldsymbol{\delta}_{MK}^{opt}, n_s))$.

| | Mean | Coefficient of variation | Probability $\in [0.92, 1.08]$ |
|---|---|---|---|
| first eigenfrequency | 0.85 | 7.4 $\%$ | 0.38 |
| second eigenfrequency | 0.91 | 6.8 $\%$ | 0.71 |
| third eigenfrequency | 1.13 | 9.8 $\%$ | 0.35 |

Table 2: Statistics for the three first dimensionless eigenfrequencies with $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and $\boldsymbol{\delta}_{MK} = (0, 0)$.

## 7 Conclusions

We have constructed and experimentally identified an uncertain model of uncertainties using experimental measurement of eigenfrequencies. We have used a generalized probabilistic approach of uncertainties to construct a prior probability model which takes into account model-parameter uncertainties and model uncertainties. The dispersion parameters have been identified using the maximum likelihood method with observations which are adapted to identify separately the dispersion parameters relative to model-parameters

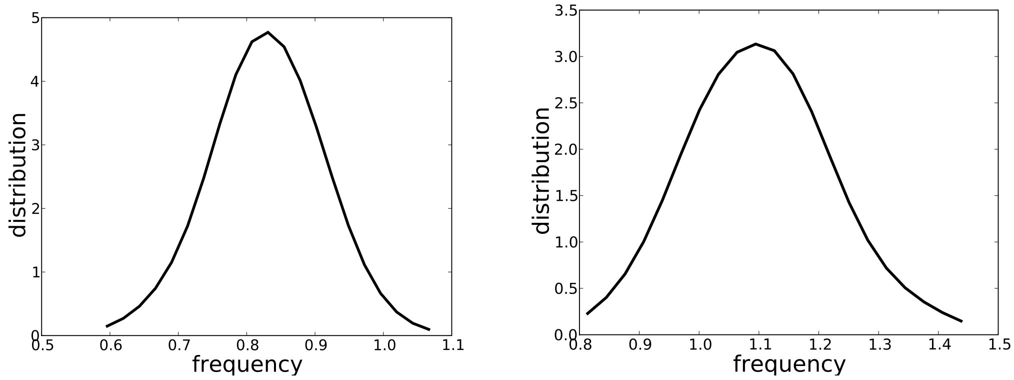


Figure 7: Probability density function of the first (left figure) and the third (right figure) dimensionless eigenfrequencies.

| | Mean | Coefficient of variation | Probability $\in [0.92, 1.08]$ |
|---|---|---|---|
| first eigenfrequency | $0.83$ | $8.6$ % | $0.23$ |
| second eigenfrequency | $0.92$ | $8.4$ % | $0.69$ |
| third eigenfrequency | $1.10$ | $10.1$ % | $0.42$ |

Table 3: Statistics for the three first dimensionless eigenfrequencies with $\boldsymbol{\delta}_X = \boldsymbol{\delta}_X^{opt}$ and $\boldsymbol{\delta}_{MK} = \boldsymbol{\delta}_{MK}^{opt}$.

uncertainties and the dispersion parameters relative to model uncertainties. Finally, statistics on the three first eigenfrequencies have been constructed using the identified stochastic model.

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