On mesoscopic probabilistic modeling of random anisotropic media under material symmetry constraints

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DRAFT: ON MESOSCOPIC PROBABILISTIC MODELING OF RANDOM ANISOTROPIC MEDIA UNDER MATERIAL SYMMETRY CONSTRAINTS

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ABSTRACT

In this study, we consider the probabilistic modeling of random media at mesoscale. More specifically, we address both the construction of a probabilistic model and the definition of a methodology allowing the numerical simulation (and consequently, the inverse experimental identification) of random elasticity tensors whose mean distance to a given class of material symmetry is specified. Following the eigensystem characterization of the material symmetries, the proposed approach relies on the probabilistic model derived in [1], allowing the variance of selected eigenvalues of the elasticity tensor to be partially prescribed. In this context, a new methodology is defined and applied to a transversely isotropic material. It is shown that the methodology allows the mean of the distance to this symmetry class to be reduced as the overall level of statistical fluctuations increases, no matter the initial distance of the mean model used in the simulations. A comparison between this approach and the non-parametric probabilistic model introduced in [2] is finally provided.

INTRODUCTION

This study focuses on the probabilistic modeling of random media at mesoscale. Such a problematic arises in experimental identification and/or industrial applications whose scale is basically close to the characteristic size of the heterogeneities (this is typically the case of concretes, long fiber reinforced composites or living tissues). While such a modeling has received a quite large attention (at least, in linear elasticity) within a micromechanical framework (see [3] [4]), yielding to the definition of bounds and hierarchies between the elasticity tensors of specimens of various sizes, the construction of probabilistic models allowing the mesoscopic randomness to be taken into account is still a challenging task and is precisely addressed in this paper. More specifically, this research deals with both the construction of a probabilistic model and the definition of a methodology allowing the numerical simulation (and consequently, the inverse experimental identification) of random elasticity tensors whose mean distance (in a sense to be defined) to a given class of material symmetry (for instance, the symmetry class that is usually assumed for the random media under consideration) is specified. It is worth noticing that such a framework is also suitable for the macroscopic modeling of elasticity tensors exhibiting uncertainties on the material symmetry class to which they belong.

SUMMARY OF THE APPROACH

Clearly, such a modeling can not be achieved, neither by using a parametric approach (which would imply by definition a null distance to the symmetry class, no matter the realization of the random elasticity tensor) nor by considering the non-parametric approach introduced in [2] and referred to as the *nonparametric approach for anisotropic media* (for which

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the anisotropic statistical fluctuations make the distance to a given material symmetry class increase with the overall level of fluctuation).

In this work, we consider the eigensystem coordinate-free characterization of the material symmetries [5], according to which a material symmetry class can be defined by both the multiplicities of the eigenvalues and constraints on the related eigenspaces. In this context, it should be pointed out that (**i**) the use of the classical random ensembles from the Random Matrix Theory generally implies all the stochastic eigenvalues to be of multiplicity one, and that (**ii**) the corresponding random eigenspaces cannot be explicitly constrained nor described. It then follows that the mean distance of the random elasticity tensor to a given material symmetry class can be partially controlled by imposing constraints on the variance of a few selected random eigenvalues. The proposed approach thus relies on the probabilistic model for symmetric positive-definite random matrices with prescribed variance on several eigenvalues, derived in [1]. It is shown that the model can be parameterized by:

- a parameter, denoted by α, controlling the overall level of statistical fluctuations $\delta_{\mathbf{C}}$;
- a set of m parameters, gathered in a deterministic vector $\tau \in \mathbb{R}^m$, allowing the variances of m selected stochastic eigenvalues (and consequently, the mean distance of the random elasticity tensor to a given material symmetry class) to be partially prescribed.

Furthermore, the distance $d\left([\mathbf{C}], [\mathbf{C}^{\mathrm{Sym}}]\right)$ of each realization of the random elasticity matrix representation $[\mathbf{C}]$ to a given material symmetry class is computed by considering its projection $[\mathbf{C}^{\mathrm{Sym}}]$ onto the class of elasticity tensors $\mathcal{C}^{\mathrm{Sym}}$ exhibiting given material symmetries. While several metrics (such as the Euclidean, Log-Euclidean and Riemannian metrics, denoted by d_E, d_{LE} and d_R respectively) have been introduced in the literature, the use of the Euclidean metric yields a closed-form expression of the projection (which is more suitable for the probabilistic analysis) and thus, the Euclidean projection has been retained in this research.

APPLICATION

In this application, we consider the following mean model (corresponding to a small distance to transverse isotropy):

$$[\underline{C}_S] = \begin{bmatrix} 10.1036 & 0.5391 & 2.9625 & -0.0040 & 0.0071 & -0.0165 \\ 0.5391 & 10.1061 & 2.9782 & -0.0041 & -0.0070 & -0.0036 \\ 2.9625 & 2.9782 & 182.690 & 0.0197 & 0.0016 & 0.0145 \\ -0.0040 & -0.0041 & 0.0197 & 14.0339 & 0.0068 & 0.0008 \\ 0.0071 & -0.0070 & 0.0016 & 0.0068 & 14.0121 & -0.0103 \\ -0.0165 & -0.0036 & 0.0145 & 0.0008 & -0.0103 & 9.5552 \end{bmatrix}$$

The mean distance to transverse isotropy is controlled by enforcing a small variance on the random eigenvalues λ_1, λ_2, λ_4 and λ_5. Setting all the parameters controlling the variances to the same value τ, the capability of the proposed approach to reduce the mean Riemannian distance to the considered symmetry class is clearly illustrated on Fig. 1, where the plot of $\tau \mapsto \mathrm{E}\left\{d_R\left([\mathbf{C}], [\mathbf{C}^{\mathrm{TI}}]\right)\right\}$ is reported in semi-log scale for $\alpha = 60$ (corresponding to $\delta_{\mathbf{C}} = 0.15$) and $\tau \in [10^{-1}, 10^4]$ ($\mathrm{E}\{\cdot\}$ denoting the mathematical expectation).

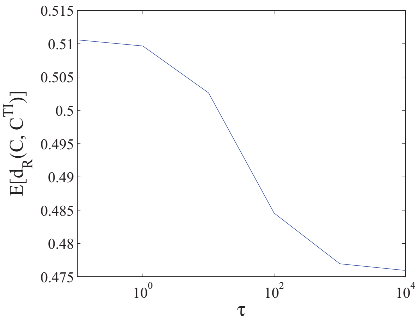

Figure 1. Plot of $\tau \mapsto \mathrm{E}\left\{d_R\left([\mathbf{C}], [\mathbf{C}^{\mathrm{TI}}]\right)\right\}$ ($\alpha = 60$).

Finally, the proposed approach and the nonparametric probabilistic model for anisotropic media are compared. It is shown that the methodology presented allows the reduction of the mean distance to transverse isotropy, no matter the mean model or the level of statistical fluctuations $\delta_{\mathbf{C}}$ used in the simulations.

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