Probabilistic model of human cortical bones with uncertain mechanical properties: Modelling and identification with experimental measurements in ultrasonic range

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PROBABILISTIC MODEL OF HUMAN CORTICAL BONES WITH UNCERTAIN MECHANICAL PROPERTIES FOR MODELING RANDOM EXPERIMENTAL MEASUREMENTS IN ULTRASONIC RANGE

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The biomechanical materials are among the most complex mechanical systems. Most often, their micro-structure are complex and random. This is the case for the human cortical bones which are considered in this paper. For such a system, the micro-structure can be altered near its interface with the marrow (osteoporosis). A gradient of porosity is then observed in the thickness direction but, in this case, none usual theory of porous materials can be applied. For this reason, we present a simplified model with gradient for the elasticity tensor. The predictability of this model is improved by taking into account uncertainties. The elasticity tensor is then modeled by a random field. This random model is well adapted for the modeling of the random experimental measurements in ultrasonic range for the human cortical bone.

Keywords: Probabilistic model, uncertainties, ultrasonic range, cortical bones, porous media.

1. Introduction

The biomechanical materials are among the most complex mechanical systems. Modeling such media is a challenge and the main difficulty is given rise to by the complexity level of their micro-structures. This is the case for the human cortical bones which are considered in this paper. For such a system, the micro-structure can be altered near its interface with the marrow (osteoporosis). A gradient of porosity is then observed in the thickness direction but, in this case, none usual theory of porous materials can be applied. For these reasons, these systems are often modeled using a simplified mechanical model which corresponds to a rough approximation of the real system. Nevertheless, the predictability of such a simplified model can be improved by taking into account the uncertainties introduced by these approximations. In this paper, a model for the human cortical bone is constructed. It consists of a fluid-solid semi-infinite multilayer system in which the solid layer (the cortical bone) is a non-homogeneous transverse isotropic elastic material and the two

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others semi-infinite layers (skin/muscles and marrow) are modeled by acoustical fluids. A gradient of the elasticity properties of the cortical bone is introduced in order to take into account the alterations of the cortical bone microstructure. Thus, inside the solid layer, the constitutive equation of the solid goes to the constitutive equation of the fluid (the marrow).

The uncertainties related to such a model are taken into account by modeling the elasticity tensor by a random field. The parameters of this probabilistic model are (1) the mean value of the effective thickness and the mean value of the elasticity tensor of the cortical bone and (2) the parameters controlling the level of uncertainties which depends on the spatial coordinates. The purpose is to present such a probabilistic model constructed within the framework of the theory of information. This probabilistic model should be adapted to the experimental identification of this parameters.

2. Simplified model

A simplified model of the biomechanical system made up of the coupling gel, the skin, the cortical bone and the marrow has been developed in Naili et al. (2010); Desceliers et al. (2009). This simplified model is composed of an elastic solid semi-infinite layer between two acoustic fluid semi-infinite layers (see fig. 1). Let $\mathbf{R}(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the reference Cartesian frame where O is the origin of the space and $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is an orthonormal basis for this space. The coordinates of the generic point $\mathbf{x}$ in $\mathbb{R}^3$ are (x_1, x_2, x_3). The thicknesses of the layers are denoted by h_1, h and h_2. The first acoustic fluid layer occupies the open unbounded domain Ω_1 , the second acoustic fluid layer occupies the open unbounded domain Ω_2 and the elastic solid layer occupies the open unbounded domain Ω. Let $\partial\Omega_1 = \Gamma_1 \cup \Sigma_1$, $\partial\Omega = \Sigma_1 \cup \Sigma_2$ and $\partial\Omega_2 = \Sigma_2 \cup \Gamma_2$ (see Fig. 1) be respectively the boundaries of Ω_1, Ω and Ω_2 in which $\Gamma_1, \Sigma_1, \Sigma_2$ and Γ_2 are the planes defined by

$$\begin{aligned}
\Gamma_1 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z_1\} \\
\Sigma_1 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = 0\} \\
\Sigma_2 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z\} \\
\Gamma_2 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z_2\}
\end{aligned}$$

in which $z_1 = h_1, z = -h$ and $z_2 = -(h + h_2)$. Therefore, the domains Ω_1, Ω and Ω_2 are unbounded along the transversal directions $\mathbf{e}_1$ and $\mathbf{e}_2$ whereas they are bounded along the vertical direction $\mathbf{e}_3$. A line source modeling an acoustical impulse is applied in domain Ω_1. This line source is defined with a source density Q_1 such that

$$\frac{\partial Q_1}{\partial t}(\mathbf{x}, t) = \rho_1 F(t) \delta_0(x_1 - x_1^S) \delta_0(x_3 - x_3^S)$$

in which $F(t) = F_1 \sin(2\pi f_c t) e^{-4(t f_c - 1)^2}$ where $f_c = 1\,\text{MHz}$ is the central frequency and $F_1 = 100\,\text{N}$; ρ_1 is the mass density of domain Ω_1; δ_0 is the Dirac function at the origin and x_1^S and x_3^S are the coordinates of a line source modeling the acoustical impulse. At time $t = 0$, the system is assumed to be at rest. Let $\rho(x_3)$ and $[C(x_3)]$ be the mass density and the effective elasticity matrix of the solid layer at a point x_3 in Ω_1. For a given effective elasticity matrix field $[C(\cdot)]$, the displacement field $\mathbf{u}$ in the solid layer Ω and the pressure fields p_1 and p_2 in the two fluids Ω_1 and Ω_2 respectively, are calculated using the fast and efficient hybrid solver presented in Desceliers et al. (2008).

3. Simplified model for a porous medium with gradient

It is well-known that bone medium are made of porous material. However, for the human cortical bones, the pore sizes are not small with respect to the thickness of the cortical layer. In addition,

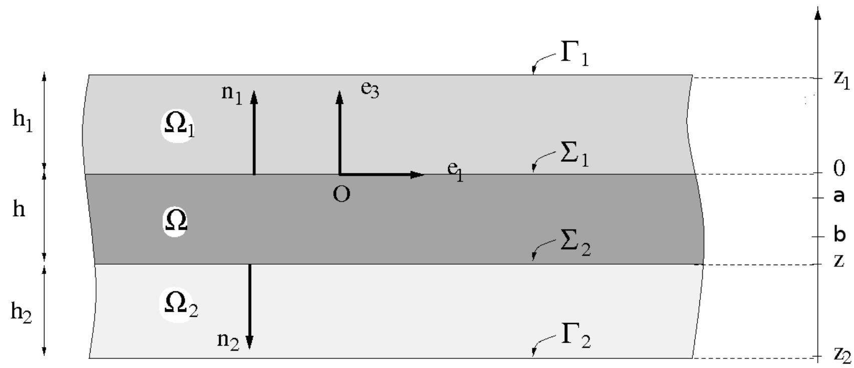

Fig. 1. Geometry of the multilayer system

the pore size increases along the transverse direction x_3. In case of osteoporosis, this gradient of porosity is such that, near interface Σ_2, the cortical material is mostly made up of a fluid. No usual theory on porous medium Biot (1956,b, 1962) is suitable for modeling such properties. Hereafter, we then propose an approach that allows the modeling of the elasticity matrix $[C(x_3)]$ to be still constructed within the usual framework of the continuum mechanics. Then, for all x_3 in $[a, 0]$, the material in the cortical layer is assumed to be locally a transverse isotropic medium and it is assumed to be a fluid for all x_3 in $[z, b]$. Consequently, (1) for all x_3 in $[0, a]$, we have $[C(x_3)] = [C^S]$ and $\rho(x_3) = \rho^S$; (2) for all x_3 in $[z, b]$ we have $[C^F]$ and $\rho(x_3) = \rho_2$; where $[C^S]$ is the elasticity matrix of a transverse isotropic medium, $[C^F]$ is the elasticity matrix of a fluid medium, ρ^S is the mass density of the cortical layer without taking into account the porosity and ρ_2 is the mass density of the second fluid (the marrow). All components of $[C^S]$ are zeros except the following

$$[C^S]_{11} = \frac{e_L^2(1-\nu_T)}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad (1)$$

$$[C^S]_{22} = \frac{e_T(e_L - e_T\nu_L^2)}{(1+\nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad (2)$$

$$[C^S]_{12} = \frac{e_T e_L \nu_L}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad (3)$$

$$[C^S]_{23} = \frac{e_T(e_L\nu_T + e_T\nu_L^2)}{(1+\nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad (4)$$

$$[C^S]_{44} = g_T \quad , \quad [C^S]_{55} = g_L \quad (5)$$

with $[C^S]_{22} = [C^S]_{33}$, $[C^S]_{12} = [C^S]_{13} = [C^S]_{21} = [C^S]_{31}$, $[C^S]_{23} = [C^S]_{32}$ and $[C^S]_{55} = [C^S]_{66}$ and where e_L and e_T are the longitudinal and transversal Young modulus, g_L and g_T are the longitudinal and transversal shear modulus and ν_L and ν_T are the longitudinal and transversal Poison coefficients such that $g_T = e_T/2(1+\nu_T)$. All components of $[C^F]$ are zero except $[C^F]_{11}$, $[C^F]_{12}$, $[C^F]_{13}$, $[C^F]_{21}$, $[C^F]_{22}$, $[C^F]_{23}$, $[C^F]_{31}$, $[C^F]_{32}$,$[C^F]_{33}$ that are all equal to $\rho_2\, c_2^2$. The proposal model of $[C(x_3)]$ and $\rho(x_3)$ is the following

$$[C(x_3)] = (1 - f(x_3))\,[C^S] + f(x_3)\,[C^F]$$
$$\rho(x_3) = (1 - f(x_3))\,\rho^S + f(x_3)\,\rho_2$$

where $f(x_3) = 1$ if $x_3 < b$, $f(x_3) = 0$ if $x_3 > a$ and $f(x_3) = c_0 + c_1\,x_3 + c_2\,x_3^2 + c_3\,x_3^3$ if $b \le x_3 \le a$ in which $c_0 = a^2\,(a - 3\,b)/(a-b)^3$, $c_1 = 6\,a\,b/(a-b)^3$, $c_2 = -3(a+b)/(a-b)^3$ and $c_3 = 2/(a-b)^3$.

This model has been constructed such that, for $x_3 = a$ or $x_3 = b$,

$$\frac{\partial[C(x_3)]}{\partial x_3} = 0 \quad \text{and} \quad \frac{\partial\rho(x_3)}{\partial x_3} = 0$$

4. Probabilistic model of the thickness and elasticity matrix of the cortical layer

The modeling these biomechanical materials is tricky due to the lack of knowledge on the micro-structure which is random and complex. In the two previous sections, a simplified model has been presented. The predictability of this model can be improved by taking into account these uncertainties. In this section, the probabilistic model of the elasticity matrix field is constructed by substituting the elasticity matrix field $x_3 \mapsto [C(x_3)]$ by a matrix-valued random field $x_3 \mapsto [\mathbf{C}(x_3)]$. The probabilistic model of random elasticity matrix field $x_3 \mapsto [\mathbf{C}(x_3)]$ is constructed using the maximum entropy principle Jaynes (1957a,b) within the framework of the theory of the information Shannon (1948) . We then consider the following available information (1) the random matrix $[\mathbf{C}(x_3)]$ is a second-order random variable with values in the set of all the (6×6) real symmetric positive-definite matrices; (2) the mean value of random matrix $[\mathbf{C}(x_3)]$ is the mean elasticity matrix $[C(x_3)]$; (3) the norm of the inverse matrix of $[\mathbf{C}(x_3)]$ is a second-order random variable. It has been shown in Soize (2006, 2008) that the random matrix $[\mathbf{C}(x_3)]$ can then be written as, for all $b < x_3 < 0$

$$[\mathbf{C}(x_3)] = [L(x_3)]^T[\mathbf{G}(x_3)][L(x_3)]$$

and since, for $x_3 < b$ the medium is not uncertain (it is a well-known fluid medium) then, for all $x_3 < b$

$$[\mathbf{C}(x_3)] = [C(x_3)]$$

in which the (6×6) upper triangular matrix $[L(x_3)]$ corresponds to the Cholesky factorization $[C(x_3)] = [L(x_3)]^T[L(x_3)]$ and where probability model of matrix-valued random field $x_3 \mapsto [\mathbf{G}(x_3)]$ is defined as the non-linear mapping of 21 second-order centered homogeneous Gaussian random fields $U_{jj'}(x_3)$ with $1 \leq j \leq j' \leq 6$. The explicit expression of this non-linear mapping can be found in [2, 3]. The stochastic germs $U_{jj'}(x_3)$ are then defined by the autocorrelation functions $R_{U_{jj'}}(\tau) = E\{U_{jj'}(x_3+\tau)U_{jj'}(x_3)\}$ such that

$$R_{U_{jj'}}(\tau) = (2\,\ell/\pi\,\tau)^2 \sin^2(\pi\,\tau/2\,\ell)$$

where the spatial correlation length ℓ is a parameter of the probabilistic model. The random field $x_3 \mapsto [\mathbf{G}(x_3)]$ also depends on an additional parameter $0 < \delta < (7/11)^{1/2}$ that is independent of x_3. This parameter controls the statistical fluctuations of $[\mathbf{G}(x_3)]$ and $[\mathbf{C}(x_3)]$ since it can be shown that

$$E\{\|[\mathbf{G}(x_3)]\|_F^2\} = 6(\delta^2+1)$$

$$\delta_C(x_3) = \frac{\delta}{\sqrt{7}}\left(1 + \frac{(\mathrm{tr}\,[C(x_3)])^2}{\mathrm{tr}\,[C(x_3]^2}\right)^{1/2} \qquad (1)$$

where $\delta_C(x_3)^2 = E\{\|[\mathbf{C}(x_3)] - [C(x_3)]\|_F^2\}/\|[C(x_3)]\|_F^2$ and $\|\cdot\|$ is the Frobenius norm. Finally, the spatial correlation length ℓ_C of random field $x_3 \mapsto [C(x_3)]$ is such that

$$\ell_C = \int_0^{+\infty} |r_c(\tau)|\, d\tau$$

where

$$\begin{aligned} r_c(\tau) = \mathrm{tr}\, E\{&([\mathbf{C}(x_3+\tau)] - [C(x_3])\\ &\times([\mathbf{C}(x_3)] - [C(x_3]])\}\\ &\times(E\{\|[\mathbf{C}(x_3)] - [C(x_3]\|_F^2\})^{-1} \end{aligned}$$

Then, the displacement of the solid layer and the two pressure of the fluid layers are random fields denoted by $\mathbf{U}, P_1$ and P_2.

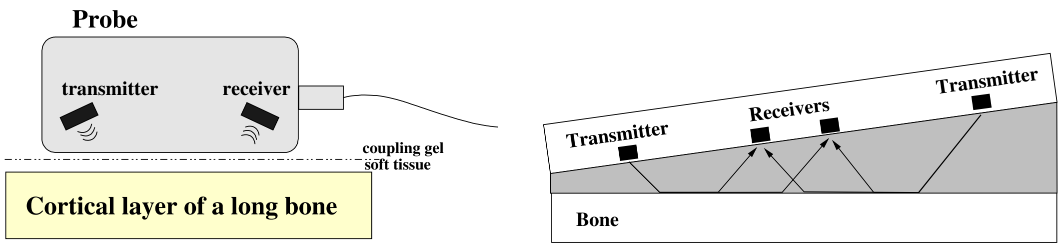

Fig. 2. Experimental configuration

5. APPLICATION

In a previous paper Desceliers et al. (2009), the components of matrix $[C^S]$ have been identified with an experimental database. The ultrasonic axial transmission technique has been used to construct this experimental database. The experimental configuration is described by Fig. 2. A device has been designed and is made up of $n_R = 11$ receivers and 2 transmitters. A coupling gel is applied at the interface between the device and the skin of the patient. Each transmitter generates an acoustical impulse in the ultrasonic range that propagates in the coupling gel, the skin, the muscle, the cortical bone and the marrow. The axial transmission technique consists in recording these signals at the $n_R = 11$ receivers receivers located in the device. The first arriving contribution of the signal (FAS) is considered. Following the signal processing method used with the experimental device, the velocity of FAS is determined from the time of flight of the first extremum of the contribution. This experimental database allows the components of matrix $[C^S]$ to be identified (see Desceliers et al. (2009)) and we obtained $\rho^S = 1598.8$ kg.m^{-3}, $e_L = 17.717$ GPa, $\nu_L = 0.3816$, $g_L = 4.7950$ GPa, $e_T = 9.8254$ GPa, $\nu_T = 0.4495$ and $\delta_C(0) = 0.1029$. Using Eq. (1) yields $\delta = 0.0575$. In this paper, we are interested by the propagation of the uncertainties to the first fluid domain Ω_1 for the cortical bone system in the context of the axial transmission technique. We then introduce the random variable Q that is such that

$$Q = \int_0^T \sum_k^{n_R} |P_2(t, x_1^k)|^2 \, dt$$

where T is the duration of an experimental signal and x_1^k, with $k = 1, \ldots, n_R$ are the positions of the receivers along direction $\mathbf{e}_1$. Let $p_Q(a, b, L; q)$ be the probability density function of random variable Q. In Fig. 3, the graph of $x_3 \mapsto \delta_C(x_3)$ is shown with $a = 0, b = z$ (thin solid line) and $a = z/2, b = z$ (thick solid line) and $a = 0, b = z/2$ (dashed thin line). It can been seen that the value of the dispersion coefficient $\delta_C(x_3)$ of the random matrix $[\mathbf{C}(x_3)]$ decreases when the constitutive equations of the material go to the constitutive equations of a fluid. In Fig. 4, the graph of $q \mapsto p_Q(a, b, L; q)$ is shown in logscale with $a = 0$, $b = z$, $L = h/10$ (thick solid line), with $a = z/2$, $b = z$, $L = h/10$ (thin solid line), with $a = 0$, $b = z$, $L = h/20$ (thick dashed line), with $a = z/2$, $b = z$, $L = h/20$ (thin dashed line). It can be seen that the probability density function is sensitive with respect to the thickness a and to the correlation length L.

6. CONCLUSION

In this paper we have considered the transient dynamical response of a multilayered system submitted to an impulse

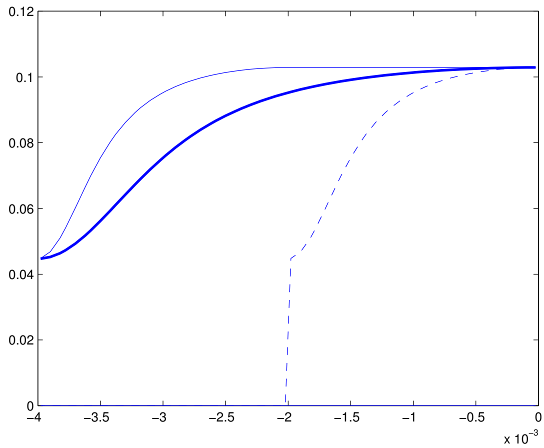

Fig. 3. Graph of $x_3 \mapsto \delta_C(x_3)$ with $a = 0, b = z$ (thin solid line) and $a = z/2, b = z$ (thick solid line) and $a = 0, b = z/2$ (dashed thin line)

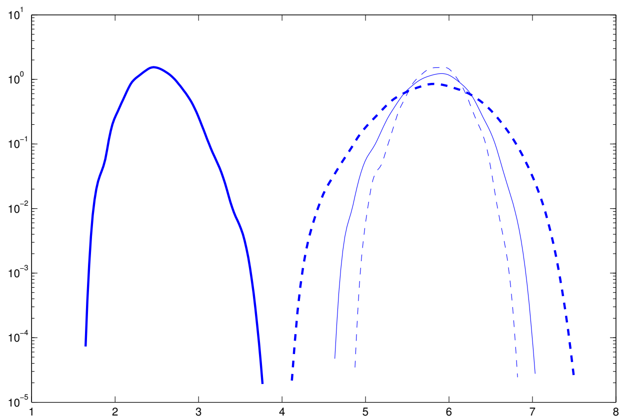

Fig. 4. Graph of $q \mapsto p_Q(a, b, L; q)$ in logscale with $a = 0$, $b = z$, $L = h/10$ (thick solid line), with $a = z/2$, $b = z$, $L = h/10$ (thin solid line), with $a = z/2$, $b = z$, $L = h/20$ (thin dashed line), with $a = z/2$, $b = z$, $L = h$ (thick dashed line).

in the ultrasonic range. The application concern a biomechanical system: the human cortical bone. This system is really tricky to be modeled due to the lack of knowledge on its micro-structure. For such a system, the micro-structure can be altered near its interface with the marrow (osteoporosis). A gradient of porosity is then observed in the thickness direction but, in this case, none usual theory of porous materials can be applied. This is the reason why we have proposed a simple model of the elasticity tensor for media with a gradient of the porosity in order to take into account the alterations of the cortical bone micro-structure. Thus, inside the solid layer, the constitutive equation of the solid goes to the constitutive equation of the fluid (the marrow). Then, in order to improve the predictability of this simplified model, we take into account the uncertainties by substituting the elasticity tensor with a random field for which the probabilistic model has been constructed using the maximum entropy principle. An application has been proposed to study the propagation of these uncertainties on the pressure field inside the first fluid domain (the skin). Results show that the total energy of the random pressure pressure field is very sensitive to the gradient and the spatial correlation length of the random elasticity tensor in the cortical layer. Consequently, experimental measurements in the context of the axial transmission technique can be used in order to identify the parameters of this probabilistic model.

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