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# DRILL-STRING WITH UNCERTAINTY IN THE BIT ROCK INTERACTION

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**Abstract.** In this paper a probabilistic model is proposed for the bit-rock interaction model of a drill-string system. A new strategy to take into account uncertainties in a local constitutive nonlinear equation using the nonparametric probabilistic approach is developed. The deterministic model considers the main forces that are applied to the column such as bit-rock interaction, fluid-structure interaction and impact forces. The nonlinear Timoshenko beam theory is applied and the system is discretized by means of the Finite Element Method.

**Keywords:** drill-string dynamics, nonlinear dynamics, bit-rock probabilistic model, local nonlinearity

## 1. INTRODUCTION

We are interested in the analysis of flexible dynamical systems taken into account uncertainties [Sampaio and Ritto, 2008]. In the present work we analyze a drill-string system with uncertainties in the bit-rock interaction model.

The drill-string is composed by thin tubes called drill-pipes that together measure some kilometers and some thicker tubes called drill-collars that together have some hundred meters. The region composed by the thicker tubes is called Bottom-Hole-Assembly (BHA). Figure 1 shows the general scheme of the system analyzed. The forces taken into account are the motor torque (as a constant rotational speed at the top  $\Omega_x$ ), the supporting force  $f_{hook}$ , the torque  $t_{bit}$  and force  $f_{bit}$  at the bit, the weight of the column, the fluid forces, the impact and rubbing between the column and the borehole, the forces due to the stabilizer, and also the elastic and kinetic forces due to the deformation and to the motion of the structure.

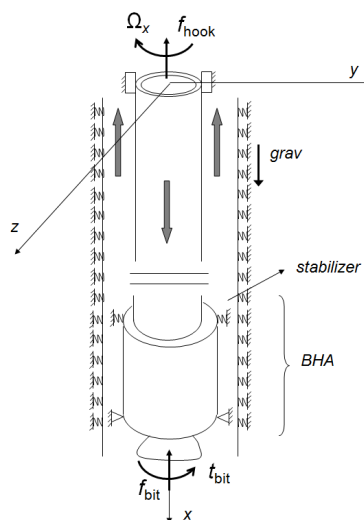


Figure 1. General scheme of the drill-string system.

There are some ways to model the nonlinear dynamics of a drill-string.

In a drilling operation there are many sources of uncertainties such as material properties (column and drilling fluid), dimensions of the system (especially the borehole), fluid-structure interaction and bit-rock interaction. The uncertainty analysis of the present paper is focused on the bit-rock interaction because it seems to be one of the most

important sources of uncertainties in this problem. There are few articles treating the stochastic problem of the drill-string dynamics, in especial we may cite [Spanos and Chevallier, 2000, Kotsonis and Spanos, 1997, Ritto et al., 2009b]. In [Spanos and Chevallier, 2000], stochastic lateral forces are analyzed at the bit, in [Kotsonis and Spanos, 1997], a random weight-on-bit is analyzed using a simple two degrees of freedom drill-string model and in [Ritto et al., 2009b], the weight-on-hook is taken as random.

The bit-rock interaction model used in the analysis is the one developed in [Tucker and Wang, 2003]. This model is able to reproduce the main phenomena and describes the penetration of the bit into the rock. Thus, it allows the analysis of the rate of penetration (ROP). Usually, the bit is assumed to be fixed or an average rate of penetration is assumed.

The nonparametric probabilistic approach [Soize, 2000, Soize, 2001, Soize, 2005] is used to model the uncertainties in the bit-rock interaction, which is represented by a nonlinear operator. It should be noticed that a new strategy is developed to take into account uncertainties for a local nonlinear operator.

The paper is organized as follows. In Section 2 the mean model is presented then, in Section 3, the probabilistic model of the bit-rock interaction model is developed. The results are shown in Section 4 and the concluding remarks are made in Section 5.

## 2. DETERMINISTIC MODEL

To derive the equations of motion, the extended Hamilton Principle is applied. For short, the expressions of the kinetic energy, strain energy, impact forces, and fluid forces are omitted, but they can be found in [Ritto et al., 2009c]. The system is discretized by means of the Finite Element Method.

### 2.1 Bit-rock interaction model

The model used in this work is the one developed in [Tucker and Wang, 2003], which can be written as

$$\begin{aligned} f_{\text{bit}} &= -\frac{\dot{u}_{\text{bit}}}{a_2 Z(\omega_{\text{bit}})^2} + \frac{a_3 \omega_{\text{bit}}}{a_2 Z(\omega_{\text{bit}})} - \frac{a_1}{a_2}, \\ t_{\text{bit}} &= -\frac{\dot{u}_{\text{bit}} a_4 Z(\omega_{\text{bit}})^2}{\omega_{\text{bit}}} - a_5 Z(\omega_{\text{bit}}), \end{aligned} \quad (1)$$

where  $f_{\text{bit}}$  is the axial force (also called weight-on-bit),  $t_{\text{bit}}$  is the torque about the  $x$ -axis,  $Z(\omega_{\text{bit}})$  is the regularization function and  $a_1, \dots, a_5$  are positive constants that depend on the bit and rock characteristics as well as on the average weight-on-bit. Note that these forces couple the axial and torsional vibrations.

### 2.2 Final discretized system of equations

The dynamics is considered about the following prestressed configuration:

$$\mathbf{u}_S = [K]^{-1} (\mathbf{f}_g + \mathbf{f}_c + \mathbf{f}_f). \quad (2)$$

where  $\mathbf{f}_g$  is the gravity,  $\mathbf{f}_c$  is the reaction force at the bit and  $\mathbf{f}_f$  is the fluid axial force. After assemblage, the final discretized system is written as

$$([M] + [M_f])\ddot{\bar{\mathbf{u}}} + ([C] + [C_f])\dot{\bar{\mathbf{u}}} + ([K] + [K_f] + [K_g(\mathbf{u}_S)])\bar{\mathbf{u}} = \mathbf{f}_{\text{NL}}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}), \quad (3)$$

in which  $\bar{\mathbf{u}} = \mathbf{u} - \mathbf{u}_S$ . The response  $\bar{\mathbf{u}}$  is represented in a subspace  $V_m \subset \mathbb{R}^m$ , where  $m$  equals the number of degrees of freedom of the system.  $[M]$ ,  $[C]$  and  $[K]$  are the usual mass, damping and stiffness matrices,  $[M_f]$ ,  $[C_f]$  and  $[K_f]$  are the fluid mass, damping and stiffness matrices,  $\mathbf{f}_f$  is the fluid force vector,  $[K_g(\mathbf{u}_S)]$  is the geometric stiffness matrix and  $\mathbf{f}_{\text{NL}}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}})$  is the nonlinear force vector that is written as

$$\mathbf{f}_{\text{NL}}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}) = \mathbf{f}_{\text{ke}}(\bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}) + \mathbf{f}_{\text{se}}(\bar{\mathbf{u}}) + \mathbf{f}_{\text{ip}}(\bar{\mathbf{u}}) + \mathbf{f}_{\text{br}}(\dot{\bar{\mathbf{u}}}) + \mathbf{g}(t). \quad (4)$$

where  $\mathbf{f}_{\text{ke}}$  is composed by the quadratic terms of the kinetic energy,  $\mathbf{f}_{\text{se}}$  is composed by the quadratic and higher order terms of the strain energy,  $\mathbf{f}_{\text{ip}}$  is the force vector due to the impact and rubbing between the column and the borehole,  $\mathbf{f}_{\text{br}}$  is the force vector due to the bit-rock interactions (see Section 2.) and  $\mathbf{g}(t)$  is the force that corresponds to the Dirichlet boundary condition (rotational speed imposed at the top).

### 2.3 Reduced model

Usually the final discretized FE system have big matrices (dimension  $m \times m$ ) and the dynamical analysis may be time consuming, which is the case of the present analysis. One way to reduce the system is to project the nonlinear dynamical equation on a subspace  $V_n$ , with  $n \ll m$ , in which  $V_n$  is spanned by an algebraic basis of  $\mathbb{R}^n$ . In the present paper, the basis used for the reduction corresponds to the normal modes projection. The normal modes are obtained from the following generalized eigenvalue problem

$$([K] + [K_f] + [K_g(\mathbf{u}_S)])\phi = \omega^2([M] + [M_f])\phi, \quad (5)$$

where  $\phi_i$  is the  $i$ -th normal mode and  $\omega_i$  is the  $i$ -th natural frequency. Using the representation  $\bar{\mathbf{u}} = [\Phi] \mathbf{q}$ , where  $[\Phi]$  is a  $(m \times n)$  real matrix composed by  $n$  normal modes, and projecting Eq. (3) on the subspace spanned by these normal modes yields

$$[M_r] \ddot{\mathbf{q}}(t) + [C_r] \dot{\mathbf{q}}(t) + [K_r] \mathbf{q}(t) = [\Phi]^T \mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}), \quad (6)$$

where

$$\begin{aligned} [M_r] &= [\Phi]^T([M] + [M_f])[\Phi], & [C_r] &= [\Phi]^T([C] + [C_f])[\Phi], \\ [K_r] &= [\Phi]^T([K] + [K_f] + [K_g(\mathbf{u}_S)])[\Phi] \end{aligned} \quad (7)$$

are the reduced matrices.

### 3. PROBABILISTIC MODEL FOR THE BIT-ROCK INTERACTION

The parametric probabilistic approach allows physical-parameter uncertainties to be modeled. It should be noted that the underlying deterministic model defined by Eq. (1) exhibits parameters  $a_1, a_2, a_3, a_4$  and  $a_5$  which do not correspond to physical parameters. Consequently, it is difficult to construct an *a priori* probabilistic model using the parametric probabilistic approach. For instance, there is no available information concerning the statistical dependence of these parameters. Then, we propose to apply the nonparametric probabilistic approach to model uncertainties [Soize, 2000] which consists in modeling the operator of the constitutive equation (Eq. (1)) by a random operator. Such an approach allows both system-parameters uncertainties and modeling errors to be globally taken into account.

Let  $f_{bit}(\dot{\mathbf{x}})$  and  $\dot{\mathbf{x}}$  be such that

$$f_{bit}(\dot{\mathbf{x}}) = \begin{pmatrix} f_{bit}(\dot{\mathbf{x}}) \\ t_{bit}(\dot{\mathbf{x}}) \end{pmatrix} \quad \text{and} \quad \dot{\mathbf{x}} = \begin{pmatrix} \dot{u}_{bit} \\ \omega_{bit} \end{pmatrix}. \quad (8)$$

Equation (1) can be rewritten as

$$f_{bit}(\dot{\mathbf{x}}) = -[A_b(\dot{\mathbf{x}})]\dot{\mathbf{x}} = - \begin{pmatrix} \frac{a_1}{a_2} + \frac{\dot{u}_{bit}}{a_2 Z(\omega_{bit})^2} - \frac{a_3 \omega_{bit}}{a_2 Z(\omega_{bit})} \\ \frac{a_4 Z(\omega_{bit})^2 \dot{u}_{bit}}{\omega_{bit}} + a_5 Z(\omega_{bit}) \end{pmatrix}. \quad (9)$$

We can deduce (see [Ritto et al., 2009c])

$$\begin{aligned} [A_b(\dot{\mathbf{x}})]_{11} &= \frac{a_1}{a_2 \dot{u}_{bit}} + \frac{1}{a_2 Z(\omega_{bit})^2} - \frac{a_3 \omega_{bit}}{a_2 Z(\omega_{bit}) \dot{u}_{bit}}, \\ [A_b(\dot{\mathbf{x}})]_{22} &= \frac{a_4 Z(\omega_{bit})^2 \dot{u}_{bit}}{\omega_{bit}^2} + \frac{a_5 Z(\omega_{bit})}{\omega_{bit}}, \\ [A_b(\dot{\mathbf{x}})]_{12} &= [A_b(\dot{\mathbf{x}})]_{21} = 0. \end{aligned} \quad (10)$$

It can be seen that for all  $\dot{\mathbf{x}}$  belonging to its admissible space  $\mathcal{C}$ ,  $[A_b(\dot{\mathbf{x}})]$  is positive-definite.

For all  $\dot{\mathbf{x}}$  in  $\mathcal{C}$ , the constitutive equation defined by Eq. (9) becomes a random constitutive equation which can be written as

$$\mathfrak{F}_{\text{bit}}(\dot{\mathbf{x}}) = -[\mathbf{A}_b(\dot{\mathbf{x}})]\dot{\mathbf{x}}. \quad (11)$$

Random matrix  $[\mathbf{A}_b(\dot{\mathbf{x}})]$  is defined by

$$[\mathbf{A}_b(\dot{\mathbf{x}})] = [L_b(\dot{\mathbf{x}})]^T [\mathbf{G}_b] [L_b(\dot{\mathbf{x}})]. \quad (12)$$

In the above equation,  $[\mathbf{G}_b]$  is a random matrix and the probability density function constructed by means of the Maximum Entropy Principle [Shannon, 1948, Jaynes, 1957a, Jaynes, 1957b] can be found in [Soize, 2005]. In the construction proposed, random matrix  $[\mathbf{G}_b]$  neither depends on  $\dot{\mathbf{x}}$  nor on  $t$ . Let the dispersion parameter  $\delta$  be such that

$$\delta = \left\{ \frac{1}{2} E \{ \| [\mathbf{G}_b] - [I] \|_F^2 \} \right\}^{\frac{1}{2}}. \quad (13)$$

#### 4. NUMERICAL RESULTS

The drill-string is discretized with 56 finite elements. For the construction of the reduced dynamical model, 158 lateral modes, 4 torsional modes, 3 axial modes and also the two rigid body modes of the structure (axial and torsional) are used. As boundary conditions, the lateral displacements and the rotations about the  $y$  and  $z$ -axis are zero at the top. The lateral displacements at the bit are also zero. A constant rotational speed about the  $x$ -axis  $\Omega_x$  is imposed at the top. Stabilizers are considered as elastic elements:  $F_y|_{x=x_{\text{stab}}} = k_{\text{stab}} v|_{x=x_{\text{stab}}}$  and  $F_z|_{x=x_{\text{stab}}} = k_{\text{stab}} w|_{x=x_{\text{stab}}}$ , where  $x_{\text{stab}}$  is the stabilizer location and  $k_{\text{stab}}$  is the stabilizer stiffness. As initial conditions, all the points move with constant axial speed and constant rotational speed about the  $x$ -axis, and the column is deflected laterally.

For the time integration procedure, the implicit Newmark integration scheme has been implemented with a predictor and a fix point procedure to equilibrate the system response at each time step. The data used for computations are representative values that are found in the literature (see [Ritto et al., 2009c]). All the numerical results presented below correspond to the forced response (deterministic case) and to the stationary response (stochastic case) for which the transient part of the response induced by the initial conditions has vanished. The results presented are the time response and the frequency spectrum defined as the modulus of its Fourier transform. The dynamical system is excited by a constant rotational speed about the  $x$ -axis at the top and with value 0.83 Hz.

##### 4.1 Stochastic response

The stochastic response is computed for two values of  $\delta$  which are 0.001, 0.1. Figure 2 displays the random ROP for  $\delta = 0.001$ . This figure shows the response of the mean model together with the mean response of the stochastic model and the 95% statistical envelope.

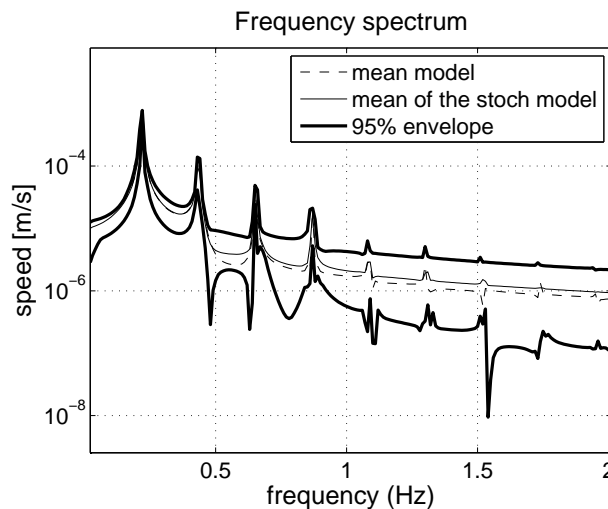


Figure 2. Stochastic response for  $\delta = 0.001$ . Frequency spectrum of the ROP.

Fig. 2 shows that the dispersion of the random ROP is already significant in the high part of the frequency band. However, the stochastic response in the low part of the frequency band is robust for the level of uncertainties considered.

Figure 3 shows the random radial displacement at  $x = 700$  m (middle point of the drill pipe). It can be seen that the lateral vibrations are also affected by the probabilistic model of the bit-rock interaction.

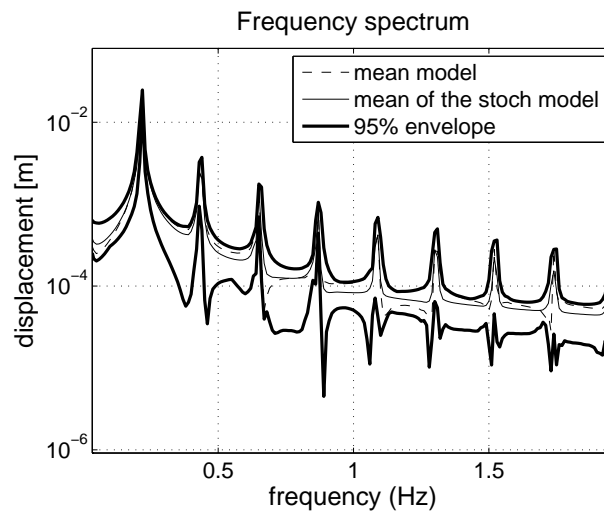


Figure 3. Stochastic response for  $\delta = 0.001$ . Frequency spectrum of the radial displacement at  $x = 700$  m.

Figure 4 shows the random rotational speed of the bit for  $\delta = 0.1$ . It can be noted that, for this level of uncertainty, the dispersion of the stochastic response is significant for all the frequency band analyzed.

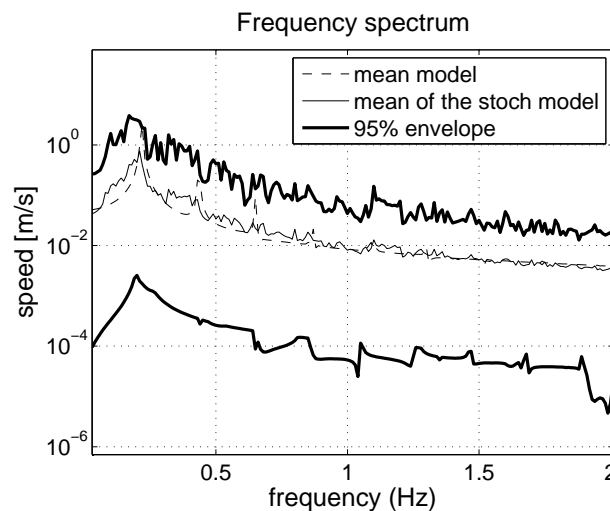


Figure 4. Stochastic response for  $\delta = 0.1$ . Frequency spectrum of the rotational speed of the bit  $\omega_{bit}$ .

The probabilistic model proposed for the bit-rock interaction model allows us to simulate cases such as the column impacting the borehole and the bit losing contact with the soil. The nonparametric probabilistic approach permits both parameters and modeling errors to be taken into account for the bit-rock interaction model.

## 5. CONCLUDING REMARKS

A computational nonlinear dynamical model taking into account uncertainties has been developed to simulate the drill-string dynamics and it has been shown to be well suited to describe the problem. A probabilistic model has been proposed for the bit-rock interaction model. Since the parameters of the bit-rock interaction do not correspond to physical parameters, these parameters are not adequate to the use of the parametric probabilistic approach. Then, the nonparametric probabilistic approach has been applied. This corresponds to a completely novel approach to take into account model uncertainties in a nonlinear constitutive equation. Since the dynamical system is globally nonlinear, an adapted strategy has been developed to implement the stochastic simulation.

The Timoshenko beam model has been used and the main forces that affect the dynamics of the drill-string have been considered such as the bit forces, the fluid forces and impacts. Finite strains have been taken into account without

neglecting the higher order terms and the vibration has been computed about a prestressed configuration.

## 6. ACKNOWLEDGEMENTS

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