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MODELLING THE VOICE PRODUCTION PROCESS USING A NONPARAMETRIC APPROACH

Edson Cataldo[1], Rubens Sampaio[2], Christian Soize[3] and Jorge Lucero[4]

[1]Universidade Federal Fluminense
Applied Mathematics Department – Graduate Program in Telecommunications Engineering
Rua Mário Santos Braga, S/N – Centro – Niterói – RJ – 24020-140 – Brazil
e-mail: ecataldo@im.uff.br

[2], PUC-Rio
Mechanical Engineering Department
Rua Marquês de São Vicente, 225 – Gávea – Rio de Janeiro – RJ – Brazil
e-mail:rsampaio@puc-rio.br

[3] Université Paris-Est
Laboratoire Modélisation et Simulation Multi Echelle – MSME FRE 3160 CNRS – 5, Bd Descartes,
77454 – Marne–la–Vallée – France
e-mail: christian.soize@univ-paris-est.fr

[4] Universidade de Brasília
Mathematics Department
Brasília – DF – 70190-900–Brazil
e-mail: lucero@unb.br

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Abstract. *The aim of this work is to discuss model uncertainties, in the case of the biomechanics of phonation. A number of mechanical models of voice production have been proposed in past years; but, in general, they have a deterministic nature. In previous works, data uncertainties were incorporated to a two-mass model of the vocal folds to perform a probabilistic analysis of the fundamental frequency of the oscillation. The present work is intended as a follow-up, to explore the effect of model uncertainties, using a non-parametric probabilistic approach based on the use of a probabilistic model for symmetric positive-definite real random matrices applying the Maximum Entropy Principle. The theory is discussed and numerical examples are presented to show that the predictability of the model may be improved when model uncertainties are taken into account. It is shown that some realizations of the output radiated pressure obtained are similar with samples of voice signals obtained from people with some pathologies, as nodulus and papilomas, in their vocal folds.*

1 INTRODUCTION

To improve the predictability of models of mechanical systems uncertainties from different sources must be considered. In general, two types of sources are discussed: (1) The first one concerns the data from which the various parameters of the model are derived. Usually these are uncertainties related to the geometry and properties of the material, and other similar parameters. One way to incorporate them is to consider the parametric probabilistic approach, in which the parameters are expressed as random variables; (2) The second source of uncertainties concerns the structure of the model itself. Those uncertainties arise from the fact that, to compute the response of a mechanical problem, a particular model is chosen, which does not necessarily capture accurately the real behavior of the mechanical system. Model uncertainties are difficult to quantify because they depend heavily on the type of problem, and their effect on the estimated response of a mechanical system is not obvious. Recently, a general non-parametric probabilistic approach of model uncertainties for dynamical systems has been proposed using random matrix theory [8].

In this work model uncertainties applied to a biomechanic model of phonation is discussed. In previous works, Cataldo et al [1, 2] incorporated data uncertainties, using a parametric approch, to a two-mass model of the vocal folds [3], to perform a probabilistic analysis of the fundamental frequency of the oscillation. Here, the aim is to follow-up the previous ideas and to explore the effect of model uncertainties using a nonparametric approach.

2 MEAN MODEL

The two-mass model of the vocal folds, originally proposed by Ishizaka and Flanagan [3], has provided a simple and effective representation of that system to study the underlying dynamics of voice production. Figure 1 shows a diagram of the model.

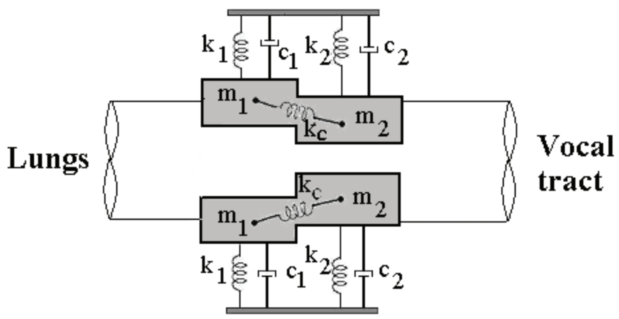

Figure 1: Two-mass model of the vocal folds.

For a complete representation of the vocal system, a vocal tract model must be coupled to the vocal fold model. Here, we adopt a simple two-tube aproximation of the vocal tract. Here, only uncertainties in the vocal fold system are taken into account. The parameters of the vocal tract will be considered deterministic.

The dynamics of the system is given by Eqs. (1) and (2) [1]:

$$\psi_1(\mathbf{w})\dot{u}_g + \psi_2(\mathbf{w})|u_g|u_g + \psi_3(\mathbf{w})u_g + \frac{1}{\tilde{c}_1}\int_0^t (u_g(\tau) - u_1(\tau))d\tau - y = 0 \qquad (1)$$

$$[M]\ddot{\mathbf{w}} + [C]\dot{\mathbf{w}} + [K]\mathbf{w} + \mathbf{h}(\mathbf{w}, \dot{\mathbf{w}}, u_g, \dot{u}_g) = 0 \tag{2}$$

where $\mathbf{w}(t) = (x_1(t), x_2(t), u_1(t), u_2(t), u_r(t))^t$, the functions x_1 and x_2 are the displacements of the masses, u_1 and u_2 describe the air volume flow through the (two) tubes that model the vocal tract and u_r is the air volume flow through the mouth. The subglottal pressure is denoted by y and u_g is the function that represent the glottal pulses signal. The function output radiated pressure p_r is given by $p_r(t) = u_r(t)r_r$, in which $r_r = \frac{128\rho v_c}{9\pi^3 y_2^2}$, ρ is the air density, v_c is the sound velocity, and y_2 is the radius of the second tube.

As the objective is to discuss uncertainties in the model of the vocal folds, the matrices $[M]$, $[C]$ and $[K]$ will be written as block matrices:

$$[M] = \begin{bmatrix} [M_{vf}] & 0 \\ 0 & [M_{vt}] \end{bmatrix} \;,\; [C] = \begin{bmatrix} [C_{vf}] & 0 \\ 0 & [C_{vt}] \end{bmatrix} \;,\; [K] = \begin{bmatrix} [K_{vf}] & 0 \\ 0 & [K_{vt}] \end{bmatrix} . \tag{3}$$

The functions ψ_1, ψ_2, ψ_3, $\mathbf{h}$, and also the matrices $[M_{vf}]$, $[M_{vt}]$, $[C_{vf}]$, $[C_{vt}]$, $[K_{vf}]$ and $[K_{vt}]$ are described in the appendix. Figure 2 shows the output radiated pressure, considering the following values of the parameters:

$\widehat{m}_1 = 0.125\,\text{g}$, $\widehat{m}_2 = 0.125\,\text{g}$, $\widehat{k}_c = 25\,\text{N/m}$, $\widehat{k}_1 = 80\,\text{N/m}$, $\widehat{k}_2 = 8\,\text{N/m}$, $\xi_1 = 0.1$, $\xi_2 = 0.6$, $\ell_g = 1.4\text{cm}$, $d_1 = 0.25\,\text{cm}$, $d_2 = 0.05\,\text{cm}$ and for the vocal tract model: $S_1 = 1\,\text{cm}^2$, $S_2 = 7\,\text{cm}^2$, $L_1 = 8.9\,\text{cm}$, $L_2 = 8.1\,\text{cm}$.

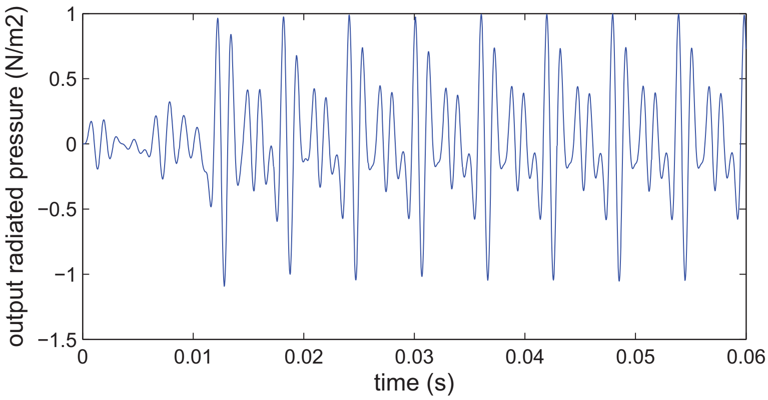

Figure 2: Output radiated pressure (normalized) computed from the mean model.

STOCHASTIC MODEL

To construct the corresponding stochastic model, a nonparametric approach is used. Probability density functions are constructed for the mass, damping and stiffness matrices, in order to incorporate uncertainties present on the linear part of the system.

Other quantities might be also considered as uncertain, but we restrict our analysis to the above parameters for simplicity. The general goal is to investigate the limits and the application of the non-parametric probabilistic approach to model voice production.

The matrices $[M_{vf}]$, $[C_{vf}]$ and $[K_{vf}]$ are substituted by random matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$ and $[\mathbf{K}_{vf}]$, respectively. Probability density functions are constructed for the matrices, based on a non-parametric approach developed by [8]. Consequently, the matrices $[M]$, $[C]$ and $[K]$ in Eq.(2) are substituted by the random matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$, respectively. Vectors $\mathbf{h}$ and $\mathbf{w}$ are substituted by the random vectors $\mathbf{H}$ and $\mathbf{W}$, respectively. Function u_g is also substituted by the random process U_g. Therefore, Eq. (6) becomes:

$$[\mathbf{M}]\ddot{\mathbf{W}} + [\mathbf{C}]\dot{\mathbf{W}} + [\mathbf{K}]\mathbf{W} + \mathbf{H}(\mathbf{W}, \dot{\mathbf{W}}, U_g, \dot{U}_g) = 0 \tag{4}$$

where matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ are given by

$$[\mathbf{M}] = \begin{bmatrix} [\mathbf{M}_{vf}] & 0 \\ 0 & [M_{vt}] \end{bmatrix} \;,\; [\mathbf{C}] = \begin{bmatrix} [\mathbf{C}_{vf}] & 0 \\ 0 & [C_{vt}] \end{bmatrix} \;,\; [\mathbf{K}] = \begin{bmatrix} [\mathbf{K}_{vf}] & 0 \\ 0 & [K_{vt}] \end{bmatrix} . \tag{5}$$

There are some parameters of stiffness and damping that are present in the nonlinear part of the system, that is, in the definition of function $\mathbf{h}$, but for simplicity of the analysis, they will not be considered as uncertain.

Probability density functions will be constructed for the matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$ and $[\mathbf{K}_{vf}]$ (see next section) and the Monte Carlo Method will be applied. Thus, for each realization θ, Eq. (6) will be written as

$$[\mathbf{M}(\theta)]\ddot{\mathbf{W}}(\mathbf{t},\theta)+[\mathbf{C}(\theta)]\dot{\mathbf{W}}(\mathbf{t},\theta)+[\mathbf{K}(\theta)]\mathbf{W}(\mathbf{t},\theta)+\mathbf{H}(\mathbf{W}(\mathbf{t},\theta), \dot{\mathbf{W}}(\mathbf{t},\theta), \mathbf{U_g}(\mathbf{t},\theta), \dot{\mathbf{U}}_\mathbf{g}(\mathbf{t},\theta)) = \mathbf{0} \,. \tag{6}$$

3 NONPARAMETRIC APPROACH

The non-parametric approach, rather than assessing the uncertainties on given parameters, tries to provide a quantification of the uncertainties on a higher level and specifically, at the level of the matrices of mass, damping and stiffness of the system. The method therefore relies on random matrices. The corresponding probability density functions are constructed by the Maximum Entropy Theorem [7, 4, 5], using algebraic properties on the matrices and their mean values as constraints. An unique parameter for each matrix is introduced to control the dispersion level.

The construction of the nonparametric probabilistic model is based on replacing the matrices $[M_{vf}]$, $[C_{vf}]$, $[K_{vf}]$ by the random matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$, $[\mathbf{K}_{vf}]$. The probabilistic model of each one of these matrices is described in the following.

Let $[\mathbf{A}]$ be the matrix that represents each one of the matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$ and $[\mathbf{K}_{vf}]$. When applied to mechanical systems, in general, the random matrix $[\mathbf{A}]$ should verify the following properties: **(i)** it is a real symmetric positive-definite matrix, almost surely; **(ii)** it is a second-order random variable; that is, $E\{||[\mathbf{A}]||_F^2\} < +\infty$; **(iii)** the mean value $E\{[\mathbf{A}]\} = [\underline{A}]$ is a real definite-positive matrix and **(iv)** $E\{ln(det[\mathbf{A}])\} = \nu_A$, $|\nu_A| < +\infty$.

However, for the random matrices $[\mathbf{M}_{vf}]$ and $[\mathbf{C}_{vf}]$ another restriction should be considered: their realizations must be always diagonal matrices. The realizations will be generated followng the general rule and the the elements extra diagonal will be set to zero.

Since $[\underline{A}]$ is positive definite, there is an upper triangular 2×2 matrix $[\underline{L}]$ such that $[\underline{A}] = [\underline{L}]^T[\underline{L}]$ and the random matrix $[\mathbf{A}]$ can be written as $[\mathbf{A}] = [\underline{L}]^T[\mathbf{N}][\underline{L}]$ in which $[\mathbf{N}]$ is a real random matrix definite-positive, almost surely, such that: **(i)** it is a second-order random variable; that is $E\{||[\mathbf{N}]||_F^2\} < +\infty$; **(ii)** its mean value is the 2×2 identity matrix and **(iii)** $E\{ln(det[\mathbf{N}]\} = \nu_N$, $|\nu_N| < +\infty$.

Realizations of the random matrix $[\mathbf{A}]$ can be computed from the realizations of the random matrix $[\mathbf{N}]$, as described in the following. It is defined with respect to the measure $\tilde{d}N$ given by

$$\tilde{d}N = 2^{1/2} \prod_{1 \leq i \leq j \leq 2} d[N]_{ij} \tag{7}$$

where $dN = \prod_{i \leq i,j \leq n} d[N]_{ij}$ is the Lebesgue measure on $\mathbb{R}^2$. With the usual normalization condition on the probability density function, it can be shown to be (Soize, 2000, 2001):

$$p_{[\mathbf{N}]}([N]) = \mathbf{1}_{\mathbb{M}_2^+(\mathbb{R})}([N]) \times C_{[\mathbf{N}]} \times (det[N])^{3(1-\delta^2)/(2\delta^2)} \times exp\left\{-\frac{3}{2\delta^2} tr[N]\right\} \tag{8}$$

in which $[N] \mapsto \mathbf{1}_{\mathbb{M}_2^+(\mathbb{R})}([N])$ is a function from $\mathbb{M}_2(\mathbb{R})$ (set of all real matrices 2×2) into $\{0, 1\}$ that is equal to 1 when $[N]$ is in $\mathbb{M}_2^+(\mathbb{R})$ and 0 otherwise, and where constant $C_{[\mathbf{N}]}$ is equal to

$$C_{[\mathbf{N}]} = \frac{(2\pi)^{-1/2} \left(\frac{3}{2\sigma^2}\right)^{3/\sigma^2}}{\prod_{j=1}^{n} \Gamma\left(\frac{3}{2\sigma^2} + \frac{1-j}{2}\right)} \tag{9}$$

with $z \mapsto \Gamma(z)$ the gamma function defined for $z > 0$ by $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt.$

The dispersion parameter δ_N is a real parameter defined, for any random matrix $[\mathbf{N}]$, with mean value $[\underline{N}]$, by

$$\delta_N = \left\{ \frac{E\{||[\mathbf{N} - I_2]||_F^2\}}{||[\underline{N}]||_F^2} \right\}^{1/2}, \tag{10}$$

where $||.||_F$ is the Frobenius norm. This dispersion parameter should be chosen independent of n and such that $0 < \delta < \sqrt{\frac{3}{7}}$, to ensure that the condition on the integrability of the inverse of the random matrices is verified.

The probability density functions of the three matrices of mass, damping and stiffness have been described as if they were independent. But, they correspond to the same physical problem and it could be argued that they should be dependent. In fact, they are, through their mean values, which are computed using the same mechanical parameters. But concerning their probabilistic model, since we did not specify any particular condition of dependence, the Maximum Entropy Theorem describes the matrices as independent (Soize,2000).

Then, one can write $[\mathbf{N}] = [\mathbf{L}^T][\mathbf{L}]$, where $[\mathbf{L}]$ is a real upper triangular random matrix. Let us introduce $\sigma_2 = \frac{\delta}{\sqrt{3}}$. It can be shown that

(i) random variables $([\mathbf{L}])_{1 \leq i \leq j \leq n}$ are independent;

(ii) for $i < j$, $[\mathbf{L}]$ can be written $[\mathbf{L}]_{ij} = \sigma_2 U_{ij}$, where U_{ij} is a Gaussian random variable with real values, zero mean and unit variance.

(iii) for $i = j$, $[\mathbf{L}]_{ij}$ can be written $[\mathbf{L}]_{ii} = \sigma_2\sqrt{2V_i}$, where V_i is a gamma random variable with positive real values and a probability density function $p_{V_i}(v)$ (with respect to dv) in the form

$$p_{V_i}(v) = \mathbf{1}_{\mathbb{R}^+}(v)\frac{1}{\Gamma\left(\frac{3}{2\sigma^2}+\frac{1-i}{2}\right)}v^{\frac{3}{2\sigma^2}-\frac{1+i}{2}}e^{-v}\,. \tag{11}$$

This algebraic structure of $[\mathbf{N}]$ allows an efficient procedure to be defined for the Monte Carlo numerical simulation of random matrix $[\mathbf{N}]$.

The nonparametric approach to stochastic modeling consists in setting the probability density functions of these matrices, ensuring that certain algebraic properties are verified. For the construction of these probability distributions, the mean value and a dispersion parameter have to be supplied for each of the matrices. We discarded the problem of the identification of the dispersion parameters, but we still have to address that of the mean value of the matrices $[\underline{M}_{vf}]$, $[\underline{C}_{vf}]$ and $[\underline{K}_{vf}]$.

4 RESULTS

The Monte Carlo method is a very general resolution technique, that can deal with complicated systems, with many random variables or processes. Its basic steps are: (1) generate samples of the input random parameter vector following its prescribed set of marginal laws, (2) compute the response of the system for each realization independently, (3) compute statistics of the response using these response samples. Then, the Monte Carlo method will be used. The main problem - and limitation- of this method derives from the computational time required to assess the statistics of the response, as it is the time necessary for one deterministic computation multiplied by the number of trials. The corresponding stochastic solver is based on a Monte Carlo numerical simulation. Realizations $[\mathbf{M}_{vf}](\theta)$, $[\mathbf{C}_{vf}](\theta)$, $[\mathbf{K}_{vf}](\theta)$, $A_{g0}(\theta)$ and $Y_s(\theta)$ of the random matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ are obtained from the probability density functions defined before.

One of the goals here is to construct probability density functions of the fundamental frequency when uncertainties in the model are taken into account using a nonparametric probabilistic approach. Then, for each realization of the stochastic problem associated, the fundamental frequency of the voice signal produced has to be calculated.

Let $F_0(\theta)$ be the fundamental frequency of the radiated pressure $P_r(t,\theta)$ evaluated for each realization. Let $T(\theta)$ be the period of the signal $U_g(t,\theta)$ for each realization θ. Then, $F_0(\theta) = 1/T(\theta)$.

The convergence analysis with respect to n is carried out in studying the convergence of the estimated second-order moment of F_0 defined by

$$\text{Conv}(n) = \frac{1}{n}\sum_{j=1}^{n} F_0(\theta_j)^2. \tag{12}$$

This convergence analysis is performed for $\delta_{[\mathbf{M_{vf}}]} = \delta_{[\mathbf{C_{vf}}]} = \delta_{[\mathbf{K_{vf}}]} = 0.05$ and Fig. 3 shows the graph of the function $n \mapsto \text{conv}(n)$. It can be noted that a reasonable convergence is reached for $n \geq 300$.

The estimation of the probability density function p_{F_0} of random variable F_0 is constructed as follows. Let M be the number of intervals. Let $I_j = [\nu_j, \nu_j + \Delta\nu[$ for $j = 1, \ldots, M$ with

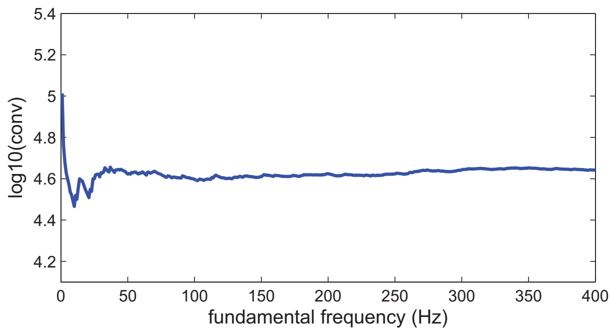

Figure 3: Convergence: graphs of functions $n \mapsto log_{10}$Conv.

$\nu_1 = \widetilde{f}_1$ and $\Delta\nu = (\widetilde{f}_n - \widetilde{f}_1)/M$. An estimation $\widehat{p}_{F_0}$ of the probability density function of F_0 is given by

$$\widehat{p}_{F_0}(f_0) = \sum_{j=1}^{M} \mathbf{1}_{I_j}(f_0) \frac{N_j}{n\Delta\nu} \,. \tag{13}$$

It can be noted that the mean value is near $140\,$Hz, which is inside the frequency band for fundamental frequencies for men (according to the data used).

The goal of this work is to discuss the capability of the nonparametric probabilistic approach of predicting the responses of a system, because it considers the model uncertainties.

Then, some realizations corresponding to the output radiated pressure were chosen and results are shown in Figs. 4, 5, 6, 7, 8, 9, which will be called signal A up to signal F. Herzel [6] showed some cases of output radiated pressure with pathological characteristics. Based on his ideas, from the realizations, six signals were chosen and they will be tested to verify if some characteristics of pathological signals can be found.

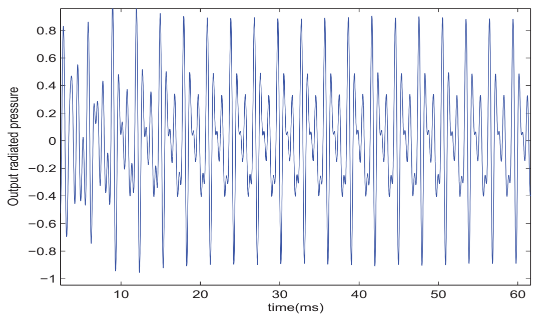

Figure 4: Output radiated pressure - signal A.

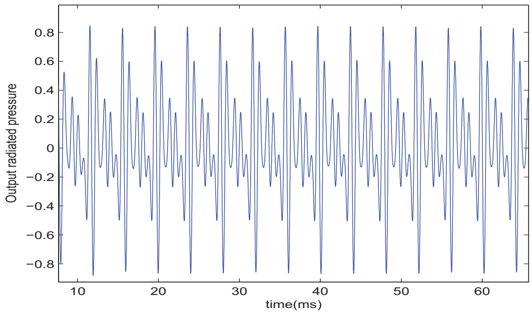

Figure 5: Output radiated pressure - signal B.

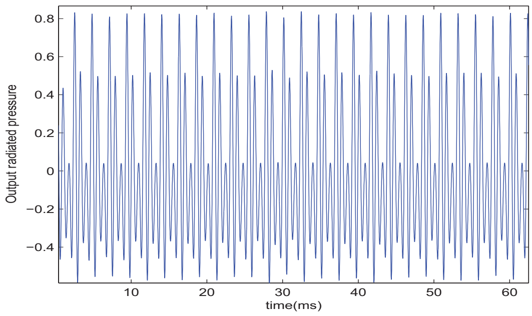

Figure 6: Output radiated pressure - signal C.

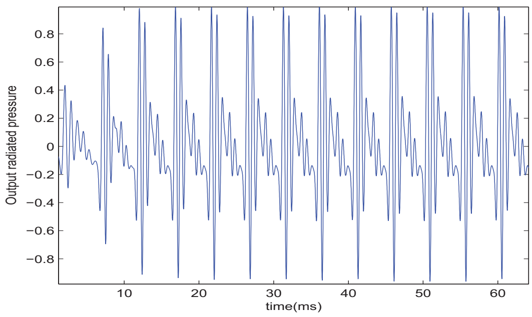

Figure 7: Output radiated pressure - signal D.

5 TESTING THE SIGNALS USING AN ARTIFICIAL NEURAL NETWORK

The signals chosen in the last section were tested by using an artificial neural network (ANN) described in the following. A multi-layer perceptron network was chosen to classify the voice signals, which inputs were the values obtained from the following acoustic measures : Jitter (local), Jitter (rap), Jitter (ppq5), Jitter (ddp), Shimmer (local), Shimmer (apq3), Shimmer (apq5), Shimmer (apq11), Shimmer (dda), HNR (Harmonic-noise ratio) and Pitch (fundamental frequency). The description of each acoustic measure can be found, for example, in [10]. To train the network were used 107 voice signals recorded from patients with normal and pathological voices, asking them to repeat a vowel /a/. The patologies used were: nodules, papilommas and

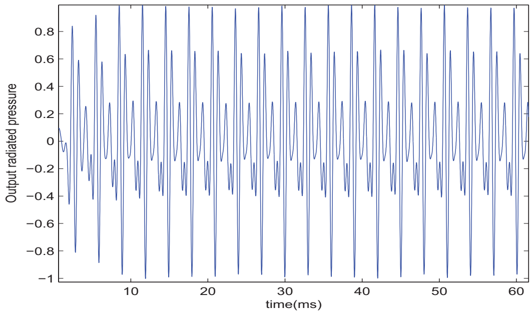

Figure 8: Output radiated pressure - signal E.

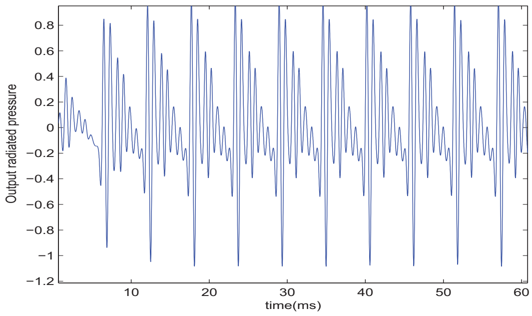

Figure 9: Output radiated pressure - signal F.

unilateral paralysis. After training the network, the six chosen signal showed before were tested and the results showed in Tab.1 were achieved:

Signal	**Classification**
A	nodule
B	normal
C	normal
D	papilomma
E	nodule
F	papilomma

Table 1: Signals classification.

These results show the possibility of understanding phenomena of voice production related to pathological cases. A possibility is to identify what were the realizations of the mass, stiffness and damping random matrices that generated the results obtained and then to reconstruct the corresponding mechanical model. This could be a good help for preventing or diagnosing pathologies related to voice production.

6 CONCLUSIONS

A non-parametric probabilistic approach has been proposed to model uncertainties in a model of vocal folds for voice production. As it has been presented in the literature, model uncertainties cannot be modeled by using parametric probabilistic approach, that is, if probability density functions are constructed directly for chosen uncertain parameters. The results obtained showed that the non-parametric probabilistic approach, applied to the model used, is capable of predicting realizations of the output radiated pressure which matches experimental data, both in cases of normal phonation and in voice disorders. By using the experience that has been acquired with this work and other, it can be said that some results obtained here could not be obtained by using the parametric probabilistic approach. But, this should be better investigated.

7 ACKNOWLEDGEMENTS

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APPENDIX

$\psi_1(\mathbf{w}) = (\frac{\rho d_1}{a_{g_0}+2\ell_g x_1} + \frac{\rho d_2}{a_{g_0}+2\ell_g x_2} + \tilde{\ell}_1)$

$\psi_2(\mathbf{w}) = (\frac{0.19\rho}{a_{g_0}+2\ell_g x_1} + 2\ell_g x_1) + \frac{\rho}{(a_{g_0}+2\ell_g x_2)^2}\left[0.5 - \frac{a_{g_0}+2\ell_g x_2}{a_1}\left(1 - \frac{a_{g_0}+2\ell_g x_2}{a_1}\right)\right]$

$\psi_3(\mathbf{w}) = (12\mu\ell_g \frac{d_1}{(a_{g_0}+2\ell_g x_1)^3} + 12\ell_g^2 \frac{d_2}{(a_{g_0}+2\ell_g x_2)^3} + r_1)$

$$[M_{vf}] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \;,\; [M_{vt}] = \begin{bmatrix} \tilde{\ell}_1 + \tilde{\ell}_2 & 0 & 0 \\ 0 & \tilde{\ell}_2 + \tilde{\ell}_r & -\tilde{\ell}_r \\ 0 & -\tilde{\ell}_r & \tilde{\ell}_r \end{bmatrix}$$

$$[C_{vf}] = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \;,\; [C_{vt}] = \begin{bmatrix} r_1 + r_2 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_r \end{bmatrix} \;,$$

$$[K_{vf}] = \begin{bmatrix} k_1 + k_c & -k_c \\ -k_c & k_2 + k_c \end{bmatrix} \;,\; [K_{vt}] = \begin{bmatrix} \frac{1}{\tilde{c}_1} + \frac{1}{\tilde{c}_2} & -\frac{1}{\tilde{c}_2} & 0 \\ -\frac{1}{\tilde{c}_2} & \frac{1}{\tilde{c}_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} ,$$

$$\mathbf{h}(\mathbf{w}, \dot{\mathbf{w}}, u_g, \dot{u}_g) = \begin{bmatrix} s_1(x_1) + t_1(x_1)\dot{x}_1 - f_1(x_1, u_g, \dot{u}_g) \\ s_2(x_2) + t_2(x_2)\dot{x}_2 - f_2(x_1, x_2, u_g, \dot{u}_g) \\ -\frac{1}{\tilde{c}_1} u_g \\ 0 \\ 0 \end{bmatrix} ,$$

where
$\tilde{\ell}_n = \frac{\rho \ell_n}{2\pi y_n^2}$, $\tilde{\ell}_r = \frac{8\rho}{3\pi^2 y_n}$, $r_n = \frac{2}{y_n}\sqrt{\rho\mu\frac{\omega}{2}}$, $\omega = \sqrt{\frac{k_1}{m_1}}$, $a_n = \pi y_n^2$, $\tilde{c}_n = \frac{\ell_n \pi y_n^2}{\rho v_c^2}$, ℓ_n is the length of the n th tube, y_n is the radius of the n th tube, and μ is the shear viscosity coefficient.

$$s_\alpha(w_\alpha) = \begin{cases} k_\alpha \eta_{k_\alpha} x_\alpha^3 \;,\; x_\alpha > -\frac{a_{g_0}}{2\ell_g} \\ k_\alpha \eta_{k_\alpha} x_\alpha^3 + 3k_\alpha \left\{ \left(w_\alpha + \frac{a_{g_0}}{2\ell_g} \right) + \eta_{h_\alpha} \left(w_\alpha + \frac{a_{g_0}}{2\ell_g} \right)^3 \right\} \;,\; x_\alpha \le -\frac{a_{g_0}}{2\ell_g} \end{cases} , \alpha = 1, 2.$$

$$t_\alpha(x_\alpha) = \begin{cases} 0 \;,\; x_\alpha > -\frac{a_{g_0}}{2\ell_g} \\ 2\xi\sqrt{m_1 k_1} \;,\; x_\alpha \le -\frac{a_{g_0}}{2\ell_g} \end{cases} , \alpha = 1, 2.$$

$$f_1(x_1, u_g, \dot{u}_g) = \begin{cases} \ell_g d_1 p_{m_1}(x_1, u_g, \dot{u}_g) \;,\; x_1 > -\frac{a_{g_0}}{2\ell_g} \\ 0 \;,\; otherwise \end{cases}$$

$$f_2(x_1, x_2, u_g, \dot{u}_g) = \begin{cases} \ell_g d_2 p_{m_2}(w_1, w_2, u_g, \dot{u}_g) \;,\; x_1 > -\frac{a_{g_0}}{2\ell_g} \text{ and } x_2 > -\frac{a_{g_0}}{2\ell_g} \\ \ell_g d_2 p_s \;,\; x_1 > -\frac{a_{g_0}}{2\ell_g} \text{ and } x_2 \le -\frac{a_{g_0}}{2\ell_g} \\ 0 \;,\; otherwise \end{cases}$$

$$p_{m_1}(x_1, u_g, \dot{u}_g) = p_s - 1.37\frac{\rho}{2}\left(\frac{u_g}{a_{g_0}+2\ell_g x_1}\right)^2 - \frac{1}{2}\left(12\mu\ell_g \frac{d_1}{(a_{g_0}+2\ell_g x_1)^3} + \frac{\rho d_1}{a_{g_0}+2\ell_g x_1}\right)\dot{u}_g$$

$$
\begin{aligned}
&p_{m_2}(x_1, x_2, u_g, \dot{u}_g) = p_{m_1} - * \\
&\quad * = \tfrac{1}{2}\left\{(12\mu\ell_g \tfrac{d_1}{(a_{g_0}+2\ell_g x_1)^3} + 12\ell_g^2 \tfrac{d_2}{(a_{g_0}+2\ell_g x_2)^3})u_g + (\tfrac{\rho d_1}{a_{g_0}+2\ell_g x_1} + \tfrac{\rho d_2}{a_{g_0}+2\ell_g x_2})\dot{u}_g\right\} \\
&\quad -\tfrac{\rho}{2}u_g^2\left(\tfrac{1}{(a_{g_0}+2\ell_g x_2)^2} - \tfrac{1}{(a_{g_0}+2\ell_g x_1)^2}\right)
\end{aligned}
$$