Information Theory for stochastic modeling of uncertainties in high dimension. Application to a new construction of the challenging inverse problem relative to the generation of accelerograms associated with SRS

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COMPDYN 2009
ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, N.D. Lagaros, M. Fragiadakis (eds.)
Rhodes, Greece, 22-24 June 2009

INFORMATION THEORY FOR STOCHASTIC MODELING OF UNCERTAINTIES IN HIGH DIMENSION. APPLICATION TO A NEW CONSTRUCTION OF THE CHALLENGING INVERSE PROBLEM RELATIVE TO THE GENERATION OF ACCELEROGRAMS ASSOCIATED WITH SRS

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Keywords: Generation of Accelerograms, Shock Response Spectrum, Information Theory, Stochastic Modeling.

Abstract. *In transient nonlinear structural dynamics, the dynamical levels of transient vibrations can be defined in terms of a shock response spectrum (SRS) in order to specify the transient loads which are applied to an equipment or to a secondary subsystem. A fundamental problem is then to construct a generator of the non-stationary stochastic process (the transient signal) satisfying a given SRS. This problem has been looked at by many scientists in using specific representations of the non-stationary stochastic process (the accelerogram). In this paper, we propose to solve this challenging stochastic inverse problem by another way in using Information Theory. In the approach proposed, the target SRS is taken as the mean value of the unknown random SRS spanned by the unknown non-stationary stochastic accelerogram for which the probabilistic model has to be constructed. We present the construction of the probability model which allows the confidence region of the random SRS to be carried out. The method presented is validated with an example.*

1 INTRODUCTION

In transient nonlinear structural dynamics, the response spectrum method is often used to characterize the dynamical response in a point of a structure (for instance, in a point of a floor of a building, in a point of the structure where an equipment is connected or in a small region of the structure where a secondary system is attached, etc). In such an approach, a transient response is characterized either by the displacement response spectrum, either by the pseudo-velocity response spectrum simply called the velocity response spectrum (VRS) or by the pseudo-acceleration response spectrum also called the shock response spectrum (SRS). These quantities can directly be constructed using the acceleration transient signals (accelerograms) calculated in different points with the computational nonlinear dynamical model of the structure which is submitted to a given transient loads due, for instance, to a shock wave, to an earthquake, etc. Such a response spectrum is often used by engineering to specify the transient loads which is applied to an equipment or to a secondary subsystem. If the equipment or the secondary subsystem has a nonlinear dynamical behavior, the transient signal accelerations (accelerograms) satisfying the given VRS (or SRS) must be constructed in order to analyze the transient responses using a computational nonlinear dynamical model of the equipment or of the secondary subsystem. This problem consisting in constructing acceleration transient signals (accelerograms) from a given VRS (or SRS) is a challenging inverse problem. Many works have been devoted to this inverse problems and we propose another way using Information Theory.

The response spectrum method was introduced by Biot in 1932 (see of instance [38]) in the context of earthquake engineering and has intensively been studied to extend its domain of applicability to many different situations and applications in the domain of structural, mechanical, earthquake, civil, nuclear and offshore engineering (see for instance, [23, 10, 20, 1, 39, 26, 11, 19, 2, 5] and [15].

For a given deterministic transient signal $\{z(t), t \in \mathcal{J}\}$ (accelerogram in a given direction) with a finite duration T and where $\mathcal{J} = [0\,, T]$, the construction of the VRS consists [3] in evaluating the maximum of the dynamical response of a family of single degree of freedom (SDOF) linear damped oscillators excited at their bases with this transient signal. Let m, ω and ξ be the mass, the eigenfrequency (pulsation in rad/s) and the damping rate of an oscillator of this family. Let $\{x(t), t \in \mathcal{J}\}$ be the displacement of the mass with respect to the basis. For all t in $]0\,, T]$, the displacement x is such that $m\,\ddot{x}(t) + 2\,\xi\,m\,\omega\,\dot{x}(t) + m\,\omega^2\,x(t) = -m\,z(t)$ with the initial condition $x(0) = \dot{x}(0) = 0$. Let $\Omega = [\omega_{min}\,, \omega_{max}]$ be the frequency band of analysis for the eigenfrequency ω and let $\Xi = [\xi_{min}\,, \xi_{max}]$ be the admissible domain for the damping rate ξ. The deterministic transient signal is then characterized by the displacement response spectrum $s_d(\omega, \xi)$ which is defined by

$$s_d(\omega, \xi) = \max_{t \in [0\,, T]} |x(t)| \quad , \tag{1}$$

in which the displacement $x(t)$ can be written as

$$x(t) = \int_0^t h(t - \tau)\, z(\tau)\, d\tau \quad . \tag{2}$$

For all ω in Ω, for all ξ in Ξ and for all $t \geq 0$, the impulse response function h is such that

$$h(t) = -1_{[0, +\infty[}(t)\, \frac{1}{\omega\sqrt{1 - \xi^2}}\; e^{-\xi\omega t} \sin(\omega\sqrt{1 - \xi^2}\, t) \quad , \tag{3}$$

in which $1_{[0,+\infty[}(t) = 1$ if $t \geq 0$ and $= 0$ if $t < 0$. The VRS $s_v(\omega, \xi)$ and the SRS $s_a(\omega, \xi)$ are such that $\omega^2\, s_d(\omega, \xi) = \omega\, s_v(\omega, \xi) = s_a(\omega, \xi)$ which shows that the VRS is such that

$$s_v(\omega, \xi) = \omega\, s_d(\omega, \xi) \quad . \tag{4}$$

For a given deterministic transient signal $\{z(t), t \in \mathcal{J}\}$, the VRS $s_v(\omega, \xi)$ is calculated using Eqs. (1) to (4). For the more realistic situation corresponding to the usual stochastic case that we consider in this paper, the deterministic transient signal $\{z(t), t \in \mathcal{J}\}$ is then replaced by a real-valued non-stationary second-order centered stochastic process $\{Z(t), t \in \mathcal{J}\}$. For all t fixed in $\mathcal{J}$, since the random variable $Z(t)$ is centered, we have $E\{Z(t)\} = 0$ and its variance is $\sigma_Z^2(t) = E\{Z(t)^2\}$ in which E denotes the mathematical expectation. Then, for all ω in Ω and for all ξ in Ξ, Eqs. (1), (2) and (4) define a second-order random variable (random VRS) denoted by $S_v(\omega, \xi)$, such that

$$S_v(\omega, \xi) = \omega \max_{t \in [0\,,T]} |X(t)| \quad , \quad X(t) = \int_0^t h(t - \tau)\, Z(\tau)\, d\tau \quad , \tag{5}$$

and for which $\underline{s}_v(\omega, \xi) = E\{S_v(\omega, \xi)\}$ is the given mean value of the random VRS. We then have to construct a generator of independent realizations $\{Z(t, \theta), t \in \mathcal{J}\}$ of the stochastic process $\{Z(t), t \in \mathcal{J}\}$. Concerning the generation of transient signals from a given response spectrum, we refer the reader to [23, 20, 19, 30, 7, 12, 22, 41, 9]. It should be noted that the majority of such approaches uses *a priori* given representations of the non-stationary stochastic process which has to be identified (see for instance [18, 37, 21, 24, 4, 40, 36, 27, 14]). Nevertheless, it seems that Information Theory has not been used yet to construct the probability model of the non-stationary stochastic process for which an available information for the random VRS is given.

In this paper, the system of marginal probability distributions of stochastic process $\{Z(t), t \in \mathcal{J}\}$ is explicitly constructed without giving an *a priori* stochastic representation but using Information Theory [31] and the maximum entropy principle [16] for which the available information is made up of the given functions $\{\sigma_Z(t), t \in \mathcal{J}\}$ and $\{\underline{s}_v(\omega, \xi)\,, \omega \in \Omega\,, \xi \in \Xi\}$. In addition, the generator of independent realizations $\{Z(t, \theta), t \in \mathcal{J}\}$ of stochastic process $\{Z(t), t \in \mathcal{J}\}$ is developed using its system of marginal probability distributions.

2 TIME SAMPLING, NORMALIZATION AND AVAILABLE INFORMATION

2.1 Time sampling of the stochastic process

The sampling points in the time domain are $t_j = j\, \Delta t$ for $j = 1, \ldots, N$ in which N is such that $T = t_N = N\, \Delta t$ and where Δt is the sampling time step. We then introduce the finite length time series $\Gamma_1, \ldots, \Gamma_N$ and the associated $\mathbb{R}^N$-valued random variable $\mathbf{\Gamma}$ such that

$$\mathbf{\Gamma} = (\Gamma_1, \ldots, \Gamma_N) \quad , \quad \Gamma_j = Z(t_j) \quad , \quad j = 1, \ldots, N \quad . \tag{6}$$

The random vector $\mathbf{\Gamma}$ defined on the probability space $(\Theta, \mathcal{T}, \mathcal{P})$ is completely defined by the probability density function $p_{\mathbf{\Gamma}}(\boldsymbol{\gamma}) = p_{\Gamma_1, \ldots, \Gamma_N}(\gamma_1, \ldots, \gamma_N)$ on $\mathbb{R}^N$ with respect to the volume element $d\boldsymbol{\gamma} = d\gamma_1 \ldots d\gamma_N$. This probability density function is unknown and the first objective of this paper is to construct it. It can easily be seen that $\mathbf{\Gamma}$ is such that

$$\mathbf{m}_{\mathbf{\Gamma}} = E\{\mathbf{\Gamma}\} = 0 \quad , \quad E\{\|\mathbf{\Gamma}\|^2\} = \sum_{j=1}^{N} \sigma_j^2 < +\infty \quad , \tag{7}$$

in which $\|\mathbf{\Gamma}\|^2 = \Gamma_1^2 + \ldots + \Gamma_N^2$ and where

$$\sigma_j^2 = E\{\Gamma_j^2\} = E\{Z(t_j)^2\} = \sigma_Z^2(t_j) \quad , \quad j = 1, \ldots, N \quad . \tag{8}$$

The vector $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_N)$ is assumed to be an available information for random vector $\mathbf{\Gamma} = (\Gamma_1, \ldots, \Gamma_N)$.

2.2 Construction of a finite representation of the VRS

Let $\omega_1, \ldots, \omega_{\nu_\omega}$ be the sampling points of the frequency band of analysis $\Omega = [\omega_{min}, \omega_{max}]$ and let $\xi_1, \ldots, \xi_{\nu_\xi}$ be the sampling points of the admissible domain $\Xi = [\xi_{min}, \xi_{max}]$ for the damping rate. Let $\nu = \nu_\omega \times \nu_\xi$. We then introduce the given vector $\underline{\boldsymbol{\mathcal{S}}} = \{\underline{s}_v(\omega_i, \xi_n) \ , \ i = 1, \ldots, \nu_\omega \ , \ n = 1, \ldots, \nu_\xi\}$ in $\mathbb{R}^\nu$ and the $\mathbb{R}^\nu$-valued random vector $\boldsymbol{\mathcal{S}} = \{S_v(\omega_i, \xi_n) \ , \ i = 1, \ldots, \nu_\omega \ , \ n = 1, \ldots, \nu_\xi\}$. The sampling/discretization of Eq. (5) for the random VRS yields

$$\boldsymbol{\mathcal{S}} = \mathbf{S}(\mathbf{\Gamma}) = (S_1(\mathbf{\Gamma}), \ldots, S_\nu(\mathbf{\Gamma})) \quad , \tag{9}$$

in which $\boldsymbol{\gamma} \mapsto \mathbf{S}(\boldsymbol{\gamma}) = (S_1(\boldsymbol{\gamma}), \ldots, S_\nu(\boldsymbol{\gamma}))$ is a perfectly defined and known nonlinear mapping from $\mathbb{R}^N$ into $\mathbb{R}^\nu$ which is such that $\mathbf{S}(-\boldsymbol{\gamma}) = \mathbf{S}(\boldsymbol{\gamma})$. The mean value $E\{\mathbf{S}(\mathbf{\Gamma})\} = \underline{\boldsymbol{\mathcal{S}}}$ is then given.

2.3 Normalization

Let $\mathbf{A} = (A_1, \ldots, A_N)$ be the random vector with values in $\mathbb{R}^N$ constructed as the normalization of the random vector $\mathbf{\Gamma}$, such that

$$\mathbf{\Gamma} = \sqrt{N}\, [\sigma]\, \mathbf{A} \quad , \quad [\,\sigma\,]_{jj'} = \sigma_j\, \delta_{jj'} \quad , \tag{10}$$

in which $[\,\sigma\,]$ is a $(N \times N)$ real diagonal matrix.

2.4 Definition of the available information

The available information for the random variable $\mathbf{A}$ is defined as follows. From $E\{\mathbf{A}\} = N^{-1/2}\, [\sigma]^{-1}\, E\{\mathbf{\Gamma}\}$, it can be deduced that $\mathbf{A}$ is a centered random variable,

$$E\{\mathbf{A}\} = 0 \quad . \tag{11}$$

For all j in $\{1, \ldots, N\}$, the second-order moment of random variable A_j is such that

$$E\{A_j^2\} = \frac{1}{N} \quad , \tag{12}$$

and then $E\{\|\mathbf{A}\|^2\} = 1$. Let $\mathbf{a} \mapsto \mathbf{s}(\mathbf{a}) = (s_1(\mathbf{a}), \ldots, s_\nu(\mathbf{a}))$ be the nonlinear mapping from $\mathbb{R}^N$ into $\mathbb{R}^\nu$ such that

$$s_k(\mathbf{a}) = \frac{S_k(\sqrt{N}\, [\sigma]\, \mathbf{a})}{\underline{\mathcal{S}}_k} \quad , \quad \forall\, k = 1, \ldots, \nu \quad , \tag{13}$$

in which $S_k(\boldsymbol{\gamma})$ is defined in Eq. (9). Since $\mathbf{S}(-\boldsymbol{\gamma}) = \mathbf{S}(\boldsymbol{\gamma})$ (see Section 2.2), we have $\mathbf{s}(-\mathbf{a}) = \mathbf{s}(\mathbf{a})$. Finally, it can then easily be deduced that

$$E\{\mathbf{s}(\mathbf{A})\} = \underline{\mathbf{s}} \quad , \tag{14}$$

in which the vector $\underline{\mathbf{s}} = (\underline{s}_1, \ldots, \underline{s}_\nu)$ is such that $\underline{s}_k = 1$ for all $k = 1, \ldots, \nu$. Therefore, the available information which allows the probability distribution of random vector $\mathbf{A}$ to be constructed, is made up of Eqs. (11), (12) and (14).

3 CONSTRUCTION OF THE PROBABILITY MODEL

3.1 Construction of the probability density function of A using the maximum entropy principle

Let $\mathbf{a} = (a_1, \ldots, a_N)$ be any vector in $\mathbb{R}^N$. Let $\mathbf{A} = (A_1, \ldots, A_N)$ be the $\mathbb{R}^N$-valued second-order random variable for which the probability density function is $\mathbf{a} \mapsto p_{\mathbf{A}}(\mathbf{a})$ with respect to the volume element $d\mathbf{a} = da_1 \ldots da_N$ of $\mathbb{R}^N$. Using the maximum entropy principle ([31, 16]) under the constraints defined by the available information (see Eqs. (11), (12) and (14), it can be proven (see [33]) that for all $\mathbf{a}$ in $\mathbb{R}^N$,

$$p_{\mathbf{A}}(\mathbf{a}) = c_0^{\text{sol}} \, \exp(- < \boldsymbol{\lambda}^{\text{sol}}, \, \mathbf{g}(\mathbf{a}) >) \quad , \tag{15}$$

in which c_0^{sol} is the constant of normalization, where $\boldsymbol{\lambda}^{\text{sol}}$ is the Lagrange multiplier in $\mathbb{R}^\mu$ with $\mu = N + \nu$, where $< \boldsymbol{\lambda}^{\text{sol}}, \, \mathbf{g}(\mathbf{a}) >= \lambda_1^{\text{sol}} \, g_1(\mathbf{a}) + \ldots + \lambda_\mu^{\text{sol}} \, g_\mu(\mathbf{a})$ and where $\mathbf{a} \mapsto \mathbf{g}(\mathbf{a})$ is the function from $\mathbb{R}^N$ into $\mathbb{R}^\mu$ such that $\mathbf{g}(\mathbf{a}) = (\mathbf{a}^2, \mathbf{s}(\mathbf{a}))$. Introducing the vector $\mathbf{f} \in \mathbb{R}^\mu$ such that $\mathbf{f} = (\mathbf{h}, \underline{\mathbf{s}})$ in which the vector $\mathbf{h} = (h_1, \ldots h_N)$ is the vector in $\mathbb{R}^N$ such that $h_j = 1/N$ for all $j = 1, \ldots, N$, then the constraints equations can then be rewritten as

$$E\{\mathbf{g}(\mathbf{A})\} = \int_{\mathbb{R}^N} \mathbf{g}(\mathbf{a}) \, p_{\mathbf{A}}(\mathbf{a}) \, d\mathbf{a} = \mathbf{f} \quad . \tag{16}$$

3.2 Computation of the vector-valued Lagrange multiplier

For $\boldsymbol{\lambda}$ fixed in $\mathbb{R}^\mu$, let $\mathbf{B}_{\boldsymbol{\lambda}}$ be the $\mathbb{R}^N$-valued random variable whose probability density function $\mathbf{b} \mapsto p(\mathbf{b}, \boldsymbol{\lambda})$ with respect to the volume element $d\mathbf{b}$ of $\mathbb{R}^N$ is written, for all $\mathbf{b}$ in $\mathbb{R}^N$, as

$$p(\mathbf{b}, \boldsymbol{\lambda}) = c_{\boldsymbol{\lambda}} \, \exp(- < \boldsymbol{\lambda}, \, \mathbf{g}(\mathbf{b}) >) \quad , \tag{17}$$

in which $c_{\boldsymbol{\lambda}}$ is a finite positive constant depending on $\boldsymbol{\lambda}$ defined by the normalization condition. Taking $c_{\boldsymbol{\lambda}^{\text{sol}}} = c_0^{\text{sol}}$, Eqs. (15) and (17) show that, for all $\mathbf{a}$ in $\mathbb{R}^N$,

$$p_{\mathbf{A}}(\mathbf{a}) = p(\mathbf{a}, \boldsymbol{\lambda}^{\text{sol}}) \quad , \tag{18}$$

which means that $\mathbf{A} = \mathbf{B}_{\boldsymbol{\lambda}^{\text{sol}}}$. From Eq. (16), it can then be deduced that $\boldsymbol{\lambda}^{\text{sol}}$ is the solution in $\boldsymbol{\lambda}$ of the equation,

$$E\{\mathbf{g}(\mathbf{B}_{\boldsymbol{\lambda}})\} = \mathbf{f} \quad , \tag{19}$$

in which the integral $E\{\mathbf{g}(\mathbf{B}_{\boldsymbol{\lambda}})\}$ is such that

$$E\{\mathbf{g}(\mathbf{B}_{\boldsymbol{\lambda}})\} = \int_{\mathbb{R}^N} \mathbf{g}(\mathbf{b}) \, p(\mathbf{b}, \boldsymbol{\lambda}) \, d\mathbf{b} \quad . \tag{20}$$

We then have to construct the solution $\boldsymbol{\lambda}^{\text{sol}}$ of Eq. (19) in $\boldsymbol{\lambda}$. It is assumed that the constraints are such that Eq. (19) has a unique solution $\boldsymbol{\lambda}^{\text{sol}}$. Consequently, for such a solution, the normalization condition and Eq. (16) are verified and the probability density function $p_{\mathbf{A}}$ is given by Eq. (15) with $c_0^{\text{sol}} = c_{\boldsymbol{\lambda}^{\text{sol}}}$. Equation (19) can be solved in $\boldsymbol{\lambda}$ with the interior-reflective Newton method.

3.3 Estimating the integrals in high dimension

The calculation of $\boldsymbol{\lambda}^{\text{sol}}$ as the solution of Eq. (19) in $\boldsymbol{\lambda}$ requires to calculate the following integral in high dimension $\int_{\mathbb{R}^N} \mathbf{g}(\mathbf{b}) \, p(\mathbf{b}, \boldsymbol{\lambda}) \, d\mathbf{b}$ in which $p(\mathbf{b}, \boldsymbol{\lambda}) = c_{\boldsymbol{\lambda}} \, \exp(- < \boldsymbol{\lambda}, \, \mathbf{g}(\mathbf{b}) >)$.

This problem is difficult for the high dimension case (for instance, in the application presented in Section 4, $N = 128$ and $\mu = 168$).

For any $\boldsymbol{\lambda}$ fixed in $\mathbb{R}^{\mu}$, the calculation of $E\{\mathbf{g}(\mathbf{B}_{\boldsymbol{\lambda}})\}$ defined by Eq. (20) can be performed using the Markov Chain Monte Carlo method (MCMC) [17, 35, 25]. The transition kernel of the homogeneous Markov chain of the MCMC method can be constructed using the Metropolis-Hastings algorithm [13] or the Gibbs sampling [6] which is a slightly different algorithm for which the kernel is directly deduced from the probability density function and for which the Gibbs samplers are always accepted. These two algorithms allow the transition kernel to be constructed for which the invariant measure is $p(\mathbf{b}, \boldsymbol{\lambda})\, d\mathbf{b}$. In general, these two algorithms are efficient, but can also be not efficient if there exists attraction regions which do not correspond to the invariant measure under consideration. These cases cannot be easily detected and are time consuming. We then use the method developed in [33] which looks like to the Gibbs approach but corresponds to a more direct construction of a random generator of independent realizations of the random variable $\mathbf{B}_{\boldsymbol{\lambda}}$ whose probability distribution is $p(\mathbf{b}, \boldsymbol{\lambda})\, d\mathbf{b}$. The difference between the Gibbs algorithm and the proposed algorithm is that the convergence in the proposed method can be studied with all the mathematical results concerning the existence and uniqueness of Itô stochastic differential equation. In addition, a parameter f_0 is introduced which allows the transient part of the response to be killed in order to get more rapidly the stationary solution corresponding to the invariant measure. The construction of the transition kernel by using the detailed balance equation is replaced by the construction of an Itô Stochastic Differential Equation (ISDE) (depending on $\boldsymbol{\lambda}$) which admits $p(\mathbf{b}, \boldsymbol{\lambda})\, d\mathbf{b}$ defined by Eq. (17) as a unique invariant measure [32]. Finally, the ergodic method is used to estimate $E\{\mathbf{g}(\mathbf{B}_{\boldsymbol{\lambda}})\}$ in order to calculate $\boldsymbol{\lambda}^{\text{sol}}$. Once the Lagrange multiplier $\boldsymbol{\lambda}^{\text{sol}}$ is calculated, the random generator of independent realizations of random vector $\mathbf{A}$ is constructed as explained in [33].

4 APPLICATION

4.1 Data and parameters

(i) It is assumed that the mean VRS is given for only one value $\xi_1 = 0.01$ of the damping rate (therefore $\nu_{\xi} = 1$)). The frequency band of analysis (see Section 2.2) is $[0.25\,, 10]$ Hz with $f_{max} = 10$ Hz and $\nu = \nu_{\omega} = 40$. The sampling points of the frequency band of analysis are then $\Delta f, 2\,\Delta f, \ldots, \nu\,\Delta f$ with $\Delta f = 0.25$ Hz. Let $\underline{\boldsymbol{\mathcal{S}}} = (\underline{\mathcal{S}}_1, \ldots, \underline{\mathcal{S}}_{\nu})$ be the mean VRS. Figure 1 (right) displays the graph of the function $k \mapsto \underline{\mathcal{S}}_k$ from $\{1, \ldots, \nu\}$ into $\mathbb{R}^+$.

(ii) Stochastic process $\{Z(t), t \in \mathcal{J}\}$ is indexed by $\mathcal{J} = [0\,, T]$ with $T = 12.8$ s. The support of its instantaneous spectral density function is the bounded interval $[-F_{max}, F_{max}]$ with $F_{max} = 5$ Hz. The sampling time step is such that $\Delta t = 0.1$ s and consequently, the number of sampling points in the time domain is $N = 128$. The sampling points are $t_j = j\,\Delta t$ for $j = 1, \ldots, N$. We have $\boldsymbol{\Gamma} = (\Gamma_1, \ldots, \Gamma_N)$ with $\Gamma_j = Z(t_j)$ (see Eq. (6)). Figure 1 (left) displays the graph of the standard-deviation function $j \mapsto \sigma_j$ from $\{1, \ldots, N\}$ into $\mathbb{R}^+$ such that (see Eq. (8)) $\sigma_j^2 = \sigma_Z^2(t_j)$.

4.2 Computation of the vector-valued Lagrange multipliers

The Lagrange multiplier $\boldsymbol{\lambda}^{\text{sol}} = (\boldsymbol{\lambda}_2^{\text{sol}}, \boldsymbol{\lambda}_3^{\text{sol}}) \in \mathbb{R}^N \times \mathbb{R}^{\nu} = \mathbb{R}^{\mu}$ with $N = 128$, $\nu = 40$ and $\mu = 128 + 40 = 168$ is computed in using Sections 3.2 and 3.3. The mathematical expectation defined by Eq. (20) is estimated by using Section 3.3. The interior-reflective Newton method is used to solve Eq. (19)

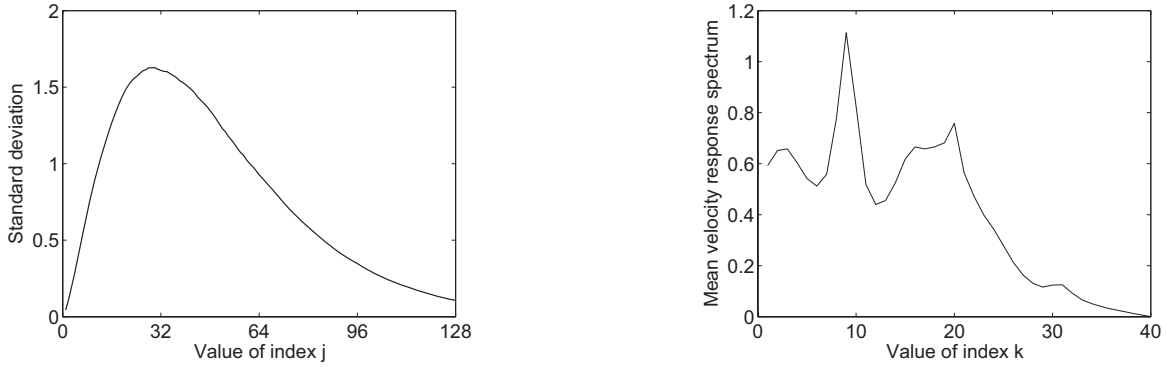

Figure 1: Graph of $j \mapsto \sigma_j = \sqrt{E\{\Gamma_j^2\}}$ (left figure) and graph of $k \mapsto \underline{\mathcal{S}}_k$ (right figure).

4.3 Validation

Solution $\boldsymbol{\lambda}^{\text{sol}}$ of Eq. (19) being known, n_s independent realizations of the random variable $\mathbf{A} = \mathbf{B}_{\boldsymbol{\lambda}^{\text{sol}}}$ are constructed using the method presented in [33]. As a first element of validation of the method proposed, we have to verify that the constraints are satisfied. The quantities $k \mapsto E\{\mathcal{S}_k\} = E\{S_k(\mathbf{\Gamma})\}$ and $j \mapsto E\{\Gamma_j^2\}$ are estimated using the Monte Carlo simulation method and are compared with $k \mapsto \underline{\mathcal{S}}_k$ and $j \mapsto \sigma_j^2$ respectively. The estimations are calculated by $E\{\mathcal{S}_k\} \simeq n_s^{-1} \sum_{\ell=1}^{n_s} S_k(\mathbf{\Gamma}(\theta_\ell))$ and $E\{\Gamma_j^2\} \simeq n_s^{-1} \sum_{\ell=1}^{n_s} \Gamma_j^2(\theta_\ell)$ in which $\mathbf{\Gamma}(\theta_\ell) = \sqrt{N}\,[\sigma]\,\mathbf{A}(\theta_\ell)$. The n_s independent realizations $\mathbf{A}(\theta_1), \ldots, \mathbf{A}(\theta_{n_s})$ of random vector $\mathbf{A}$ are calculated using the generator with $n_s = 300$. Figure 2 (left) displays the correspond-

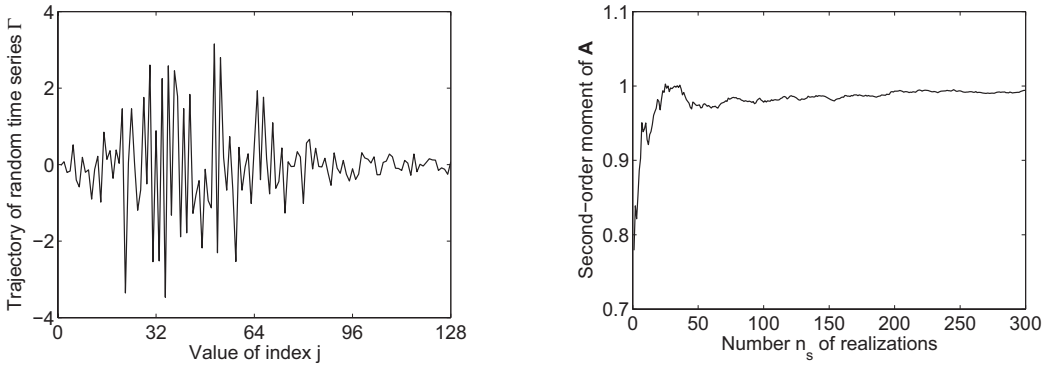

Figure 2: Graph of $j \mapsto \Gamma_j(\theta_\ell)$ for a realization θ_ℓ (left figure) and graph of $n_s \mapsto \text{convMC}(n_s)$ (right figure).

ing trajectory of the random time series $j \mapsto \Gamma_j$, that is to say the graph of the realization $j \mapsto \Gamma_j(\theta_\ell)$ in which $\mathbf{\Gamma}(\theta_\ell) = \sqrt{N}\,[\sigma]\,\mathbf{A}(\theta_\ell)$. (ii) Concerning the value of n_s, Figure 2 (right) shows the graph of $n_s \mapsto \text{convMC}(n_s) = {n_s}^{-1} \sum_{\ell=1}^{n_s} \|\mathbf{A}(\theta_\ell)\|^2$ which is an estimation of the second-order moment $E\{\|\mathbf{A}\|^2\} = E\{\|\mathbf{B}_{\boldsymbol{\lambda}^{\text{sol}}}\|^2\}$ of the random variable $\|\mathbf{A}\|$. This figure shows that $n_s = 300$ is a reasonable value for n_s to reach mean-square convergence. Figure 3 shows the estimation of the constraints (standard deviation and mean VRS) constructed with the random generator and compares these estimations with the references defined in Figure 1. Figure 3 (left) compares the graph of the standard-deviation function $j \mapsto \sigma_j$ with the estimation $j \mapsto E\{\Gamma_j^2\} \simeq n_s^{-1} \sum_{\ell=1}^{n_s} \Gamma_j^2(\theta_\ell)$. Figure 3 (right) compares the graph of the mean velocity response spectrum $k \mapsto \underline{\mathcal{S}}_k$ with the estimation $k \mapsto E\{\mathcal{S}_k\} \simeq n_s^{-1} \sum_{\ell=1}^{n_s} S_k(\mathbf{\Gamma}(\theta_\ell))$. The comparisons validate the method proposed. The small fluctuations of the estimation of the

standard-deviation function computed by the Monte Carlo method using the random generator can be reduced in increasing the value of n_s.

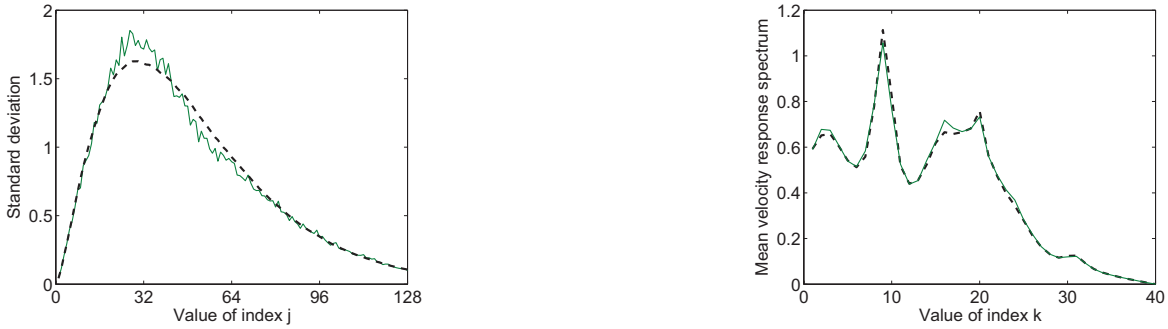

Figure 3: Graph of $j \mapsto \sigma_j = E\{\Gamma_j^2\}^{1/2}$ (left figure) and graph of $k \mapsto \underline{\mathcal{S}}_k$ (right figure). Reference (dashed lines). Estimation with the random generator (solid lines).

4.4 Properties of the constructed probability model

Random vector $\mathbf{\Gamma} = (\Gamma_1, \ldots, \Gamma_N)$ is defined by $\mathbf{\Gamma} = \sqrt{N}\,[\sigma]\,\mathbf{A}$ for which the probability density function on $\mathbb{R}^N$ of the non-Gaussian random vector $\mathbf{A}$ is $p_{\mathbf{A}}(\mathbf{a}) = c_0^{\rm sol} \exp(- < \boldsymbol{\lambda}^{\rm sol}, \mathbf{g}(\mathbf{a}) >)$ with $N = 128$ and where $\mathbf{a} \mapsto \mathbf{g}(\mathbf{a})$ is a nonlinear mapping from $\mathbb{R}^N$ into $\mathbb{R}^\mu$ with $\mu = 168$. Random VRS $\boldsymbol{\mathcal{S}} = (\mathcal{S}_1, \ldots, \mathcal{S}_\nu)$ with $\mathcal{S}_k = S_v(\omega_k, \xi_1)$ is such that $E\{\boldsymbol{\mathcal{S}}\} = \underline{\boldsymbol{\mathcal{S}}}$ (see Figure 3 (right)). It is interesting to define the statistical fluctuations of $\boldsymbol{\mathcal{S}}$ induced by the probability model of $\mathbf{\Gamma}$. For that we construct the confidence region of the time series $\{\mathcal{S}_1, \ldots, \mathcal{S}_\nu\}$ which is delimited by the upper envelope $k \mapsto \mathcal{S}_k^+$ and the lower envelope $k \mapsto \mathcal{S}_k^-$ such that,

$$\text{Proba}\{\mathcal{S}_k^- < \mathcal{S}_k \leq \mathcal{S}_k^+\} = P_c \quad , \quad \forall\, k \in \{1, \ldots, \nu\} \quad , \tag{21}$$

with $P_c < 1$. The envelopes are constructed by using the method of quantiles. Figure 4 displays the graph of the confidence region of the time series $\{\mathcal{S}_1, \ldots, \mathcal{S}_\nu\}$ for $P_c = 0.98$.

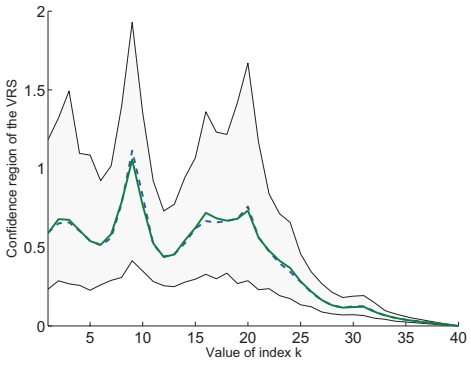

Figure 4: Random VRS $k \mapsto \mathcal{S}_k$. Graph of the mean value $k \mapsto \underline{\mathcal{S}}_k$ (dashed line). Graph of the mean function $k \mapsto E\{\mathcal{S}_k\}$ estimated with the random generator (solid line). Confidence region of the time series $\{\mathcal{S}_1, \ldots, \mathcal{S}_\nu\}$ with $P_c = 0.98$ estimated with the random generator (colored region).

5 CONCLUSIONS

In many applications, the dynamical levels of transient vibrations in structures are defined in terms of shock response spectrum (SRS). Such a response spectrum is used by engineering to specify the transient loads which are applied to an equipment or to a secondary subsystem. A fundamental problem is then to construct a generator of the non-stationary stochastic process satisfying a given SRS. In this paper, we have not imposed a stochastic representation for the non-stationary stochastic accelerogram and we have proposed to solve this challenging stochastic inverse problem by another way based on the use of Information Theory. The target SRS is taken as the mean value of the unknown random SRS spanned by the unknown non-stationary stochastic accelerogram. This approach allows to construct the confidence region of the random SRS for which the target SRS is the mean value. The method presented has been validated with an example.

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