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# EXPERIMENTAL IDENTIFICATION OF STOCHASTIC LOADS USING A NONLINEAR DYNAMICAL SYSTEM COUPLED WITH AN UNCERTAIN LINEAR ONE

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**Abstract.** The problem presented here concerns fuel assemblies for which the stochastic loads induced by turbulent fluid forces have to be identified. The simplified model used for this identification is composed of two coupled subsystems:(1) A nonlinear subsystem which is accurate and contains the quantity of interest (2) A linear subsystem which is less accurate and contains model uncertainties which have to be taken into account. Others sources of uncertainties are to be taken into account in the stochastic loads identification process: uncertainties concerning the stochastic loads and measurements errors. The identification is carried out using the moments method and the maximum likelihood method. Finally, the stochastic simplified computational model on which the identified stochastic loads are applied is used to construct statistics of the fretting wear of the fuel rods.

## 1 INTRODUCTION

A fuel assembly is made up of thousands of fuel rods and tubes which are held in position by grids. This dynamical system bathes in a flow of a liquid (water)which induces turbulent forces that are likely to induce fretting-wear of the fuel rods. A fuel assembly is a very complex nonlinear dynamical system for which an accurate computational model (called the reference computational model) would be time consuming and generally, would induce many numerical problems due to the high modal density of such a structure. Therefore, the computational model must be simplified from an engineering design point of view. The model uncertainties are thus due to the simplification introduced by the mathematical-mechanical modeling process. The measurements are realized with an experimental setup which is constituted of a half fuel assembly which bathes in a turbulent fluid. The objectives of this paper are to identify the parameters of the mathematical model of the stochastic forces induced by the turbulent fluid which are applied to the experimental setup, using an uncertain stochastic simplified computational model and

experimental responses. The general methodology used to solve this problem has been presented in . The identified stochastic model is then used to analyze the robustness of the predictions and allows the fretting-wear of the rods to be estimated.

The uncertainties introduced in this methodology are summarized on Figure ?? . In

Figure 1: Designed system, experimental setup, uncertain computational model

the problem under consideration, there are four sources of uncertainties:(1) The model uncertainties induced by the introduction of simplifications in the model. This type model uncertainties are taken into account using the nonparametric probabilistic approach which consists in modeling the reduced mass and stiffness matrices by full random matrices defined on a probability space  $(\Theta, \mathcal{T}, \mathcal{P})$  (2) The mean model of the stochastic loads (induced by the statistical fluctuations of the turbulent pressure applied to the structure) is a vector-valued Gaussian centered second-order stationary stochastic process defined on a probability space  $(\Theta', \mathcal{T}', \mathcal{P}')$  (3) The uncertainties concerning the stochastic loads are taken into account by replacing the nominal value of the matrix-valued spectral density function (defined above) by a random matrix-valued spectral density function defined on a probability space  $(\Theta'', \mathcal{T}'', \mathcal{P}'')$  (4) The uncertainties induced by measurement errors.

## 2 Experimental measurements

The experimental setup is composed of a half fuel assembly. All the structure bathes in a flow of a liquid (water) whose velocity is approximatively  $1m/s$ . One of the fuel rod is equipped with 12 strain sensors. The matrix-valued spectral density function of the vector-valued measured random signal is estimated by the periodogram method.

Figure 2: PSD for the strain measured by sensors  $J1x$  (red line) and  $J5x$  (black line).

## 3 Reference computational model

The reference computational model is developed in order to construct an observation which will be useful for the identification of the dispersion parameters  $\delta_M^A$  and  $\delta_K^A$  controlling the level of uncertainties in the linear subsystem of the simplified computational model. In the reference model, all the guide tubes, the fuel rods and the grids are modeled by Timoshenko's beams. The bumps and springs are modeled by springs elements. For the fuel rod equipped with sensors for measurements, the bumps and springs are modeled accurately by elastic stops. The reference computational model is composed of two sub-systems. The first one is linear and composed of all the guide tubes, the non-equipped

fuel rods and the grids. The second one is the nonlinear fuel rod which is equipped with the sensors for measurements. The modal density of the reference computational model is not homogeneous at all in the frequency band of analysis and have locally high values. Such a situation induces many numerical problem for the calculation of the stationary response of the stochastic nonlinear dynamical system with random parameters and random excitation. For this reason, the reference model must be simplified from the engineering design point of view.

#### 4 Mean simplified computational model

The mean simplified computational model is derived from the reference computational model. Indeed, the linear subsystem of the reference computational model is replaced by an equivalent linear subsystem composed of two Timoshenko beams. The first one is equivalent to the 25 guide tubes and the other one is equivalent to the 263 non-equipped fuel rods. The nonlinear subsystem of the simplified computational model is the same that the nonlinear subsystem of the reference computational model. The linear subsystem and the linear part of the nonlinear subsystem are reduced using the Craig & Bampton method.

#### 5 Stochastic simplified computational model

The simplifications introduced in the simplified computational model induce model uncertainties which have to be taken into account. In [?], the model uncertainties on the linear subsystem of the reference computational model are taken into account using the nonparametric probabilistic approach. For the linear subsystem, this method consists in replacing the reduced mass and the reduced stiffness matrices of the mean reduced simplified computational model by random matrices. The probability density functions of these full random matrices depend on the dispersion parameters  $\delta_M^A$  and  $\delta_K^A$  which are identified using the maximum likelihood method and the reference computational model as an observation. Then, the stationary stochastic process  $\mathbf{Q}(t)$  which is a vector whose components are the physical DOF at the coupling interface and are the generalized DOF for the two subsystems with fixed coupling interface satisfies the random differential equation

$$[\mathbf{M}]\ddot{\mathbf{Q}}(t) + [\mathbf{D}]\dot{\mathbf{Q}}(t) + [\mathbf{K}]\mathbf{Q}(t) + \mathcal{F}^{NL}(\mathbf{Q}(t), \dot{\mathbf{Q}}(t)) = \mathcal{F}(t) \quad . \quad (1)$$

In this equation, the vector  $\mathcal{F}^{NL}(\mathbf{Q}(t), \dot{\mathbf{Q}}(t))$  is the generalized localized nonlinear forces due to the elastic stops. The detailed construction of the different terms in Eq. (??) can be found in [?]. The stochastic equation (??) is solved using the Monte Carlo simulation method.

## 6 Identification of the uncertain stochastic loads

For the construction of stochastic process  $\tilde{\mathbf{F}}^{\text{unc}}$ , first we introduce a stochastic process  $\{\tilde{\mathbf{F}}(t), t \in \mathbb{R}\}$  of the stochastic loads without uncertainties. It is then assumed that stochastic process  $\tilde{\mathbf{F}}$  is a Gaussian stationary centered second-order stochastic process defined on a probability space  $(\Theta', \mathcal{T}', \mathcal{P}')$  for which the matrix-valued spectral density function is  $\{[S_{\tilde{\mathbf{F}}}(\omega)], \omega \in \mathbb{R}\}$ . The uncertain stochastic process  $\tilde{\mathbf{F}}^{\text{unc}}$  is then constructed as the stochastic process  $\tilde{\mathbf{F}}$  for which the deterministic function  $\{[S_{\tilde{\mathbf{F}}}(\omega)], \omega \in \mathbb{R}\}$  is replaced by a random function  $\{[\mathbf{S}_{\tilde{\mathbf{F}}}(\omega)], \omega \in \mathbb{R}\}$  defined on a probability space  $(\Theta'', \mathcal{T}'', \mathcal{P}'')$ . The probability distribution of random function  $\{[\mathbf{S}_{\tilde{\mathbf{F}}}(\omega)], \omega \in \mathbb{R}\}$  is constructed using the maximum entropy principle and depends on a dispersion parameter  $\delta_F$ . Such a stochastic process and its generator of independent realizations are completely defined by (1) the spectral density function  $[S_{\tilde{\mathbf{F}}^{\text{unc}}}]$  and (2) its dispersion parameter  $\delta_F$ . So the identification of the stochastic loads consists in identifying these two quantities.

### 6.1 Identification of the matrix-valued spectral density function of the uncertain stochastic loads

We introduce the vector-valued stochastic process  $\{\Xi^{\text{exp}}(t), t \in \mathbb{R}\}$  whose components are the 7 measured strains for which the matrix-valued spectral density function  $\{[S_{\Xi^{\text{exp}}}(\omega)], \omega \in \mathbb{R}\}$  is estimated using the periodogram method. The corresponding stochastic process  $\{\Xi(t; \mathbf{r}), t \in \mathbb{R}\}$  is calculated with the stochastic simplified computational model. The matrix-valued spectral density function  $\{[S_{\Xi}(\omega; \mathbf{r})], \omega \in \mathbb{R}\}$  of the stochastic process  $\Xi(t; \mathbf{r})$  is also estimated using the periodogram method. The identification is then performed by minimizing the distance between the experimental matrix-valued spectral density function  $[S_{\Xi^{\text{exp}}}(\omega)]$  and the numerical matrix-valued spectral density function  $[S_{\Xi}(\omega; \mathbf{r})]$ .

### 6.2 Identification of the dispersion parameter $\delta_F$ .

We introduce the random variable  $J_s$  which is such that for all  $\theta \in \Theta$  and for all  $\theta'' \in \Theta''$ ,

$$J_s(\theta, \theta'') = \int_B \|[\mathbf{S}_{\Xi}(\omega, \theta, \theta'')]\|_F^2 d\omega \quad . \quad (2)$$

The measurements errors on the experimental variable  $J_s^{\text{exp}}$  are modeled by a given additive noise  $\mathcal{E}$ , defined on a probability space  $(\Theta''', \mathcal{T}''', \mathcal{P}''')$ , for which the probability density function is  $e \mapsto p_{\mathcal{E}}(e)$ . We then have

$$J_s^{\text{er}} = J_s + \mathcal{E} \quad . \quad (3)$$

The dispersion parameter  $\delta_F$  is identified using the maximum likelihood method for the random variable  $J_s^{\text{er}}$  for which the probability density function is defined by

$$p_{J_s^{\text{er}}}(y) = \int_{-\infty}^{+\infty} p_{J_s^{\text{er}}|\mathcal{E}=e}(y|e)p_{\mathcal{E}}(e)de \quad , \quad (4)$$

where  $x \mapsto p_{J_s^{\text{er}}|\mathcal{E}=e}(y|e)$  is the conditional probability density function  $J_s^{\text{er}}$  given  $\mathcal{E} = e$ . It is assumed that the additive noise  $\mathcal{E}$  is modeled by a centered Gaussian random variable for which the standard deviation is given.

## 7 Random fretting-wear estimation

The random fretting-wear in the contacts rod/bump and rod/spring for the uncertain system submitted to the uncertain stochastic excitation is based on the use of the Archard power wear. The mean value, the dispersion (ratio of the standard deviation with the mean value), and quantiles 5% and 95% of the random fretting-wear for the first grid following  $x$  direction are reported in Table ???. The estimated dispersions are lower than 61 %.

	mean	dispersion	quantile 5 %	quantile 95 %
low bump	0.024	60.1%	0.014	0.035
spring	0.029	51.7%	0.025	0.036
high bump	0.014	55.4%	0.01	0.019

Table 1: Statistics for the random fretting-wear on the first grid following  $x$  direction.

## 8 Conclusions

We have presented a complete methodology for the identification of turbulent fluid forces applied to fuel assemblies using an uncertain simplified computational model and experimental strain responses. All the sources of uncertainties have been taken into account in the identification process. The probabilistic model of model uncertainties in the simplified computational model depends on dispersion parameters which have been identified using the maximum likelihood method and a reference computational model. The uncertainties concerning the parametric representation of the uncertain stochastic loads have also been taken into account. The uncertain stochastic loads have been identified taking into account measurements errors. The identified stochastic loads has been applied to the stochastic simplified computational model in order to construct the statistics on the random fretting-wear of the fuel roads. The estimated dispersions of the random fretting-wear are about 61% that induces a relatively robustness with respect to uncertainties for this complex industrial problem.

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