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► **To cite this version:**

Christophe Desceliers, Christian Soize, Q. Grimal, M. Talmant, Salah Naili. Identification of the elasticity tensor of an uncertain biomechanical computational model using axial transmission. NOVEM 2009, Noise and Vibration: Emerging Methods, Klebe College, Oxford, Apr 2009, Oxford, United Kingdom. pp.1-7. hal-00691747

HAL Id: hal-00691747

<https://hal-upec-upem.archives-ouvertes.fr/hal-00691747>

Submitted on 26 Apr 2012

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IDENTIFICATION OF THE ELASTICITY TENSOR OF AN UNCERTAIN BIOMECHANICAL COMPUTATIONAL MODEL USING AXIAL TRANSMISSION

C. Desceliers^{1*}, C. Soize¹, Q. Grimal², M. Talmant² and S. Naili³

¹Université Paris-Est
Modélisation et Simulation Multi Echelle, MSME FRE3160 CNRS, France
Email: christophe.desceliers@univ-paris-est.fr
Email: christian.soize@univ-paris-est.fr

²Université Paris 6
Laboratoire d'imagerie paramétrique, CNRS UMR 7623, France

³Université Paris-Est
Laboratoire de mécanique physique, CNRS UMR 7052 B2OA, France

ABSTRACT

The purpose of this paper is the construction of an uncertain probabilistic model for the mechanical properties of the cortical bone. The main objective is firstly to propose a probabilistic simplified model adapted to the ultrasonic axial transmission technique and secondly to present an experimental identification using this technique. The simplified model is constructed as a fluid-solid semi-infinite multilayered system in which the solid layer (the cortical bone) is a nonhomogeneous anisotropic elastic material and the two others semi-infinite layers are fluids. The uncertainties are related to the elasticity tensor and are taken into account with a probabilistic model. In this paper, the parameters of the probabilistic model are the mean elasticity tensor and a dispersion coefficient. A complete application is presented for the human cortical bone for which an experimental database is available.

1 INTRODUCTION

Biomechanical systems such as the cortical bone, are very complex systems which are difficult to model in regard to their constitutive material at the microscopic scale. Such a biomechanical system can be modeled using a mechanical model which can be more or less sophisticated using or not a multiscale approach. Nevertheless, assumptions yielding modeling simplifications and approximations are introduced and therefore the developed model is always a rough approximation of the real biomechanical system. In this paper, such sources of uncertainties are taken into account in order to extend the domain of validity of a simplified model and, therefore, a probabilistic model is constructed to take into account uncertainties in the model of elasticity tensor of the cortical bone. This construction is carried out using Information Theory with the available information derived from the mechanical and probabilistic properties for the random elasticity tensor. The parameters of the probabilistic model are the mean elasticity tensor and a dispersion parameter characterizing the level of uncertainties. The identification of these parameters is performed by using *in vivo* measurements obtained with the axial transmission technique: an acoustic pulse

is applied on the skin layer in the ultrasonic range and the velocity of the first arriving signal is measured. Thus, the purpose of this paper is the experimental identification and validation of the probabilistic model developed to take into account uncertainties in the elasticity tensor of the cortical bone using the ultrasonic axial transmission technique. In this paper, the uncertain simplified mechanical model is constructed as a fluid-solid semi-infinite multilayered system in which the solid layer (representing a cortical bone) is a homogeneous anisotropic elastic material and the two others semi-infinite layers are fluids (representing the skin and the marrow bone). This model is obviously quite simple in regard to the real biomechanical system. Nevertheless, it allows the velocity of the first arriving signal to be accurately predicted. The solver used for this problem is presented in [1]. The experimental database is obtained by *in vivo* ultrasonic axial transmission on cortical bones of a given set of patients. A computational optimization problem is then introduced, consisting in minimizing a cost function with respect to the parameters of the probabilistic model. This cost function is defined by taking into account the type of experimental observations. The stochastic solver used to solve the optimization problem is based on the Monte Carlo method for which the simplex algorithm is used at each iteration. The complete stochastic model is presented with its experimental validation.

2 EXPERIMENTAL DATABASE

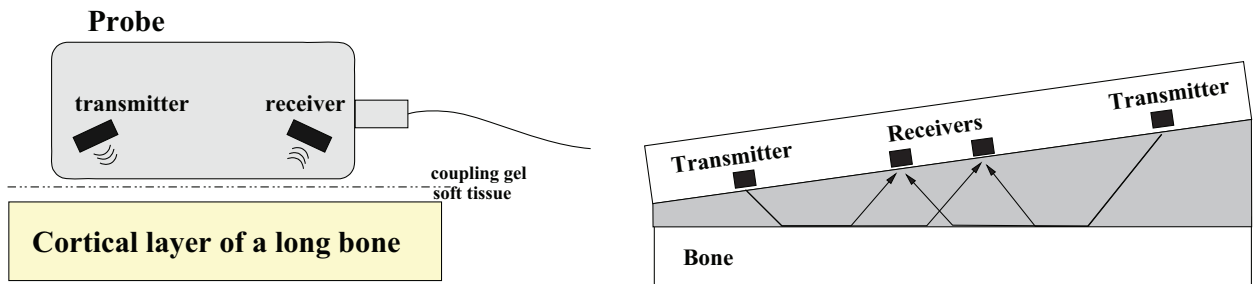


Figure 1. Experimental configuration

The ultrasonic axial transmission technique is used to construct an experimental database (see for instance [3-5]). The experimental configuration is described by Fig. 1. A device has been designed and is made up of several receivers and transmitters. A coupling gel is applied at the interface between the device and the skin of the patient. Each transmitter generates an acoustical impulse in the ultrasonic range that propagates in the coupling gel, the skin, the muscle, the cortical bone and the marrow. The axial transmission technique consists in recording these signals at several receivers located in the device. The first arriving contribution of the signal (FAS) is considered. Following the signal processing method used with the experimental device, the velocity of FAS is determined from the time of flight of the first extremum of the contribution. Figure 2 shows a part of a simulated signal and the FAS.

These *in vivo* measurements were previously performed on a population of 168 subjects examined at the third distal radius. This group is a subset of a larger group of patients who participated to a clinical evaluation of the bidirectional axial transmission device. The multi-element probe operating at a center frequency of 1 MHz recorded twenty series of axially transmitted signals without particular angular scanning protocol except natural micro-movements of the operator. The experimental database finally consisted of 2018 measurements of FAS velocity. Each velocity measurement is considered as a realization of a random variable V^{exp} . Thus, the database is made up of $N = 2747$ statistical independent realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ of random variable V^{exp} . The mean value of V^{exp} is $v^{\text{exp}} = E\{V^{\text{exp}}\}$ and its coefficient of variation Δ^{exp} is defined by $(\Delta^{\text{exp}})^2 = E\{(V^{\text{exp}})^2\}/(v^{\text{exp}})^2 - 1$ in which $E\{\cdot\}$ is the mathematical expectation. Accordingly,

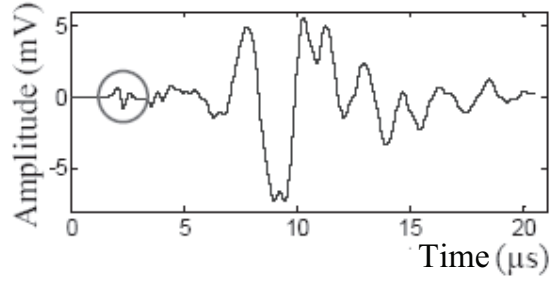


Figure 2. First arriving signal

the database consists of $N = 2018$ statistically independent realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ of random variable V^{exp} . Using the usual statistical estimators and since N is sufficiently large, v^{exp} and Δ^{exp} can be estimated by

$$v^{\text{exp}} = \frac{1}{N} \sum_{k=1}^N V^{\text{exp}}(\hat{\theta}_k) \quad , \quad \Delta^{\text{exp}} = \frac{1}{v^{\text{exp}}} \sqrt{\frac{1}{N} \sum_{k=1}^N V^{\text{exp}}(\hat{\theta}_k)^2 - (v^{\text{exp}})^2} .$$

3 SIMPLIFIED MODEL

A simplified model of the biomechanical system made up of the coupling gel, the skin, the cortical bone and the marrow has been developed in [1, 2]. This simplified model is composed of an elastic solid semi-infinite layer between two acoustic fluid semi-infinite layers (see Fig. 3). Let

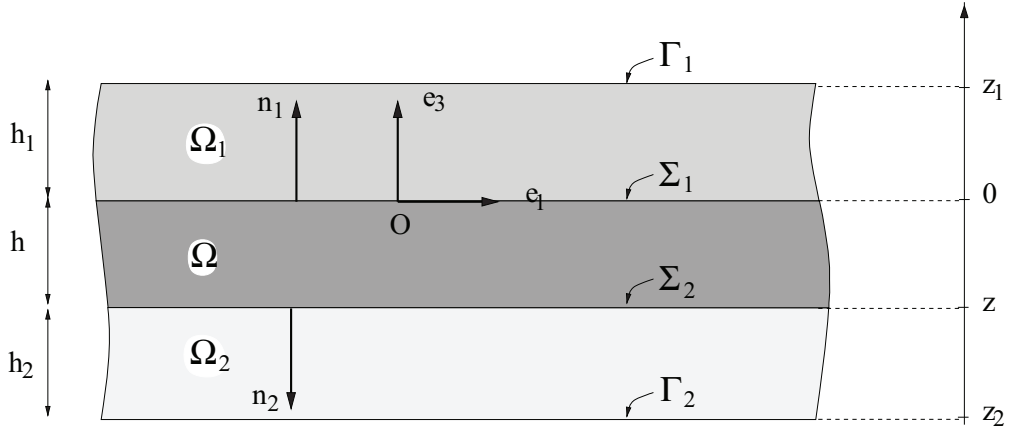


Figure 3. Geometry of the multilayer system

$\mathbf{R}(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the reference Cartesian frame where O is the origin of the space and $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is an orthonormal basis for this space. The coordinates of the generic point \mathbf{x} in \mathbb{R}^3 are (x_1, x_2, x_3) . The thicknesses of the layers are denoted by h_1 , h and h_2 . The first acoustic fluid layer occupies the open unbounded domain Ω_1 , the second acoustic fluid layer occupies the open unbounded domain Ω_2 and the elastic solid layer occupies the open unbounded domain Ω . Let $\partial\Omega_1 = \Gamma_1 \cup \Sigma_1$, $\partial\Omega = \Sigma_1 \cup \Sigma_2$ and $\partial\Omega_2 = \Sigma_2 \cup \Gamma_2$ (see Fig. 3) be respectively the boundaries of Ω_1 , Ω and Ω_2 in which $\Gamma_1, \Sigma_1, \Sigma_2$ and Γ_2 are the planes defined by

$$\Gamma_1 = \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, \quad x_3 = z_1\}$$

$$\begin{aligned}\Sigma_1 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = 0\} \\ \Sigma_2 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z\} \\ \Gamma_2 &= \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z_2\}\end{aligned}$$

in which $z_1 = h_1$, $z = -h$ and $z_2 = -(h + h_2)$. Therefore, the domains Ω_1 , Ω and Ω_2 are unbounded along the transversal directions \mathbf{e}_1 and \mathbf{e}_2 whereas they are bounded along the vertical direction \mathbf{e}_3 . Let \mathbf{u} be the displacement field of the solid elastic and, for $k = 1, 2$, let p_k be the pressure field in the fluid occupying the domain Ω_k . Let ρ and σ be the mass density and the Cauchy stress tensor field of the solid. A line source modeling the acoustical impulse is applied in domain Ω_1 . This line source is defined with a source density Q_1 such that

$$\frac{\partial Q_1}{\partial t}(\mathbf{x}, t) = \rho_1 F(t) \delta_0(x_1 - x_1^S) \delta_0(x_3 - x_3^S) \quad , \quad (1)$$

in which $F(t) = F_1 \sin(2\pi f_c t) e^{-4(t f_c - 1)^2}$ where $f_c = 1$ MHz is the central frequency and $F_1 = 100$ N; ρ_1 is the mass density of domain Ω_1 ; δ_0 is the Dirac function at the origin and x_1^S and x_3^S are the coordinates of a line source modeling the acoustical impulse. At time $t = 0$, the system is assumed to be at rest. Let $[C]$ be the effective elasticity matrix of the solid layer such that

$$[C] = \begin{pmatrix} c_{1111} & c_{1122} & c_{1133} & \sqrt{2}c_{1123} & \sqrt{2}c_{1131} & \sqrt{2}c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & \sqrt{2}c_{2223} & \sqrt{2}c_{2231} & \sqrt{2}c_{2213} \\ c_{3311} & c_{3322} & c_{3333} & \sqrt{2}c_{3323} & \sqrt{2}c_{3331} & \sqrt{2}c_{3312} \\ \sqrt{2}c_{2311} & \sqrt{2}c_{2322} & \sqrt{2}c_{2333} & 2c_{2323} & 2c_{2331} & 2c_{2312} \\ \sqrt{2}c_{3111} & \sqrt{2}c_{3122} & \sqrt{2}c_{3133} & 2c_{3123} & 2c_{3131} & 2c_{3112} \\ \sqrt{2}c_{1211} & \sqrt{2}c_{1222} & \sqrt{2}c_{1233} & 2c_{1223} & 2c_{1231} & 2c_{1212} \end{pmatrix}. \quad (2)$$

For a transverse isotropic homogeneous medium, all the components $[C]_{ij}$ are zeros except the following

$$[C]_{11} = \frac{e_L^2(1 - \nu_T)}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad [C]_{22} = \frac{e_T(e_L - e_T\nu_L^2)}{(1 + \nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad (3)$$

$$[C]_{12} = \frac{e_T e_L \nu_L}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad [C]_{23} = \frac{e_T(e_L\nu_T + e_T\nu_L^2)}{(1 + \nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad (4)$$

$$[C]_{44} = g_T \quad , \quad [C]_{55} = g_L \quad , \quad (5)$$

with $[C]_{22} = [C]_{33}$, $[C]_{12} = [C]_{13} = [C]_{21} = [C]_{31}$, $[C]_{23} = [C]_{32}$ and $[C]_{55} = [C]_{66}$ and where (1) e_L and e_T are the longitudinal and transversal Young moduli, (2) g_L and g_T are the longitudinal and transversal shear moduli and (3) ν_L and ν_T are the longitudinal and transversal Poisson coefficients such that $g_T = e_T/2(1 + \nu_T)$. For a given effective elasticity matrix $[C]$, the displacement field in the solid layer and the pressure fields in the two fluids are calculated using the fast and efficient hybrid solver presented in [1]. For a given mean elasticity matrix $[C]$, this solver allows the displacement field \mathbf{u} in Ω and the pressure fields p_1 and p_2 in Ω_1 and Ω_2 respectively, to be calculated. Then, the velocity v^{velo} of the first arriving signal is deduced. Consequently, it is possible to construct a mapping g_{velo} such that

$$v^{\text{mod}} = g_{\text{velo}}([C]) \quad . \quad (6)$$

4 STOCHASTIC SIMPLIFIED MODEL

It is assumed that uncertainties are only related to the components c_{ijkh} of the effective elasticity tensor. The introduced probabilistic model is presented in [2]. The construction of the probabilistic

model consists in substituting $[C]$ by a random matrix $[\mathbf{C}]$ for which the probability density function is constructed using the information theory (see [6, 7]) with the available information defined as follows: (1) the random matrix $[\mathbf{C}]$ is a second-order random variable with values in the set $\mathbb{M}^+(\mathbb{R})$ of all the (6×6) real symmetric positive-definite matrices; (2) the mean value of random matrix $[\mathbf{C}]$ is the mean elasticity matrix $[C]$; (3) the norm of the inverse matrix of $[\mathbf{C}]$ is a second-order random variable. It has been shown in [8, 9] that the random matrix $[\mathbf{C}]$ can then be written as

$$[\mathbf{C}] = [L]^T[\mathbf{G}][L] \quad , \quad (7)$$

in which the (6×6) upper triangular matrix $[L]$ corresponds to the Cholesky factorization $[C] = [L]^T[L]$ and where the probability density function $p_{[\mathbf{G}]}$ of random matrix $[\mathbf{G}]$ is written as

$$p_{[\mathbf{G}]}([G]) = \mathbb{1}_{\mathbb{M}^+(\mathbb{R})}([G]) c (\det[G])^b \exp\{-a \text{tr}[G]\} \quad , \quad (8)$$

where $a = 7/(2\delta^2)$, $b = a(1 - \delta^2)$, $\mathbb{1}_{\mathbb{M}^+(\mathbb{R})}([G])$ is equal to 1 if $[G]$ belongs to $\mathbb{M}^+(\mathbb{R})$ and is equal to zero if $[G]$ does not belong to $\mathbb{M}^+(\mathbb{R})$, $\text{tr}[G]$ is the trace of matrix $[G]$ and where positive constant c is such that

$$c = \frac{(2\pi)^{-15/2} a^{6a}}{\prod_{j=1}^6 \Gamma(\alpha_j)} \quad ,$$

in which $\alpha_j = 7/(2\delta^2) + (1 - j)/2$ and where Γ is the Gamma function. The parameter δ allows the dispersion of the random matrix $[\mathbf{C}]$ to be controlled. Thus, the parameters of the probabilistic model of uncertainties for the elasticity matrix are the components of $[C]$ and the coefficient δ . The velocity of the FAS constructed using this stochastic simplified model is a random variable denoted by V^{mod} that corresponds to the random experimental velocity V^{exp} introduced in Section 2 and we have (see Eq. (6))

$$V^{\text{mod}} = g_{\text{velo}}([\mathbf{C}]) \quad . \quad (9)$$

5 OPTIMIZATION PROBLEM FOR THE IDENTIFICATION

The stochastic simplified model parameters that have to be identified are the coefficients e_L , ν_L , g_L , e_T and ν_T relative to $[C]$, the mass density ρ and the coefficient δ . Let \mathbf{a} be the vector such that $\mathbf{a} = (\rho, e_L, \nu_L, g_L, e_T, \nu_T)$. The identification problem consists in finding vector \mathbf{a} and coefficient δ such that the stochastic model can represent the experimental database in a statistical sense. The optimal values $(\mathbf{a}^{\text{opt}}, \delta^{\text{opt}})$ for (\mathbf{a}, δ) is given by solving the following optimization problem

$$(\mathbf{a}^{\text{opt}}, \delta^{\text{opt}}) = \arg \min_{(\mathbf{a}, \delta)} F^{\text{cost}}(\mathbf{a}, \delta) \quad , \quad (10)$$

in which $F^{\text{cost}}(\mathbf{a}, \delta)$ is the cost function which has to be defined. The cost function F^{cost} adapted to the optimization problem is written as

$$F^{\text{cost}}(\mathbf{a}, \delta) = \frac{(v^{\text{exp}} - v^{\text{mod}}(\mathbf{a}, \delta))^2}{(v^{\text{exp}})^2} + \frac{(\Delta^{\text{exp}} - \Delta^{\text{mod}}(\mathbf{a}, \delta))^2}{(\Delta^{\text{exp}})^2} \quad ,$$

in which

$$\Delta^{\text{mod}} = \sqrt{\frac{E\{(V^{\text{mod}}(\mathbf{a}, \delta))^2\}}{(v^{\text{mod}}(\mathbf{a}, \delta))^2} - 1} \quad .$$

The optimization problem defined by Eq. (10) is solved by the simplex algorithm. For each iteration of the simplex algorithm, the cost function is calculated in solving the stochastic equations with the Monte Carlo numerical simulation method (see for instance [1, 2]).

6 EXPERIMENTAL VALIDATION OF THE STOCHASTIC SIMPLIFIED MODEL

This section is devoted to the experimental validation of the stochastic simplified model. The stochastic simplified model must be able to simulate the experimental database in a statistical sense. The experimental validation is performed with the *in vivo* experimental database presented

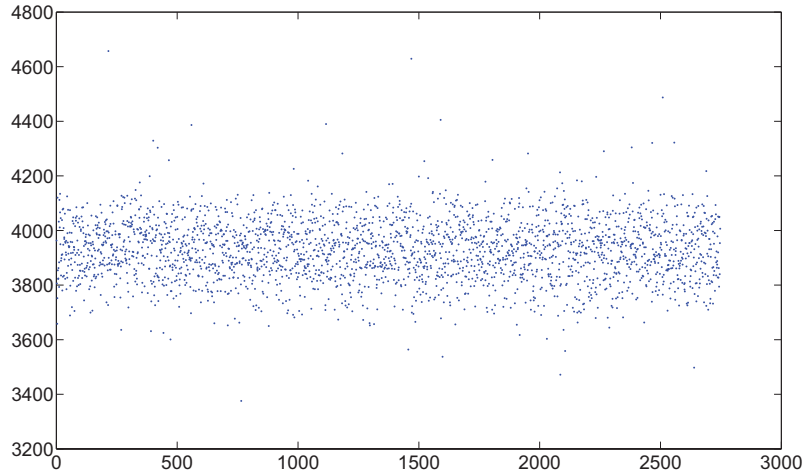


Figure 4. Graph of the realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$

in Section 2 and is made up of $N = 2747$ measurements $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ plotted in Fig. 4. The identification of the vector $\mathbf{a} = (\rho, e_L, \nu_L, g_L, e_T, \nu_T)$ and the coefficient δ is carried out using the method presented in Section 5 with $h_1 = 10^{-2}\text{m}$, $h = 4 \times 10^{-3}\text{m}$, $h_2 = 10^{-2}\text{m}$, $\rho_1 = \rho_2 = 1000 \text{ kg.m}^{-3}$ and $c_1 = c_2 = 1500 \text{ m.s}^{-1}$. The solution $\mathbf{a}^{\text{opt}} = (\rho^{\text{opt}}, e_L^{\text{opt}}, \nu_L^{\text{opt}}, g_L^{\text{opt}}, e_T^{\text{opt}}, \nu_T^{\text{opt}})$ and δ^{opt} are such that $\rho^{\text{opt}} = 1598.8 \text{ kg.m}^{-3}$, $e_L^{\text{opt}} = 17.717 \text{ GPa}$, $\nu_L^{\text{opt}} = 0.3816$, $g_L^{\text{opt}} = 4.7950 \text{ GPa}$, $e_T^{\text{opt}} = 9.8254 \text{ GPa}$, $\nu_T^{\text{opt}} = 0.4495$ and $\delta^{\text{opt}} = 0.1029$. For $\mathbf{a} = \mathbf{a}^{\text{opt}}$ and $\delta = \delta^{\text{opt}}$, the realizations $V^{\text{mod}}(\theta_1), \dots, V^{\text{mod}}(\theta_N)$ of random velocity V^{mod} are constructed with the stochastic simplified model and then, the probability density function $v \mapsto p_{V^{\text{mod}}}(v)$ of V^{mod} is estimated. Figure 5

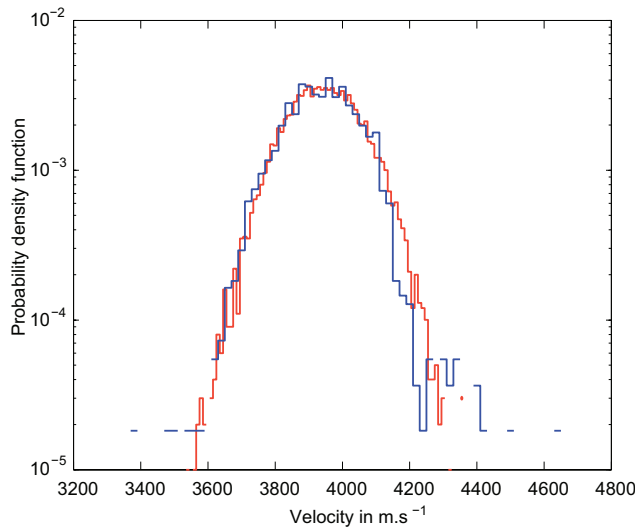


Figure 5. Graphs of $v \mapsto \log(p_{V^{\text{exp}}}(v))$ and $v \mapsto \log(p_{V^{\text{exp}}}(v; \mathbf{a}^{\text{opt}}, \delta^{\text{opt}}))$.

compares the graphs of the probability density function $v \mapsto p_{V^{\text{exp}}}(v)$ of the random variable V^{exp} estimated with the $N = 2747$ experimental realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ and the graph of $v \mapsto p_{V^{\text{mod}}}(v)$ in logarithm scale. This figure shows that the stochastic simplified model is able to predict in a statistical sense the velocity of the first arriving signal in a good accordance with the experimental tests.

7 CONCLUSION

A simplified elastoacoustic model has been developed to simulate the ultrasonic wave propagation in a complex biomechanical system made up of multilayered media. In order to improve the simplified model, the uncertainties related to the solid layer have been taken into account using a probabilistic approach. A method has been presented to identify the parameters of the stochastic simplified model. The capability of the proposed stochastic simplified model to predict the velocity of the first arriving signal in the statistical sense has been demonstrated using a large experimental *in vivo* database.

8 ACKNOWLEDGMENTS

This research was supported by the French Research National Agency (ANR) for the BONECHAR project.

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