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DRILL-STRING DYNAMICS WITH UNCERTAINTY IN THE BIT-ROCK NONLINEAR INTERACTION MODEL

Ritto, T. G.[1,2], Soize, C.[2] & Sampaio, R.[1]

[1]Mechanical Engineering Department – PUC-Rio, Rua Marquês de São Vicente, 225, Gávea, RJ, CEP: 22453-900, Brazil

[2]Université Paris-Est, Laboratoire de Modélisation et Simulation Multi Echelle, MSME FRE3160 CNRS, 5 bd Descartes, 77454 Marne-la-Vallée, France

thiagoritto@gmail.com

ABSTRACT. The stochastic dynamics of a drill-string is analyzed, where the uncertainty is in the bit-rock nonlinear interaction model. The Maximum Entropy Principle is used to construct a probabilistic model for the nonlinear operator related to the bit-rock interaction model. A numerical model is developed using the Timoshenko beam theory and it is discretized by means of the Finite Element Method. The nonlinear dynamics analyzed considers the main efforts that the column is subjected to as, for instance, imposed rotation at the top, fluid-structure interaction, impact between the column and the borehole, and finite strains (what couples axial, torsional and lateral vibrations).

KEYWORDS: probabilistic model, local nonlinearities, nonlinear stochastic dynamics, drill-string dynamics, bit-rock stochastic model.

1 INTRODUCTION

Drill-strings are slender structures used to dig into the rock in search of oil. The drill-string dynamics must be controlled to avoid failures [8], see a general vibration perspective of the oil and gas drilling process in [16]. We consider a vertical well for which the length of the column may reach some kilometers. The drill-string is composed by thin tubes called drill-pipes and some thicker tubes called drill-collars. The thicker tubes are in the bottom region which is known as the Bottom-Hole-Assembly (BHA). The forces taking into account are: the motor torque (as a constant rotation speed at the top); a constant supporting force; the torque and force at the bit; the weight of the column; the fluid forces; the impact and rubbing between the column and the borehole; the forces due to the stabilizer; plus the elastic and kinetic forces due to the deformation and to the motion of the structure.

In the literature, the nonlinear dynamics of a drill-string is modeled in several different ways, *e.g.*[2, 18, 7]. These models are able to quantify some effects that occur in a drilling operation, as the stick-slip oscillations, for instance, but they cannot correctly predict the dynamic response of a real system. This is explained, first, because the above models are too simple compared to the real system and, second, because the uncertainties are not taken into account.

Moreover, a fluid-structure interaction that takes into account the drilling fluid that flows inside and outside the column is not considered in any of the above mentioned works. This kind of fluid-structure interaction model was proposed in [10] for a plane problem in another context, and it was extended for our problem [11]. To model the column, the Timoshenko beam model is employed and the Finite Element Method is used to discretize the system. Besides, it is considered: finite strain with no simplifications (higher order terms are not neglected); quadratic terms derived from the kinetic energy; impact and rubbing between the column and the borehole; stabilizers; fluid-structure interaction; and a bit-rock interaction that models how the bit penetrates the rock.

A stochastic bit-rock interaction is analyzed because it is one of the most important sources of uncertainties of this problem. The bit-rock interaction model chosen was the one developed in [18] because it describes well the penetration of the bit into the rock (so we can analyze the rate-of-penetration-ROP).

The nonparametric probabilistic approach [13, 14] is used to model the uncertainties in the bit-rock interaction which is represented by a nonlinear operator. Note that a new strategy has to be developed

to take into account uncertainties in a local nonlinear operator. The probability density function is derived using the Maximum Entropy Principle [12, 5, 6]. Two probabilistic models are analyzed: one that allows only change in the parameters (called non-coupled model) and another that permits changes in the model (the usual nonparametric model). Then, by perturbing the bit-rock operator, the robustness of the models used is analyzed.

The paper is organized as following. In Section 2 the mean model is presented and in Section 3 the probabilistic model of the bit-rock interaction model is developed. The numerical results are shown in Section 4 and the concluding remarks are made in Section 5.

2 MEAN MODEL

In this Section the equations used to model the problem are presented. The Total Lagrangian (TL) formulation is used, six degrees of freedom are considered in the points of discretization (three translations, u, v and w, and three rotations θ_x, θ_y and θ_z), the stress tensor is the second Piola-Kirchhoff tensor and finite strains are considered (Green-Lagrange strain tensor). The main hypothesis are the following: (1) the drill-string is axisymmetric about x-axis; (2) the following strain components are neglected: $e_{yy} \sim e_{zz} \sim \gamma_{yz} \sim \gamma_{zy} \sim 0$; (3) the rotations θ_y and θ_z are small, i.e, $\sin(\theta_y) \sim \theta_y$, $\sin(\theta_z) \sim \theta_z$ and $\cos(\theta_y) \sim \cos(\theta_z) \sim 1$; (4) the stress-strain relationship is linear; and (5) the rotation θ_x is finite (the rotational speed $\dot{\theta}_x$, of course, is not constant over the element).

The strategy used in this work is, in some respects, similar to the one used in [7], but there are several important additional features, such as (1) impact and rubbing between the column and the borehole; (2) shear (Timoshenko beam model); (3) finite θ_x; (4) fluid-structure interaction; (5) all the terms of the strain energy are used in the analysis; (5) a bit-rock interaction model that allows the simulation of the bit penetration is used; and (6) constant force at the top (supporting force or weight-on-hook).

To derive the dynamic equations, the extended Hamilton Principle is used. The first variation of Π must vanish

$$\delta\Pi = \int_{t_1}^{t_2} (\delta U - \delta T - \delta W) dt = 0\,. \tag{1}$$

where U is the potential strain energy, T is the kinetic energy and W is the work done by the nonconservative forces and by any force not accounted for in the potential energy.

General equations. In the discretization by means of the Finite Element Method, a two-node approximation with six degrees of freedom per node is chosen. The nodal displacement is written as $u_e = \mathbf{N}_u \mathbf{u}_e$, $v_e = \mathbf{N}_v \mathbf{u}_e$, $w_e = \mathbf{N}_w \mathbf{u}_e$, $\theta_{xe} = \mathbf{N}_{\theta_x} \mathbf{u}_e$, $\theta_{y_e} = \mathbf{N}_{\theta_y} \mathbf{u}_e$, $\theta_{z_e} = \mathbf{N}_{\theta_z} \mathbf{u}_e$. Where $\mathbf{N}$ are the shape functions (see [11]); u_e, v_e and w_e are the displacements in x, y and z directions; θ_{xe}, θ_{ye} and θ_{ze} are the rotations about x, y and z axis. The element coordinate is $\xi = x/l_e$, and

$$\mathbf{u}_e = \begin{pmatrix} u_1 & v_1 & \theta_{z1} & w_1 & \theta_{y_1} & \theta_{x1} & u_2 & v_2 & \theta_{z2} & w_2 & \theta_{y_2} & \theta_{x2} \end{pmatrix}^T\,, \tag{2}$$

where $(\cdot)^T$ means transpose. The dynamics is computed from a prestressed configuration, $\mathbf{u}_S = [K]^{-1}(\mathbf{f}_g + \mathbf{f}_c + \mathbf{f}_f)$. After assemblage, the final discretized system is written as

$$([M] + [M_f])\ddot{\bar{\mathbf{u}}} + ([C] + [C_f])\dot{\bar{\mathbf{u}}} + ([K] + [K_f] + [K_g(\mathbf{u}_S)])\bar{\mathbf{u}} = \mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}})\,, \tag{3}$$

where $\bar{\mathbf{u}} = \mathbf{u} - \mathbf{u}_S$. $\mathbf{u}$ is the $\mathbb{R}^m$-valued response for which m is the number of degrees of freedom of the system. $[M]$, $[C]$ and $[K]$ are the usual mass, damping and stiffness matrices; $[M_f]$, $[C_f]$, $[K_f]$ are the added fluid mass, damping and stiffness matrices, and $\mathbf{f}_f$ is the fluid force vector; $\mathbf{f}_g$ is the gravity force; $\mathbf{f}_c$ is a concentrated reaction force at the bit; $[K_g(\mathbf{u}_S)]$ is the geometric stiffness matrix and $\mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}})$ is the nonlinear force vector that is decomposed in

$$\mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}) = \mathbf{f}_{ke}(\ddot{\bar{\mathbf{u}}}, \dot{\bar{\mathbf{u}}}, \bar{\mathbf{u}}) + \mathbf{f}_{se}(\bar{\mathbf{u}}) + \mathbf{f}_{ip}(\bar{\mathbf{u}}) + \mathbf{f}_{br}(\dot{\bar{\mathbf{u}}}) + \mathbf{g}(t)\,. \tag{4}$$

where $\mathbf{f}_{ke}$ is composed by the quadratic terms of the kinetic energy; $\mathbf{f}_{se}$ is composed by the quadratic and higher order terms of the strain energy; $\mathbf{f}_{ip}$ are the forces due to the impact and rubbing between the column and the borehole; $\mathbf{f}_{br}$ are the forces due to the bit-rock interactions, see section ; and $\mathbf{g}$ is the force that corresponds to the Dirichlet boundary condition (rotation imposed at the top). For a detailed explanation of each term of the nonlinear force see [11].

Fluid-structure interaction. The drilling fluid (mud) is responsible to transport the cuttings (drilled solids) from the bottom to the top to avoid clogging of the hole. It also plays an important role in cooling and stabilizing the system [1]. The rheological properties of the mud are complexes, see [3] for instance. There is no doubt that the drilling fluid influences the dynamics of a drill-string, but to solve the complete problem would be too expensive computationally. There are some works that study only the drilling fluid flow, as, for example, [4]. In this work a linear fluid-structure coupling model similar to [10] is used. In this simplified model there are the following hypotheses: the inside fluid is inviscid, while the outside flow is viscous, the flow induced by the rotation speed about x-axis is not considered in the analysis and the pressure varies linearly with x.

For short, the element equations are presented. These equations are an extension and an adaptation of the model developed in [10].

$$
\begin{aligned}
[M_f]^{(e)} &= \int_0^1 \left(M_f + \chi\rho_f A_o\right)(\mathbf{N}_w^T\mathbf{N}_w + \mathbf{N}_v^T\mathbf{N}_v) l_e d\xi\,,\\
[K_f]^{(e)} &= \int_0^1 \left(-M_f U_i^2 - A_i p_i + A_o p_o - \chi\rho_f A_o U_o^2\right)(\mathbf{N'}_w^T\mathbf{N'}_w + \mathbf{N'}_v^T\mathbf{N'}_v)\frac{1}{l_e} d\xi +\\
&\quad + \int_0^1 \left(-A_i\frac{\partial p_i}{\partial x} + A_o\frac{\partial p_o}{\partial x}\right)(\mathbf{N}_{\theta_y}^T\mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T\mathbf{N}_{\theta_z}) l_e d\xi\,,\\
[C_f]^{(e)} &= \int_0^1 \left(-2M_f U_i + 2\chi\rho_f A_o U_o\right)(\mathbf{N}_{\theta_y}^T\mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T\mathbf{N}_{\theta_z}) l_e d\xi +\\
&\quad + \int_0^1 \left(\frac{1}{2}C_f\rho_f D_o U_o + k\right)(\mathbf{N}_w^T\mathbf{N}_w + \mathbf{N}_v^T\mathbf{N}_v) l_e d\xi\,,\\
\mathbf{f}_f^{(e)} &= \int_0^1 \left(M_f g - A_i\frac{\partial p_i}{\partial x} - \frac{1}{2}C_f\rho_f D_o U_o^2\right)\mathbf{N}_u^T l_e d\xi\,.
\end{aligned}
\tag{5}
$$

in which,

M_f is the fluid mass per unit length, ρ_f is the density of the fluid, $\chi = \dfrac{(D_{ch}/D_o)^2 + 1}{(D_{ch}/D_o)^2 - 1}$ (> 1), D_{ch} is the borehole (channel) diameter, D_i, D_o are the inside and outside diameters of the column, U_i, U_o are the inlet and outlet flow velocities, p_i, p_o are the pressures inside and outside the drill-string, A_i, A_o are the inside and outside cross sectional area of the column and C_f, k are the fluid viscous damping coefficients.

It is assumed that the inner and the outer pressures (p_i and p_o) vary linearly with x

$$p_i = \left(\rho_f g\right) x + p_{cte}\,, \tag{6}$$

$$p_o = \left(\rho_f g + \frac{F_{fo}}{A_o}\right) x\,, \tag{7}$$

where p_{cte} is a constant pressure and F_{fo} is the friction force due to the external flow given by

$$F_{fo} = \frac{1}{2} C_f \rho_f \frac{D_o^2 U_o^2}{D_h} \,. \tag{8}$$

In the above equation, D_h is the hydraulic diameter ($=4A_{ch}/S_{tot}$) and S_{tot} is the total wetted area per unit length ($\pi D_{ch} + \pi D_o$). Note that the reference pressure is $p_o|_{x=0} = 0$. Another assumption is that there is no head loss when the fluid passes from the drill-pipe to the drill-collar (and vice-versa). The head loss due to the change in velocity of the fluid at the bottom (it was going down, then it goes up) is given by

$$h = \frac{1}{2g}(U_i - U_o)^2 \,. \tag{9}$$

Note that if the geometry and the fluid characteristics are given, only the inlet flow at $x = 0$ can be controlled because the fluid speed is calculated using the continuity equation and the pressures are calculated using the Bernoulli equation.

Examining Eq. (5), it can be seen that the mass matrix due to the fluid is the usual added mass that, in our case, represents a significative part of the total mass (in norm). For example, using representative values (used in our simulations), the added mass is around 50%, what changes the natural frequencies in about 20%.

The stiffness matrix due to the fluid depends on the speed of the inside and outside flow, on the pressure and on the pressure derivatives. Analyzing the signs in the equation (Eq. 5) it can be seen see that the outsize pressure tends to stabilize the system while the inside pressure and the flow tends to destabilize the system. The term $(-p_i A_i + p_o A_0)$ plays a major role on the stiffness of the system because, even though p_i is close to p_o, in the drill collar region (in the bottom) A_0 is around ten times A_i what turns the system stiffer at the bottom.

The damping matrix due to the fluid depends on the flow velocity as well as in the viscous parameter of the fluid, which are not well established values. There are uncertainties to determine the damping characteristics and a stochastic model should be developed to the damping, but in this work a detailed analysis will not be addressed. Finally, the force vector ($\mathbf{f}_f$) represents the buoyancy induced by the fluid and it is the only force in the axial direction (x-direction).

Bit-rock interaction. The model used in this work is the one developed by [18], which can be written as

$$\begin{aligned} f_{xbit} &= -\frac{\dot{u}_{bit}}{a_2 Z(\dot{\theta}_{bit})^2} + \frac{a_3 \dot{\theta}_{bit}}{a_2 Z(\dot{\theta}_{bit})} - \frac{a_1}{a_2} \\ t_{xbit} &= -\frac{\dot{u}_{bit} a_4 Z(\dot{\theta}_{bit})^2}{\dot{\theta}_{bit}} - a_5 Z(\dot{\theta}_{bit}) \end{aligned} \tag{10}$$

where f_{xbit} is the axial force (also called weight-on-bit), t_{xbit} is the torque about x-axis and $a_1, \ldots, a_5$ are positive constants that depend on the bit and rock characteristics as well as on the weight-on-bit. Note that $\dot{u}_{bit}$ is the rate-of-penetration (ROP). $Z(\dot{\theta}_{bit})$ is the regularization function so that when $\dot{\theta}_{bit}$ approaches to zero t_{xbit} and $\dot{u}_{bit}$ vanish.

$$Z(\dot{\theta}_{bit}) = \frac{\dot{\theta}_{bit}}{\sqrt{(\dot{\theta}_{bit})^2 + e^2}} \,. \tag{11}$$

Equation (10) was derived in a stable operation with $\dot{\theta}_{bit} \sim 100$ RPM and with $f_{xbit} \sim 100$ kN. In this model the bit exerts only an axial force (f_{xbit}) and a torque (t_{xbit}) about x-axis what couples axial and torsional vibrations.

Reduced model. Usually the final discretized FE system have big matrices (dimension $m \times m$) and the dynamic analysis may be time consuming, which is the case of the analysis presented. One way to reduce the system is to project the nonlinear dynamical equation on a subspace $V_n \in \mathbb{R}^m$, with $n << m$. In this paper, the basis used to generate V_n for the reduction basis is formed by the normal modes, but, as it will be pointed out later, these normal modes have to properly be chosen (they can not be taken simply in the order that they appear). The normal modes are obtained from the following generalized eigenvalue problem,

$$([K] + [K_f] + [K_g(\mathbf{u}_S)])\boldsymbol{\phi} = \omega^2([M] + [M_f])\boldsymbol{\phi}\,, \tag{12}$$

where $\boldsymbol{\phi}_i$ is the i-th normal mode and ω_i is the i-th natural frequency. We use the representation $\overline{\mathbf{u}} = [\Phi]\,\mathbf{q}$, where $[\Phi]$ is a $(m \times n)$ real matrix composed by n normal modes obtained using the prestressed configuration. Projecting the equation on the subspace spanned by these normal modes yields

$$[M_r]\,\ddot{\mathbf{q}}(t) + [C_r]\,\dot{\mathbf{q}}(t) + [K_r]\,\mathbf{q}(t) = [\Phi]^T\mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}})\,, \tag{13}$$

in which

$$[M_r] = [\Phi]^T([M] + [M_f])[\Phi], \quad [C_r] = [\Phi]^T([C] + [C_f])[\Phi]$$
$$[K_r] = [\Phi]^T([K] + [K_f] + [K_g(\mathbf{u}_S)])[\Phi] \tag{14}$$

are the reduced matrices.

3 PROBABILISTIC MODEL OF THE BIT-ROCK INTERACTION MODEL

The parametric probabilistic approach allows physical parameter uncertainties to be modeled. It should be noted that the underlying deterministic model defined by Eq. (10) exhibits parameters (a_1, a_2, a_3, a_4 and a_5) which are obtained by an identification process, so it would be difficult to propose a stochastic model for each one, moreover they are not independent from each other. We then proposed to use the nonparametric probabilistic approach of uncertainties [13] consisting in globally modeling the operator of the constitutive equation, Eq. (10), by a random operator.

In the early works, the nonparametric probabilistic approach was applied for linear operators [14]. Recently is was extended to geometrically nonlinear dynamical systems [9], but the type of problem studied here is completely different. We are dealing with a nonlinear operator that is an interaction model (bit-rock interaction), therefore it requires a different methodology, and we propose to use the nonparametric idea.

For convenience Eq. (10) is rewritten in the matrix form as

$$\mathfrak{f}_{bit}(\dot{\mathbf{x}}) = -[A_b(\dot{\mathbf{x}})]\dot{\mathbf{x}}\,, \tag{15}$$

in which $[A_b(\dot{\mathbf{x}})]$ is the positive-definite matrix depict by:

$$[A_b(\dot{\mathbf{x}})] = \begin{pmatrix} \left(\dfrac{a_1}{a_2\dot{u}_{bit}} + \dfrac{1}{a_2 Z(\dot{\theta}_{bit})^2} - \dfrac{a_3\dot{\theta}_{bit}}{a_2 Z(\dot{\theta}_{bit})\dot{u}_{bit}}\right) & 0 \\ 0 & \left(\dfrac{a_4 Z(\dot{\theta}_{bit})^2\dot{u}_{bit}}{\dot{\theta}_{bit}^2} + \dfrac{a_5 Z(\dot{\theta}_{bit})}{\dot{\theta}_{bit}}\right) \end{pmatrix}, \tag{16}$$

and

$$\mathfrak{f}_{bit}(\dot{\mathbf{x}}) = \begin{pmatrix} f_{bit} \\ t_{bit} \end{pmatrix} \quad \text{and} \quad \dot{\mathbf{x}} = \begin{pmatrix} \dot{u}_{bit} \\ \dot{\theta}_{bit} \end{pmatrix}.$$

To verify the positive-definiteness of $[A_b(\dot{\mathbf{x}})]$, in our case, we simply check if the diagonal terms are greater than zero, and this is true for the range of values that we are working with.

The nonparametric probabilistic approach consists, for all deterministic vector $\dot{\mathbf{x}}$, in modeling the matrix $[A_b(\dot{\mathbf{x}})]$ by a random matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ with values in the set $\mathbb{M}_n^+(\mathbb{R})$ of all positive-definite symmetric $(n \times n)$ real matrices, with $n = 2$. Note that for each instant the matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ will be different because it depends on $\dot{\mathbf{x}}$ that changes with time.

In order to apply the nonparametric probabilistic approach for the operator $[A_b(\dot{\mathbf{x}})]$, we have to define the available information and in a second step construct the probability density function of the random matrix using the Maximum Entropy Principle. The available information is made up of

1. $\forall\ \dot{\mathbf{x}}$, random matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ is positive definite almost surely,
2. $E\{[\mathbf{A}_b(\dot{\mathbf{x}})]\} = [A_b(\dot{\mathbf{x}})]$,
3. $E\{|||[\mathbf{A}_b(\dot{\mathbf{x}})]^{-1}||_F^2\} = \mathrm{c}_1\ ,\ |\mathrm{c}_1| < +\infty$,

in which $E\{\cdot\}$ is the mathematical expectation, $\|\cdot\|_F$ denotes the Frobenius norm of the matrix ($|||[B]|||_F = (tr\{[B][B]^T\})^{1/2}$) and $[A_b(\dot{\mathbf{x}})]$ is the matrix of the mean model. Following the methodology of the nonparametric probabilistic approach, the mean value is written, using the Cholesky decomposition, as

$$[A_b(\dot{\mathbf{x}})] = [L_b(\dot{\mathbf{x}})]^T [L_b(\dot{\mathbf{x}})]\,, \tag{17}$$

and the random matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ is defined by

$$[\mathbf{A}_b(\dot{\mathbf{x}})] = [L_b(\dot{\mathbf{x}})]^T [\mathbf{G}_b][L_b(\dot{\mathbf{x}})]\,. \tag{18}$$

In the above equation, $[\mathbf{G}_b]$ is a random matrix satisfying the following available information,

1. random matrix $[\mathbf{G}_b]$ is positive definite almost surely,
2. $E\{[\mathbf{G}_b]\} = [\,I\,]$,
3. $E\{|||[\mathbf{G}_b]^{-1}||_F^2\} = \mathrm{c}_2\ ,\ |\mathrm{c}_2| < +\infty$,

in which $[\,I\,]$ is the identity matrix. It should be noted that, in this construction, the random matrix $[\mathbf{G}_b]$ neither depends on $\dot{\mathbf{x}}$ nor on time. Taking into account the available information and applying the Maximum Entropy Principle yields an explicit expression [14] of the probability density function $p_{[\mathbf{G}_b]}$ which is written as

$$p_{[\mathbf{G}_b]}([G_b]) = \mathbb{1}_{\mathbb{M}_n^+(\mathbb{R})}([G_b]) C_{\mathbf{G}_b} det([G_b])^{(n+1)\frac{(1-\delta^2)}{2\delta^2}} \exp\left\{-\frac{(n+1)}{2\delta^2} tr[G_b]\right\}, \tag{19}$$

in which $det(\cdot)$ is the matrix determinant, $tr(\cdot)$ is the matrix trace and δ is the dispersion parameter of the distribution. The constant of normalization is written as

$$C_{\mathbf{G}_b} = \frac{(2\pi)^{-n(n-1)/4} \left(\frac{n+1}{2\delta^2}\right)^{n(n+1)(2\delta^2)^{-1}}}{\left\{\prod_{j=1}^n \Gamma\left(\frac{n+1}{2\delta^2} + \frac{1-j}{2}\right)\right\}}\,, \tag{20}$$

where $\Gamma(z)$ is the gamma function defined for $z > 0$ by $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt$ and the dispersion parameter δ is given by

$$\delta = \left\{\frac{1}{n} E\{|||[\mathbf{G_b}] - [I]||_F^2\}\right\}^{\frac{1}{2}}\,. \tag{21}$$

The random generator of independent realizations of random matrix $[\mathbf{G}_b]$, for which the probability density function is defined by Eq. (19), is given in [14]. Attention with the stochastic solver because while matrix $[A_b(\dot{\mathbf{x}})]$ changes with time, random matrix $[\mathbf{G}_b]$ is constant in time.

In the deterministic equation, we have

$$\mathcal{L}_{NL}(\overline{\mathbf{u}}(t), \dot{\overline{\mathbf{u}}}(t), \ddot{\overline{\mathbf{u}}}(t)) = \mathbf{f}_{br}(\dot{\overline{\mathbf{u}}}(t)) \,, \tag{22}$$

where $\mathcal{L}_{NL}$ represents all the terms in Eq. (3) except the bit forces $\mathbf{f}_{br}$. The only nonzero components of $\mathbf{f}_{br}$ are related to the axial and torsional d.o.f. at $x = L$, which are represented by $\mathfrak{f}_{bit}$, Eq. (15). For the stochastic equation, we have

$$\mathcal{L}_{NL}(\overline{\mathbf{U}}(t), \dot{\overline{\mathbf{U}}}(t), \ddot{\overline{\mathbf{U}}}(t)) = \mathbf{F}_{br}(\dot{\overline{\mathbf{U}}}(t)) \,, \tag{23}$$

where $\overline{\mathbf{U}}(t)$ is the random response and $\mathbf{F}_{br}$ is the random force at the bit for which the only nonzero components are related to the axial and torsional d.o.f. at $x = L$. This force is $\mathfrak{F}_{bit}$ which is the random variable related to $\mathfrak{f}_{bit}$ and is written as $\mathfrak{F}_{bit} = [L_b(\dot{\mathbf{x}})]^T [\mathbf{G}_b][L_b(\dot{\mathbf{x}})]\dot{\mathbf{x}}$. Let

$$[\mathbf{G}_b(s_1)], \ldots, [\mathbf{G}_b(s_\nu)] \tag{24}$$

be ν independent realizations of random matrix $[\mathbf{G}_b]$. For each realization s_j, we have to solve the deterministic equation

$$\mathcal{L}_{NL}(\overline{\mathbf{U}}(t, s_j), \dot{\overline{\mathbf{U}}}(t, s_j), \ddot{\overline{\mathbf{U}}}(t, s_j)) = \mathbf{F}_{br}(\dot{\overline{\mathbf{U}}}(t, s_j)) \,. \tag{25}$$

4 NUMERICAL RESULTS

The drill-string was discretized using 56 finite elements. For the dynamics analysis it was used 10 lateral modes, 10 torsional modes, 10 axial modes and also the two rigid body modes of the structure (axial and torsional), so matrix $[\Phi]$ is composed by 32 modes. For the time integration procedure, a scheme based on the Newmark method has been implemented with a procedure to equilibrate the system response in each time step. The system parameters used are representative values that are found in the literature [2, 18, 7], see appendix A.

Fig. 1 shows 50 Monte Carlo simulations and the 95% envelope for the nonparametric model using $\delta = 0.01$.

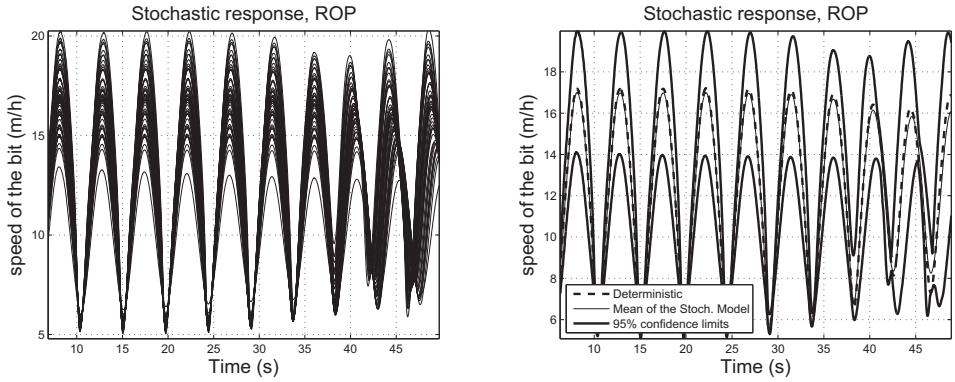


Figure 1: $\delta = 0.01$. Nonparametric model, rate-of-penetration, ROP. Left: 50 Monte Carlo simulations. Right: 95% envelope.

Fig. 2 shows the 95% envelope of the rotation speed of the bit using $\delta = 0.01$.

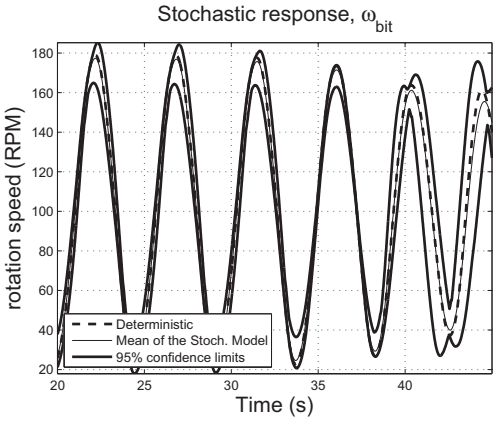


Figure 2: $\delta = 0.01$. Nonparametric model, rotation speed of the bit, 95% envelope.

Note that even for a small dispersion, $\delta = 0.01$, the stochastic response presents a significative variance. The probability space is richer when the usual nonparametric model is used, *i.e.*, there is an extra coupling and model uncertainties are taken into account. The dispersion will be increased to verify the robustness of the bit-interaction model to model uncertainties, Fig. 3.

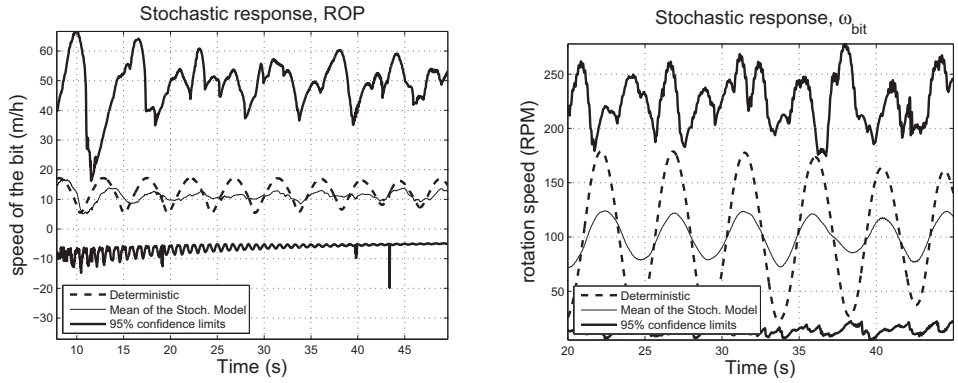


Figure 3: $\delta = 0.1$. Nonparametric model, 95% envelope. Left: rate-of-penetration, ROP. Right: rotation speed of the bit.

Fig. 3 shows that there are some realizations that present negative rate-of-penetration. See Fig. 4, where 50 Monte Carlo simulations for $\delta = 0.03$ are plotted.

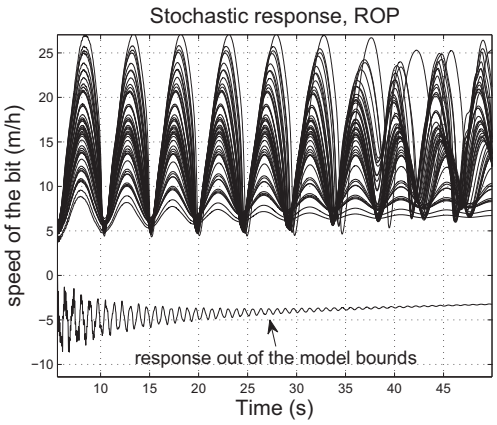


Figure 4: $\delta = 0.03$. Nonparametric model. 50 Monte Carlo simulations of the ROP.

Note that there are some realizations (the ones in the bottom of Fig. 4) that escape the limits of the model we are working with because the bit loses contact with the soil. The bit-bounce phenomenon is not considered in the current model. We may say then that the bit-rock interaction model is not robust to model uncertainties. Using the nonparametric probabilistic model, a little perturbation in the operator related to the bit-rock interaction ($\delta = 0.03$) leads to a stochastic response with large confidence region and there are some dynamic responses that get out of the model's bound.

5 CONCLUDING REMARKS

The stochastic dynamics of a drill-string was analyzed and a probabilistic model was proposed for the bit-rock interaction. As the parameters of the bit-rock interaction do not correspond to physical parameters, it can easily be concluded that it is not really adequate to use the parametric probabilistic approach to model the uncertainties, therefore the nonparametric probabilistic approach is used. This corresponds to a completely novel approach to take into account model uncertainties in a nonlinear constitutive equation. Since the dynamical system is globally nonlinear, an adapted strategy is developed to implement the stochastic simulation. The results showed that a small uncertainty in the bit-rock operator yields a very spread response and there are some realizations that get out of the model's bound. This means that this bit-rock interaction model is very sensitive to model uncertainties.

The Timoshenko beam model is used and the main forces that affect the dynamics are considered: motor torque (as a constant rotation speed at the top), supporting force, stabilizers, bit-rock interaction that describes the rate-of-penetration, impact and rubbing between the column and the borehole, fluid-structure interaction (that flows downwards then goes upwards). Finite deformations were considered without neglecting the higher order terms and the vibration was computed about a prestressed configuration.

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A DATA USED IN THE SIMULATION

$\Omega_x = 100$ [RPM] (imposed rotational speed about x-axis at $x = 0$ m), $f_c = 100$ [kN] (initial reaction force at the bit), $L_{dp} = 1400$ [m] (length of the drill pipe), $L_{dc} = 200$ [m] (length of the drill collar), $D_{odp} = .127$ [m] (outside diameter of the drill pipe), $D_{odc} = .2286$ [m] (outside diameter of the drill collar), $D_{idp} = .095$ [m] (inside diameter of the drill pipe), $D_{idc} = 0.0762$ [m] (inside diameter of the drill collar), $D_{ch} = 0.3$ [m] (diameter of the borehole (channel)), $x_{stab} = 1400$ [m] (location of the stabilizer), $k_{stab} = 17.5$ [MN/m] (stiffness of the stabilizer per meter), $E = 210$ [GPa] (elasticity modulus of the drill string material), $\rho = 7850$ [kg/m^3] (density of the drill string material), $\nu = .29$ [-] (poisson coefficient of the drill string material), $k_s = 6/7$ [-] (shearing correcting factor), $c_1 = 0.01$ [N.s/m] (friction coefficient for the axial rigid body motion), $c_2 = 0.01$ [N.s/m] (friction coefficient for the rotation rigid body motion), $k_{ip} = 1e8$ [N/m] (stiffness per meter used for the impacts), $\mu_{ip} = 0.0005$ [-] (friction coefficient between the string and the borehole), $u_{in} = 1.5$ [m/s] (flow speed in the inlet), $\rho_f = 1200$ [kg/m^3] (density of the fluid), $C_f = .0125$ [-] (fluid viscous damping coefficient), $k = 0$ [-] (fluid viscous damping coefficient), $g = 9.81$ [m/s^2] (gravity acceleration), $a_1 = 3.429e-3$ [m/s] (constant of the bit-rock interaction model), $a_2 = 5.672e-8$ [m/(N.s)] (constant of the bit-rock interaction model), $a_3 = 1.374e-4$ [m/rd] (constant of the bit-rock interaction model), $a_4 = 9.537e6$ [N.rd] (constant of the bit-rock interaction model), $a_5 = 1.475e3$ [N.m] (constant of the bit-rock interaction model), $e = 2$ [rd/s] (regularization parameter). The damping matrix is constructed using the relationship $[C] = \alpha([M] + [M_f]) + \beta([K] + [K_f] + [K_g(\mathbf{u}_S)])$ with $\alpha = .01$ and $\beta = .0003$.

REFERENCES

[1] ASME. Handbook: Drilling fluids processing. Massachusetts: Elsevier, Inc.; 2005.

[2] Christoforou AP, Yigit AS. Fully vibrations of actively controlled drillstrings. Journal of Sound and Vibration, 2003;267:1029–1045.

[3] Coussot P, Bertrand F, Herzhaft B. Rheological Behavior of drilling muds, characterization using MRI vizualization. Oil and Gas Science and Technology, 2004;59(1):23–29.

[4] Escudier MP, Oliveira PJ, Pinho FT. Fully developed laminar flow of purely viscous non-Newtonian liquids through annuli, including the effects of eccentricity and inner-cylinder rotation. International Journal of Heat and Fluid Flow, 2002;23:52–73.

[5] Jaynes E. Information theory and statistical mechanics. The Physical Review, 1957;106(4):1620–630.

[6] Jaynes E. Information theory and statistical mechanics II. The Physical Review, 1957;108:171–190.

[7] Khulief YA, Al-Sulaiman FA, Bashmal S. Vibration analysis of drillstrings with self excited stick-slip oscillations. Journal of Sound and Vibration, 2007;299:540–558.

[8] Macdonald KA, Bjune JV. Failure analysis of drillstrings. Engineering Failure Analysis, 2007;14:1641–1666.

[9] Mignolet MP, Soize C. Stochastic Reduced Order Models For Uncertain Geometrically Nonlinear Dynamical Systems. Computer Methods in Applied Mechanics and Engineering, 2008;197(45-48):3951–3963.

[10] Paidoussis MP, Luu TP, Prabhakar S. Dynamics of a long tubular cantilever conveying fluid downwards, which then flows upwards around the cantilever as a confined annular flow. Journal of Fluids and Structures, 2007;24(1):111–128.

[11] Ritto TG, Sampaio R, Soize C. Drill-string dynamics coupled with the drilling fluid dynamics. Proceedings of the XIII DINAME, Angra dos Reis, RJ, Rio de Janeiro, Brazil, Mar. 2-6; 2009.

[12] Shannon CE. A mathematical theory of communication. Bell System Tech. J., 1948;27:379-423 and 623-659.

[13] Soize C. A nonparametric model of random uncertities for reduced matrix models in structural dynamics. Probabilistic Engineering Mechanics, 2000;15:277–294.

[14] Soize C. Random matrix theory for modeling uncertainties in computational mechanics. Computer Methods in Applied Mechanics and Engineering, 2005;194(12-16),1333–1366.

[15] Spanos PD, Sengupta AK, Cunningham RA, Paslay PR. Modeling of roller cone bit lift-off dynamics in rotary drilling. Journal of Energy Resources Technology, 1995;117(3):197–207.

[16] P. D. Spanos, A. M. Chevallier, N. P. Politis, M. L. Payne. Oil and Gas Well Drilling: A Vibrations Perspective. The Shock and Vibration Digest, 2003;35(2):85–103.

[17] R. J. Serfling. Approximation Theorems of Mathematical Statistics. USA: John Wiley & Sons; 1980.

[18] Tucker RW, Wang C. Torsional vibration control and cosserat dynamics of a drill-rig assembly. Meccanica, 2003;38(1):143–159.