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▶ **To cite this version:**

M. Kassem, Christian Soize, L. Gagliardini. A sub-structuring method in low- and medium-frequency ranges based on an energy formulation and a stochastic vibroacoustic computational model. 2nd International Conference on Uncertainty in Structural Dynamics, The University of Sheffield, Jun 2009, Sheffield, United Kingdom. pp.1-8. hal-00691741

HAL Id: hal-00691741
https://hal.science/hal-00691741

Submitted on 26 Apr 2012

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A SUB-STRUCTURING METHOD IN LOW- AND MEDIUM-FREQUENCY RANGES BASED ON AN ENERGY FORMULATION AND ON A STOCHASTIC VIBROACOUSTIC COMPUTATIONAL MODEL

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ABSTRACT. A sub-structuring method of complex structures is proposed in this paper. This method is developed using an energy formulation which is based on a stochastic vibroacoustic computational model. Recent works have been done for the application of the non parametric probabilistic approach of system parameters and model uncertainties on automotive vehicle structures. The results of this work showed that, in a stochastic context, some statistical properties of the frequency response functions (FRF) in low- and medium-frequency ranges can be used in order to simplify the vibroacoustic problem. Thus, although inspired by Statistical Energy Analysis (SEA), unlike the existing statistical methods, the used energy formulation is applicable in the low- and medium-frequency ranges thanks to the stochastic vibroacoustic computational model.

KEYWORDS: Vibroacoustic, Energy Analysis, stochastic computational Model, sub-structuring.

1 INTRODUCTION

The vibroacoustic problem of industrial structures can be of a considerable complexity due to the presence of multiple acoustic and vibration sources as well as a very high number of Frequency Response Functions (FRF). In addition, the gap between the experimental measures and the numerical predictions due to the presence of uncertainties not only in the physical system parameters, but also in the numerical model itself adds to the complexity of the problem. Several methods have been developed in order to simplify the vibroacoustic response as well as the source-structure interaction of complex structures specially in the medium-frequency ranges. These methods are mainly energy or statistical methods and aim to the sub-structuring of the studied problem. One well known method is the Statistical Energy Analysis (SEA) and its derivations (see [1, 2, 3, 4, 5, 6, 7, 8]). There exist other energy methods using different types of mobility of the structure in order to simplify the response of the vibroacoustic system (e.g. [9, 10, 11]) Most of these methods use frequency or space averaging in order to simplify the problem.

The sub-structuring method presented in this paper is based on an energy formulation using the input and output driving point mobilities of the structure (see [12]). A simplified sub-structured model is obtained using mean values obtained from ensemble averaging rather than frequency or space averaging. Acoustic and structural input and output mobilities of the viroacoustic coupled system are used to normalize the FRF in order to obtain a dimensionless FRF which is independent of the direction of excitation or the direction of observation. Moreover, it is shown that this function is independent of the precise position of the observation point which leads to the sub-structuring method.

The only hypothesis needed to apply this method is that the input forces are uncorrelated. In the present application this hypothesis can be a strong one specially at lower frequencies. In order to avoid any loss of information due to this hypothesis a change of the coordinate system is done from the global coordinates system to a local coordinate system defined by the principal directions of the mean local mobilities.

The energy formulation is based on a stochastic vibroacoustic computational model constructed using the non parametric probabilistic approach of model and system-parameters uncertainties([13,

14, 15]). This model takes into account both model and system-parameters uncertainties. Thus the energy formulation is robust regarding those uncertainties.

The sub-structuring method is developed based on a simplified model which is constructed using the mean values obtained from ensemble averaging over independent Monte Carlo realizations of the probabilistic model. The sub-structuring approach considers predefined functional components of the structure to verify the belonging of the Degrees Of Freedom (DOFs) to this component.

The work is presented in five parts. First, the mean reduced vibroacoustic computational model is presented. Then, the application of a probabilistic approach for model and system parameters uncertainties is presented in a second part. In the third part the proposed energy formulation along with the simplified model are explained. The automatic sub-structuring method is then presented in a fourth part. Finally, the results of an application on an automotive vehicle structure and the conclusion are given in the last part.

2 MEAN REDUCED VIBROACOUSTIC COMPUTATIONAL MODEL

The vibroacoustic system consists in a three dimensional structure coupled with its internal acoustic cavity. The structure is a damped structure and has no rigid body displacements. for more details about the mean model the reader is referred to [16]. The frequency band of analysis is defined as $B = [\omega_{min}, \omega_{max}]$ with $\omega_{min} > 0$. Thus, for every ω belonging to B, One can write,

$$\mathbf{u}^s(\omega) = \Psi \mathbf{q}^s(\omega) \quad , \quad \mathbf{p}^f(\omega) = \Phi \mathbf{q}^f(\omega) \quad , \tag{1}$$

Which is the equation representing the mean reduced vibroacoustic computational model and in which $\mathbf{q}^s(\omega)$ is the vector of the generalized structural coordinates associated with the n first structural elastic modes and $\mathbf{q}^f(\omega)$ is the vector of the generalized acoustical coordinates associated with the m first acoustic modes which includes the constant pressure mode. $\mathbf{u}^s(\omega)$ and $\mathbf{p}^f(\omega)$ are, respectively, the vector of the structural DOFs and the vector of the acoustical DOFs. $\mathbf{q}^s(\omega)$ and $\mathbf{q}^f(\omega)$ verify the equation

$$\begin{bmatrix} \mathbb{A}^s(\omega) & \mathbb{C} \\ \omega^2\, \mathbb{C}^T & \mathbb{A}^f(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{q}^s(\omega) \\ \mathbf{q}^f(\omega) \end{bmatrix} = \begin{bmatrix} \mathbb{F}^s(\omega) \\ \mathbb{F}^f(\omega) \end{bmatrix} \quad , \tag{2}$$

where, $\mathbb{A}^s(\omega)$ and $\mathbb{A}^f(\omega)$ are the generalized dynamical stiffness matrices for the structure and for the acoustic cavity which are written as

$$\mathbb{A}^s(\omega) = -\omega^2 \mathbb{M}^s_n + i\omega \mathbb{D}^s_n + \mathbb{K}^s_n \quad , \tag{3}$$

$$\mathbb{A}^f(\omega) = -\omega^2 \mathbb{M}^f_m + i\omega \mathbb{D}^f_m + \mathbb{K}^f_m \quad , \tag{4}$$

where $\mathbb{M}^s_n, \mathbb{D}^s_n$ and $\mathbb{K}^s_n$ are positive-definite symmetric real $(n \times n)$ matrices corresponding to the generalized mass, damping and stiffness matrices, and where $\mathbb{M}^f_m$ is a positive-definite symmetric real $(m \times m)$ matrix corresponding to the generalized "mass" matrix and, $\mathbb{D}^f_m$ and $\mathbb{K}^f_m$ are the positive symmetric real $(m \times m)$ matrices corresponding to the generalized "damping" and "stiffness" matrices of the acoustic cavity. Finally, in Eq. 2, $\mathbb{C}$ is the real $(n \times m)$ matrix corresponding to the generalized vibroacoustic coupling matrix and where $\mathbb{F}^s(\omega)$ and $\mathbb{F}^f(\omega)$ are the generalized structural forces and the generalized acoustical sources applied to the vibroacoustic system.

3 NON PARAMETRIC STOCHASTIC VIBROACOUSTIC COMPUTATIONAL MODEL

As mentioned in the introduction the energy formulation is constructed based on a stochastic vibroacoustic computational model. The model is constructed using the non parametric probabilistic approach of model and system-parameters uncertainties. The non parametric approach consists in replacing the deterministic matrices of the mean reduced vibroacoustic computational model by random matrices whose probability distribution is explicitly constructed using the available information from the deterministic matrices ([13, 14, 15]). The mean values of the random matrices are, by definition,

equal to the deterministic matrices. In analogy with Eq. 2 one can write for the random vibroacoustic computational model

$$\begin{bmatrix} \mathbf{A}^s(\omega) & \mathbf{C} \\ \omega^2\mathbf{C}^T & \mathbf{A}^f(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{Q}^s(\omega) \\ \mathbf{Q}^f(\omega) \end{bmatrix} = \begin{bmatrix} \mathbb{F}^s(\omega) \\ \mathbb{F}^f(\omega) \end{bmatrix} \quad , \tag{5}$$

where $\mathbf{C}$ is a real $(n \times m)$ random matrix and where $\mathbf{A}^s(\omega)$ and $\mathbf{A}^f(\omega)$ are the random dynamical stiffness matrix of the structure and the random dynamical stiffness matrix of the acoustic cavity, respectively and which can be written, The dispersion of the random matrices are controlled by dispersion parameters which can be estimated from experimental results by performing the inverse problem. In the present work the inverse problem is not performed and the values of the dispersion parameters were taken from a previous work [17] which applied the non parametric approach to the same models as those used in Section 6.

4 INTRODUCTION OF THE ENERGY FORMULATION

After solving the random equations of the stochastic model using the Monte Carlo method, one obtains independent realizations of the random FRF. For the construction of the energy formulation and given the large number of DOFs present in a complex structure, few points are chosen for the definition of the FRF matrix. Thus, if one has a total of $\mu = n^s + n^f$ DOFs, the subset $\{j_1, \ldots, j_\alpha, \ldots, j_\nu\}$ of the ν observed and excited DOFs of the vibroacoustic system is, in general, such that $\nu \ll \mu$. The excitation vector at one DOF j_α $\mathbf{f}^\alpha(t)$ is such defined such that $\mathbf{f}^\alpha(\omega) = \{0, \ldots, f^\alpha_\alpha(\omega), \ldots, 0\}$. Let $\mathbf{T}(\omega)$ be the $(\nu \times \nu)$ random FRF such for a given the set j_α and j_β of DOF such that and having the property $\mathbf{T}(-\omega) = \overline{\mathbf{T}}(\omega)$, one can write the relation between the ν complex random vector of the velocity responses for the observed DOFs $\mathbf{V}^\alpha(\omega)$ and the input forces vector $\mathbf{f}^\alpha(\omega)$ such that,

$$\mathbf{V}^\alpha(\omega) = \mathbf{T}(\omega)\mathbf{f}(\omega) \quad . \tag{6}$$

Eq. 6 represents, the random vibroacoustic system using the random FRF matrix, the input in terms of a force vector and the output in terms of a velocity vector. As mentioned earlier the aim of the energy formulation is to simplify the vibroacoustic response of the system as well as taking into account uncertainties. Thus, in this formulation, one represents the system with a new parameter representing a type of FRF having the input and output represented in terms of spectral power functions. Introducing the $(r \times r)$ real diagonal random driving points mobility matrix $\mathbf{Y}_{\alpha\beta}(\omega)$ defined by,

$$\mathbf{Y}_{\alpha\beta}(\omega) = \begin{cases} Re(\mathbf{T}_{\alpha\alpha}(\omega)) & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases} \quad . \tag{7}$$

and defining the $(\mathbb{R}^+)^r$ vector-valued spectral force function $\mathbf{s}^f(\omega) = (s^f_1(\omega), \ldots, s^f_r(\omega))$ relative to the excited DOFs such that $s^f_\alpha(\omega) = (1/2\pi) f^\alpha_\alpha(\omega)^2$ and the $(\mathbb{R}^+)^r$ random vector-valued spectral velocity function $s^v_\alpha(\omega) = (s^v_1(\omega), \ldots, s^v_r(\omega))$ relative to the observed DOFs such that $s^v_\alpha(\omega) = \frac{1}{\pi} \parallel \mathbf{V}^\alpha(\omega) \parallel^2$, it can be shown that (see [12]) the $(\mathbb{R}^+)^r$-valued spectral input power function $\boldsymbol{\pi}_{in} = (\pi^1_{in}, \ldots, \pi^\alpha_{in}, \ldots, \pi^r_{in})$ is expressed using the equation equation,

$$\boldsymbol{\pi}_{in}(\omega) = 2\mathbf{Y}(\omega)\mathbf{s}^f(\omega) \quad . \tag{8}$$

and the random $(\mathbb{R}^+)^r$-valued spectral local response power function is defined such that,

$$\mathbf{s}^v(\omega) = \mathbf{Y}(\omega)\boldsymbol{\pi}^R(\omega) \quad , \tag{9}$$

Having defined the excitation and observation in terms of spectral power functions, one can deduce the following fundamental equation relating these functions,

$$\boldsymbol{\pi}^R(\omega) = \mathcal{E}(\omega)\, \boldsymbol{\pi}_{in}(\omega) \quad , \tag{10}$$

where $\mathcal{E}(\omega)$ is the $(r \times r)$ real full random dimensionless FRF matrix defined by,

$$\mathcal{E}(\omega) = \mathbf{Y}(\omega)^{-1}\mathbf{H}(\omega)\mathbf{Y}(\omega)^{-1} \quad , \tag{11}$$

where $\mathbf{H}(\omega)$ is the random defined by, $\mathbf{H}_{\beta\alpha}(\omega) = |\mathbf{T}_{\beta\alpha}(\omega)|^2$. Taking into account the previous equations one can deduce the following fundamental equation which represents the relation between the vector-valued spectral force function and the random vector-valued spectral velocity function such that,

$$\mathbf{s}^v(\omega) = 2\,\mathbf{Y}(\omega)\mathcal{E}(\omega)\mathbf{Y}(\omega)\,\mathbf{s}^f(\omega) \quad . \tag{12}$$

which will be used in the model simplification and the sub-structuring introduced in the following section.

representation of the system in the principal directions of the mean local mobilities It has to be noted that the assumption of uncorrelated forces necessary for the exactitude of the energy formulation is not always valid as the forces might be physically correlated in the real problem specially at low-frequencies. To overcome this problem the system is represented in a local coordinates system defined by the principal directions of the mean local mobilities. In these directions the mobility matrix is a diagonal matrix by definition and contains all the information concerning the point and the transfer mobility functions without any simplification. Thus, even if the forces are physically correlated the observed response remains exact despite of the made hypothesis.
The orthogonal base of the local coordinates system defined by the principal directions of the mean local mobilities is represented by the eigenvectors of the mean mobility matrix at one point of the structure which is such that, $\underline{\mathbb{Y}}_p(\omega) = \Re\mathrm{e}\{E\{\mathbf{T}_p(\omega)\}\}$, where $\mathbf{T}_p(\omega)$ is the symmetric complex (3×3) random matrix corresponding to the translational DOF's of the random FRF $\mathbf{T}(\omega)$ at a given point p of the structure and where E denotes the mathematical expectation. Let $\mathbb{X}_p(\omega)$ be the orthogonal real (3×3) matrix containing the eigenvectors of $\mathbb{Y}_p(\omega)$ The local coordinates at this given point, defined by the principal directions of the mean local mobility, are represented by $\mathbb{X}_p(\omega)$. One can then write,

$$\mathbf{T}_p^{loc}(\omega) = \mathbb{X}_p(\omega)^T\mathbf{T}_p(\omega)\mathbb{X}_p(\omega) \quad , \tag{13}$$

where $\mathbf{T}_p^{loc}(\omega)$ is the representation of the random FRF matrix $\mathbf{T}_p(\omega)$ in the local coordinates. All the parameters represented in Section 4 can then be represented in the local coordinates for the point p and after assembling over all the DOFs one can obtain similar equations in the local coordinates. In the following sections the a subscript or a superscript *loc* is used to represent parameters expressed in the local coordinates.

5 INTRODUCTION OF THE SUB-STRUCTURING METHOD

In this section a simplified model is constructed based on the mean values of the system parameters obtained by averaging over the independent Monte Carlo realizations defined using the mathematical expectation. In what follows an underlined parameter is a mean parameter. One can now define $(\underline{\mathbf{s}}^v_{loc}(\omega))^{ref}$ as the mathematical expectation of the random vector $\mathbf{s}^v(\omega)$ defined in the local coordinates and which is written as

$$\underline{\mathbf{s}}^v_{loc}(\omega)^{ref} = 2\,E\{\mathbf{Y}^{loc}(\omega)\mathcal{E}^{loc}(\omega)\mathbf{Y}^{loc}(\omega)\mathbf{s}^f_{loc}(\omega)\} \quad , \tag{14}$$

Let J and O be the set of excitation and observation DOF's respectively, placed on two different zones of the structure such that $J = \{k_q, q = 1, ..., \mu\}$ and $O = \{j_p, p = 1, ..., \nu\}$, where μ and ν are the number of excitation and the number of observation DOF's respectively. Assuming that the excitation and observation DOF's J and O are sufficiently distant from each others, it can be shown that (see [12]) the one can make the following approximation,

$$\underline{\mathbf{s}}^v_{loc}(\omega)^{app}_{j_p} = \underline{e}_{OJ}(\omega)\,\underline{\mathbf{Y}}^{loc}(\omega)_{j_pj_p}\,\underline{\pi}^{loc}_{in,J} \quad , \tag{15}$$

where $\underline{e}_{OJ}(\omega)$ is a positive real number representing the values of the dimensionless FRF at all the excitation and observation DOFs, and where $\underline{\pi}^{loc}_{in,J}$ is the mean value of the random total spectral local input power relative to the set J, and which is thus defined as the sum of the local input power over all DOFs of in this set. Using Eq. 15 one can obtain the following equation used to calculate the positive scalar $\underline{e}_{OJ}(\omega)$ and which is written,

$$\underline{e}_{OJ}(\omega) = \frac{\sum_{p=1}^{\nu} \underline{\mathbf{s}}^{v}_{loc}(\omega)^{ref}_{j_p}}{\underline{\pi}^{loc}_{in,J} \sum_{p=1}^{\nu} \underline{\mathbf{Y}}^{loc}(\omega)_{j_p j_p}} \quad . \tag{16}$$

It has to be noted that, the value of the mean spectral local response function introduced in Eq. 15 is only an approximated value and that, for the calculation of $\underline{e}_{OJ}(\omega)$ using Eq. 16 the reference value calculated introduced in Eq. 14 has to be used. The error introduced du to the approximation can be measured by defining, for all j_p in J and k_q in O, and for all ω in B, the error function $\varepsilon_{\mathcal{E}}(\omega)_{j_p k_q}$ such that,

$$\varepsilon_{\mathcal{E}}(\omega)_{j_p k_q} = |\underline{\mathcal{E}}^{loc}(\omega)_{j_p k_q} - \mathbb{E}_{OJ}(\omega)^{app}_{j_p k_q}| \quad , \tag{17}$$

where $\mathbb{E}_{OJ}(\omega)^{app}_{j_p k_q} = e_{OJ}(\omega)$. The sub-structuring method is based on predefined structural functional components. The error function $\varepsilon_{\mathcal{E}}(\omega)$ introduced in Eq. 17 is used as a criteria of belonging to a certain structural zone. Considers the zones grouping the excitation and observation points J and O, one calculates the value of $\varepsilon_{\mathcal{E}}(\omega)_{j_p k_q}$ at one DOF will determine if the observation point j_p belongs to the zone grouping the observation set O, such that if $\varepsilon_{\mathcal{E}}(\omega)_{j_p k_q}$ is high then the DOF belongs to the considered zone and vice-versa. In the following section the results of the application of the energy formulation and the sub-structuring method on an automotive vehicle structure are shown.

6 NUMERICAL APPLICATION

The energy formulation and the sub-structuring method were validated on an automotive vehicle structure coupled with its internal acoustic cavity. The structure have 1 042 851 DOF's and the acoustic cavity have 9157 DOF's. There are 12 acoustic sources, and 28 force excitations. The vibroacoustic analysis is performed in the low- and medium-frequency band $B = [50, 350]Hz$. For this frequency band of analysis B, the structure is represented by 1955 elastic modes and 3 rigid body translational modes ($n = 1958$), while the acoustic cavity is represented by 160 acoustic modes ($m = 160$). The convergence as a function of the mean-square convergence of the random solution is studied as a function of the number of realizations n^r. Nearly six hundred realizations are necessary for the convergence of both the structure and the acoustic cavity. Fig 1 shows the confidence regions around the mean values of the conventional and the dimensionless FRFs for an excitation in the first principal direction at one point of the structure and an observation in first principal direction at another point of the structure. It is shown than the dispersion of the dimensionless FRF is much less than that of the conventional FRF. This means that the new model is more robust regarding model and system-parameters uncertainties. Similar results were observed for the acoustic FRF (excitation and observation at a point inside the acoustic cavity) and for the vibroacoustic FRF (excitation at a DOF of the structure and observation at a point inside the acoustic cavity). Given that the dimensionless FRF is an energy parameter and that the excitation and observation parameter are expressed in terms of spectral power the dimensionless FRF is independent of the direction of excitation or the direction of observation. This is illustrated in Fig. 2 for excitations in the three principal directions at a point of the structure and an observation in the first principal direction at another point of the structure. One can see that the three dimentionless FRF have very close values and are less fluctuating than the normal FRF. The three dimensionless FRF tend towards an asymptotic value at about 250 Hz. Fig 3 show images of the error matrix $\varepsilon_{\mathcal{E}}(\omega)$ in decibels calculated for all the DOF of the system. This first sub-structuring approach shows that starting at about 70 Hz the matrix takes a block diagonal form, the blocks on the diagonal correspond to points situated at different functional parts of the structure.

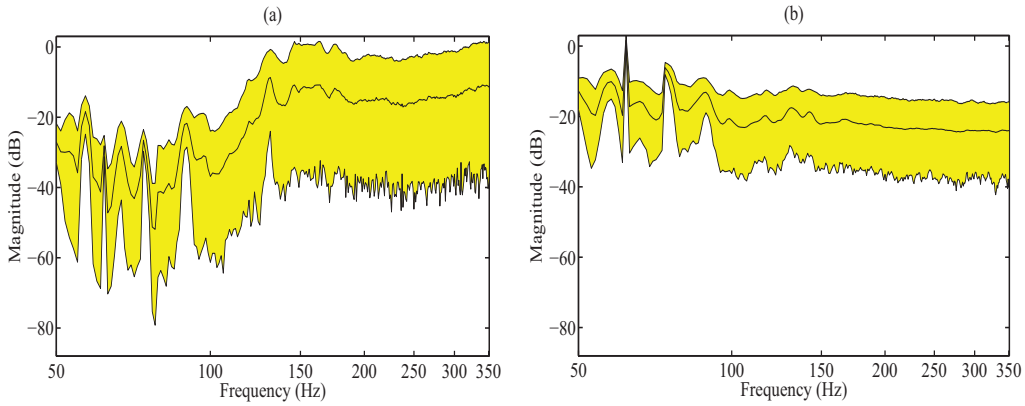

FIG. 1 – Confidence regions of $\mathbf{T}^{loc}(\omega)$ (a) and $\mathcal{E}^{loc}(\omega)$ (b). Excitation is a force applied to a given point on the structure in the first principal direction. Observation is the velocity in another point of the structure in the first principal direction. Medium line is the mean value. Upper and lower lines delimit the confidence region.

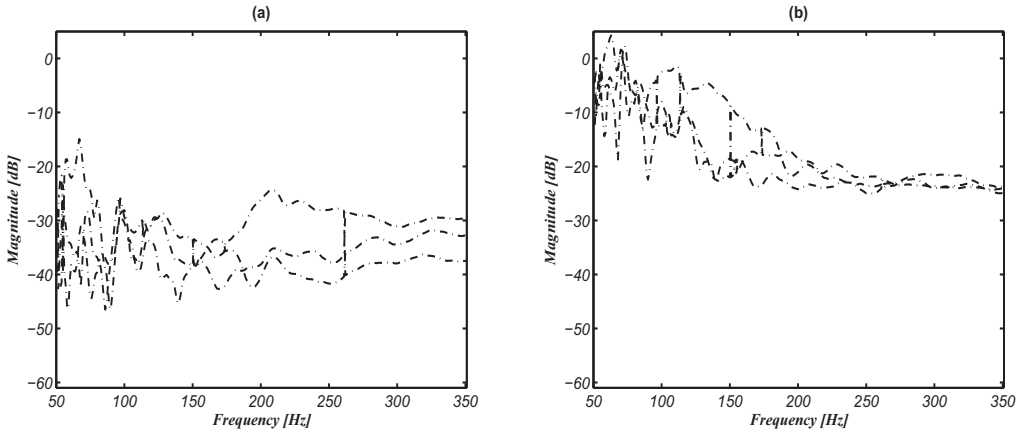

FIG. 2 – Graphs of $\underline{\mathbf{T}}^{loc}(\omega)$ (a) and of $\underline{\mathcal{E}}^{loc}(\omega)$ (b) for the structure input - structure output FRF as a function of the frequency. The structure output is the structure velocity in direction first principal direction. The three structure inputs are the structural forces applied in the three local principal directions first principal direction(thin line),second principal direction(medium line), third principal direction(thick line).

Inside some of the diagonal blocks one can observe a trend of one DOF out of three DOFs corresponding to one point on the structure having a higher error value. This trend shows that at some points there is one dominant principal direction having the highest energy participation. These points are usually placed on thin flexible structural elements. away from the diagonal zones with uniform error values show that neighbor points on different functional components have similar behavior, which means that the dimensionless FRF are independent of the precise location where the excitation or the observation point is located. In other words, points located at these zones have the same behavior towards a given excitation. Finally the error is maximum on the diagonal where the excitation point is the same as the observation point, which confirms the condition of the construction of the simplified model stating that the observation and excitation points should be far enough from each others.

7 CONCLUSION

A sub-structuring method based on an energy formulation and a stochastic vibroacoustic computational model was presented. The method and the energy formulation were validated using an automotive vehicle model. The energy formulation separates the local effects, represented by the lo-

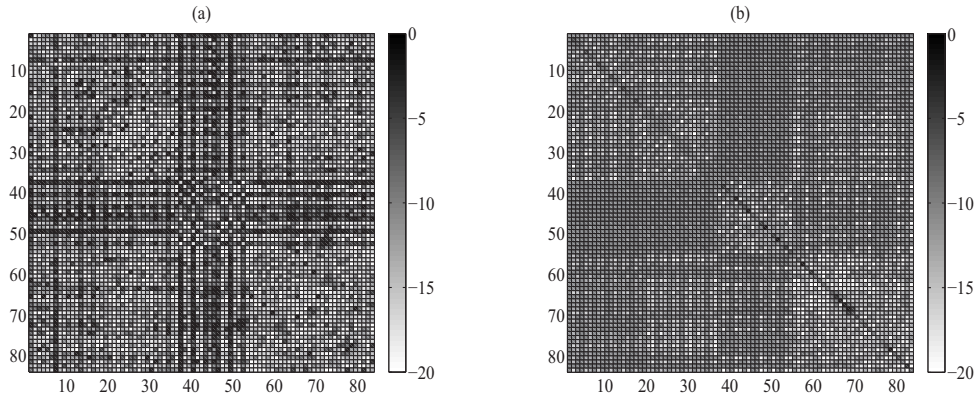

FIG. 3 – Grey scale plots of $\varepsilon_{\mathcal{E}}(\omega)$ at 70 Hz (a) and 350 Hz (b).

cal mobilities, from the global effects represented by the dimensionless FRF. The dimensionless FRF are said to represent global effects as they are not only independent from the direction of excitation or the direction of observation, but also independent from the precise location of the excitation or observation points. This conclusion leads to the construction of a simplified model. The error estimation due to the model simplification is used to determine the belonging of a system of points to a given substructure. First, predefined functional substructures are fixed and then if the value of the error at each DOF is evaluated to determine if the DOF belong to the given substructure. the energy formulation and the sub-structuring method were shown to be valid at low- and medium- frequency ranges but, the low-frequency limit is to be determined in further development of the work.

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