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Proceedings of the XIII International Symposium on Dynamic Problems of Mechanics (DINAME 2009),
Almeida, C. A. (Editor), ABCM, Angra dos Reis, RJ , Brazil, March 2nd - March 6th, 2009

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Abstract: A drill-string is a slender structure that turns and drills into the rock in search of oil. There are many sources of uncertainties in this problem, but in this article only the weight-on-hook, which is the support force exerted by the hook at the top, is considered random. In a drilling operation there are three parameters that can be continuously controlled: (1) the weight-on-hook, (2) the drilling fluid flow, and (3) the speed of the rotary table. The idea is to understand how a perturbation on the weight-on-hook affects the performance of the system, which is measured by the rate of penetration. A numerical model is developed using the Timoshenko beam theory and discretized by means of the Finite Element Method. The main efforts that the column is subjected to are considered: rotation at the top; hanging force at the top; bit-rock interaction; fluid-structure interaction that takes into account the drilling fluid that flows downwards the column then goes upwards in the annulus; shock and rubbing between the column and the borehole; and the own weight of the column. To derive the probability density function of the random variable WOH, the Maximum Entropy Principle is used, so that the probability distribution is coherent with the physics of the problem.

Keywords: nonlinear dynamics, stochastic dynamics, fluid-structure interaction, bit-rock interaction, random weight-on-hook

NOMENCLATURE

A = area of the transversal section, [m^2]
$[C]$ = damping matrix
conv = convergence function
D = diameter, [m]
E = Young Modulus, [Pa]
$\mathbf{f}$ = force vector
G = shear coefficient, [Pa]
h = head loss, [m]
I = inertia moment of the transversal section, [m^4]
$[K]$ = stiffness matrix
L = length, [m]
M = mass per unit length, [kg/m]
$[M]$ = mass matrix
$\mathbf{N}$ = shape functions
p = pressure, [Pa]
$\mathbf{q}$ = displacement vector
S = Shannon entropy measure
t = time, [s]
T = kinetic energy, [N.m]
U = potential energy of deformation, [N.m]; or fluid velocity, [m/s]
u = displacement in x-direction, [m]
v = displacement in y-direction, [m]
w = displacement in z-direction, [m]
W = work done by the external forces and work not considered in U or T, [N.m]
Z = regularizing function

Greek Symbols
$\mathbb{1}_B(x)$ = assumes value 1 if x belong to B and 0 otherwise
δ = dispersion parameter
ε and γ = deformation
ω = angular velocity vector, [rad/s]
ρ = density, [kg/m^3]
Ω_x = rotation speed at $x = 0$, [rad/s]
$[\Phi]$ = modal basis
Π = total potential of the system, [N.m.t]
σ = standard deviation
θ_x = rotation about x-axis
θ_y = rotation about y-axis
θ_z = rotation about z-axis
ξ = damping factor

Subscripts
br = bit-rock
ch = channel (or borehole)
r = reduced
e = element
f = fluid
g = geometric (for $[K]$) and gravity (for $\mathbf{f}$)
i = inside
ke = kinetic energy
o = outside
p = polar
se = strain energy
NL = nonlinear
S = static response
u = displacement in x-direction
v = displacement in y-direction
w = displacement in z-direction
θ_x = rotation about x-axis
θ_y = rotation about y-axis
θ_z = rotation about z-axis

INTRODUCTION

In a drilling operation there are many sources of uncertainties as, for instance: the material properties of the column and the drilling fluid; the dimensions of the system, specially the borehole; the fluid-structure interaction; the bit-rock interaction, among others. This paper is concerned with the stochastic model of the weight-on-hook because it is one of the three parameters that are continuously controlled in a drilling operation. In Ritto et al. (2009) the modeling of a drill-string and the main forces that act on it is done, so not much details will be presented here.

Figure 1 shows the general scheme of the system analyzed. The forces taking into account are: the motor torque (as a constant rotation speed at the top Ω_x); a constant hanging force f_{hook}; the torque t_{bit} and force f_{bit} at the bit; the weight of the column; the fluid forces; the shocks between the column and the borehole; the forces due to the stabilizer; plus the elastic and kinetic forces due to the deformation and to the motion of the structure.

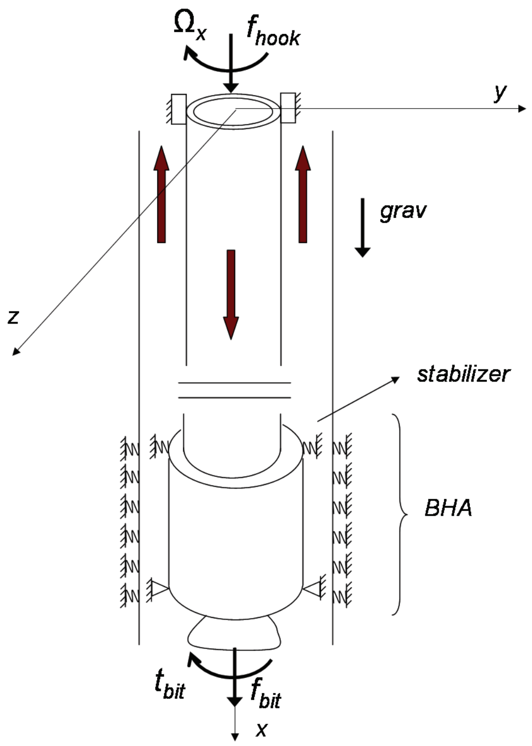

Figure 1 – General scheme.

There are not many articles treating the stochastic problem of the drill-string dynamics, in special we might cite Spanos et al. (1997 and 2000). In Kotsonis and Spanos (1997) the weight-on-bit is modeled as stochastic in a simple two degrees of freedom drill-string model, and in Spanos and Chevallier (2000) lateral forces at the bit are modeled as stochastic.

The bit-rock interaction model chosen was the one developed in Tucker and Wang (2003) basically for two reasons: (1) it is able to reproduce the main phenomena (as stick-slip oscillations); (2) it describes well the penetration of the bit into the rock (so we can analyze the rate-of-penetration-ROP). Usually the bit is considered fixed, Khulief et al. (2007), Sampaio et al. (2006 and 2007), or an average rate of penetration is assumed, Spanos et al. (1995), Christoforou and Yigit (2003).

The probability density function of the weight-on-hook is constructed by the means of the Maximum Entropy Principle (Shannon, 1948). To know more about model uncertainties in mechanical systems see Soize et al. (2000, 2001, 2001b, 2005, 2005b, and 2008)

Final discretized system

The final discretized system considering the prestressed state is written as (Ritto et al., 2009):

$$([M]+[M_f])\ddot{\bar{\mathbf{q}}}+([C]+[C_f])\dot{\bar{\mathbf{q}}}+([K]+[K_f]+[K_g(\mathbf{q}_S)])\bar{\mathbf{q}}=\mathbf{f}_{NL}(\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}},\ddot{\bar{\mathbf{q}}})+\mathbf{f}_f\,, \tag{1}$$

where the response $\mathbf{q}$ is represented in a subspace $V_m \subset \mathbb{R}^m$, where m equals the number of degrees of freedom of the system; $\bar{\mathbf{q}} = \mathbf{q} - \mathbf{q}_S$ is the configuration about which the vibration takes place, where $\mathbf{q}_S = [K]^{-1}(\mathbf{f}_g + \mathbf{f}_c)$; $\mathbf{f}_g$ is the gravity force; $\mathbf{f}_c$ is a concentrated reaction force at the bit; $[M]$, $[C]$, and $[K]$ are the classical mass, damping and stiffness matrices; $[M_f]$, $[C_f]$, $[K_f]$ are the fluid mass, damping and stiffness matrices; $\mathbf{f}_f$ is the fluid force vector; $[K_g(\mathbf{q})]$ is the geometric stiffness matrix; $\mathbf{f}_{NL}(\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}},\ddot{\bar{\mathbf{q}}})$ is the nonlinear force vector that is decomposed in four parts:

$$\mathbf{f}_{NL}(\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}},\ddot{\bar{\mathbf{q}}})=\mathbf{f}_{ke}(\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}},\ddot{\bar{\mathbf{q}}})+\mathbf{f}_{se}(\bar{\mathbf{q}})+\mathbf{f}_{sh}(\bar{\mathbf{q}})+\mathbf{f}_{br}(\dot{\bar{\mathbf{q}}})\,. \tag{2}$$

where $\mathbf{f}_{ke}(\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}},\ddot{\bar{\mathbf{q}}})$ are the quadratic forces due to the kinetic energy; $\mathbf{f}_{se}(\bar{\mathbf{q}})$ are the quadratic and higher order forces due to the strain energy; $\mathbf{f}_{sh}(\bar{\mathbf{q}})$ are the forces due to the shocks and rubbing between the column and the borehole; and $\mathbf{f}_{br}(\dot{\bar{\mathbf{q}}})$ are the forces due to the bit-rock interactions.

A reduced-order matrix model is constructed from Eq. (1) using the modal analysis. The system is reduced and the time integration is done using an explicit Runge-Kutta algorithm with a time step controller to keep the error within a given precision.

Bit-rock interaction model

In this work, the model used is the one developed by Tucker and Wang (2003), which is rewritten as

$$f_{xbit} = -\frac{\dot{u}_{bit}}{a_2 Z(\dot{\theta}_{bit})^2} + \frac{a_3\dot{\theta}_{bit}}{a_2 Z(\dot{\theta}_{bit})} - \frac{a_1}{a_2} \quad , \quad t_{xbit} = -\frac{\dot{u}_{bit} a_4 Z(\dot{\theta}_{bit})^2}{\dot{\theta}_{bit}} - a_5 Z(\dot{\theta}_{bit}) \tag{3}$$

in which f_{xbit} is the axial force, where t_{xbit} is the torque about the x-axis and where $Z(\dot{\theta}_{bit})$ is the regularizing function.

In the above equation, $a_1,\ldots,a_5$ are positive constants that depend on the bit and rock characteristics as well as on the weight-on-bit (wob). This equation was derived for a stable operation with $\dot{\theta}_{bit} \sim 100$ RPM and with $wob \sim 100$ kN.

In this model, the bit exerts only an axial force (f_{xbit}) and a torque (t_{xbit}) about the x-axis. These force and torque exerted by the rock at the bit depend on the axial speed ($\dot{u}_{bit}$) and the rotation speed ($\dot{\theta}_{bit}$) of the bit. Note that these forces at the bit couple axial and torsional vibrations.

STOCHASTIC MODEL OF THE WEIGHT-ON-HOOK

The weight-on-hook (woh) is modeled as a random variable, WOH. The Maximum Entropy Principle (Shannon, 1948; Jaynes, 1957) is used to construct the probability density function. This principle consists on finding the probability density function that maximizes the entropy (see Eq.(4)) given the available information. This is a way of being coherent with the physics of the problem (Soize, 2008).

$$S = -\int_{\mathbb{R}} p(woh)\ln p(woh)dwoh\,. \tag{4}$$

The available information we have is:

1. WOH can assume any value in the real line, $\therefore$ support $=\mathbb{R}$.
2. The mean value is known, $\therefore E\{WOH\} = \underline{woh}$.
3. The variance (or dispersion) is known, $\therefore E\{(WOH-\underline{woh})^2\} = \sigma^2$.

The distribution is then a Normal distribution with the probability density function:

$$p_{WOH}(woh) = \mathbb{1}_{\mathbb{R}}(woh)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(woh-\underline{woh})^2}{2\sigma^2}\right). \tag{5}$$

To measure the dispersion it will be used the variation coefficient: $\delta = \dfrac{\sigma}{\underline{woh}}$. The $\underline{woh}$ can be calculated as:

$$\underline{woh} = \int_0^L \rho g A dx - wob = 1.06\text{MN}\,. \tag{6}$$

NUMERICAL RESULTS

Data used in the simulations

$L_{dp} = 1400$ [m] (length of the drill pipe), $L_{dc} = 200$ [m] (length of the drill collar), $D_{odp} = .127$ [m] (outside diameter of the drill pipe), $D_{odc} = .2286$ [m] (outside diameter of the drill collar), $D_{idp} = .095$ [m] (inside diameter of the drill pipe), $D_{idc} = 0.0762$ [m] (inside diameter of the drill collar), $D_{ch} = 0.3$ [m] (diameter of the borehole (channel)), $x_{stab} = 1400$ [m] (location of the stabilizer), $k_{stab} = 17.5$ [MN/m] (stiffness of the stabilizer per meter), $E = 210$ [GPa] (elasticity modulus of the drill string material), $\rho = 7850$ [kg/m^3] (density of the drill string material), $\nu = .29$ [-] (poisson coefficient of the drill string material), $k_s = 6/7$ [-] (shearing correcting factor), $c_1 = 0.05$ [N.s/m] (friction coefficient for the axial rigid body motion), $c_2 = 0.05$ [N.s/m] (friction coefficient for the rotation rigid body motion), $\xi_i = 0.3$ [-] i-th (damping factor), $k_{sh} = 1e8$ [N/m] (stiffness per meter used for the shocks), $\mu_{sh} = 0.0005$ [-] (friction coefficient between the string and the borehole), $\Omega_x = 100$ [RPM] (constant speed at the top), $U_i = 1.5$ [m/s] (flow velocity in the inlet), $\rho_f = 1200$ [kg/m^3]

(density of the fluid), $C_f = .0125$ [-] (fluid viscous damping coefficient), $k = 0$ [-] (fluid viscous damping coefficient), $wob = 100$ [kN] (initial weight on the bit), $g = 9.81$ [m/s^2] (gravity acceleration), $a_1 = 3.429e-3$ [m/s] (constant of the bit-rock interaction model), $a_2 = 5.672e-8$ [m/(N.s)] (constant of the bit-rock interaction model), $a_3 = 1.374e-4$ [m/rd] (constant of the bit-rock interaction model), $a_4 = 9.537e6$ [N.rd] (constant of the bit-rock interaction model), $a_5 = 1.475e3$ [N.m] (constant of the bit-rock interaction model), $e = 2$ [rd/s] (regularization parameter).

Considering $\mathbf{f}_{ke} = \mathbf{f}_{se} = 0$

Fig. 2 shows the comparison of the complete dynamics (*case 1*) and the dynamics with $\mathbf{f}_{ke} = \mathbf{f}_{se} = 0$ (*case 2*).

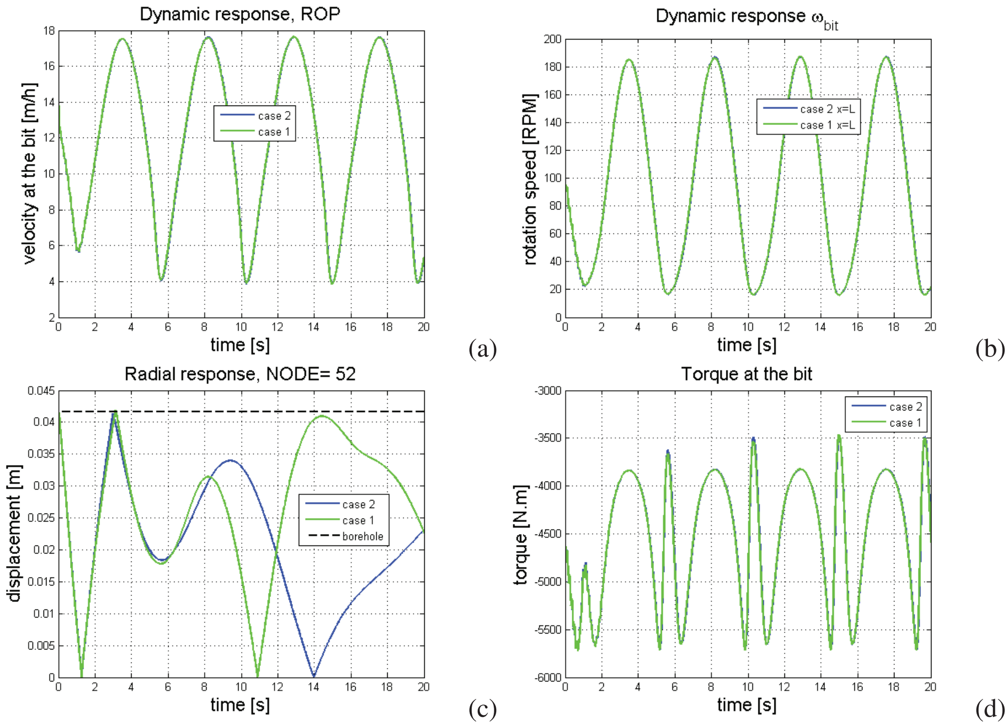

Figure 2 – *Case 1* $\times$ *case 2*. (a) axial speed at $x = L$, or rate of penetration (ROP); (b) rotation speed at $x = L$ (ω_{bit}); (c) radial displacement at $x = 1560$ m; and (d) torque at the bit.

The nonlinear forces $\mathbf{f}_{ke}$ and $\mathbf{f}_{se}$ are important to the dynamic response of the system (take a look at the radial displacement, Fig. 2 (c)), but the torsional and axial displacements are not very affected when $\mathbf{f}_{ke} = \mathbf{f}_{se} = 0$. In fact the simulations showed that $\mathbf{f}_{se}$ is very significant while $\mathbf{f}_{ke}$ is negligible for the case analyzed. We can see that the torsional and axial displacements are mainly dictated by the bit-rock interaction model. Moreover, the time to perform the numerical simulation is around: 70 minutes for *case 1*; and 80 seconds for *case 2*.

As we want to investigate the influence of the probabilistic model of the weight-on-hook (axial force) we will use *case 2* for the next simulations, knowing that it is an approximation of the problem analyzed.

Convergence of the stochastic solution

Let $[\mathbf{U}(t,s)]$ be the response of the stochastic dynamical system calculated for each realization s. The mean-square convergence analysis with respect to the number n_s of independent realizations is carried out studying the function $n_s \mapsto \text{conv}(n_s)$ defined by

$$\text{conv}(n_s) = \frac{1}{n_s} \sum_{j=1}^{n_s} \int_0^{t_f} ||\mathbf{U}_j(s,t)||^2 dt \,. \tag{7}$$

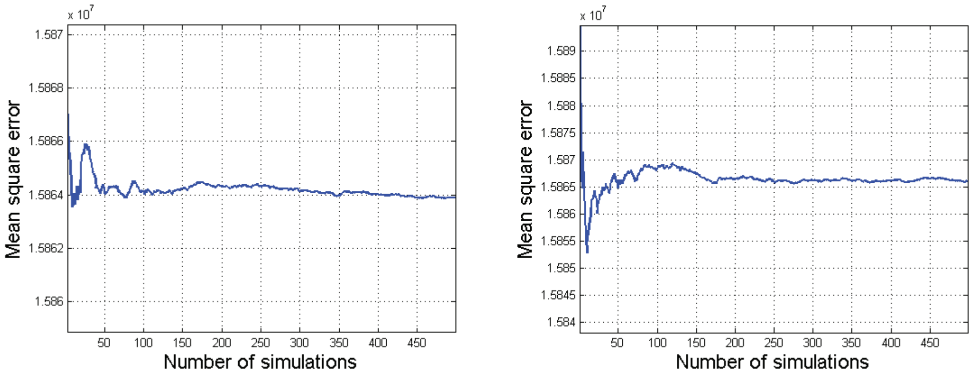

Figure 3 – Mean square convergence for $\delta = 0.01$ (left) and $\delta = 0.05$ (right).

Fig. 3 shows that 500 simulations are sufficient to reach the mean-square convergence.

RESPONSE OF THE STOCHASTIC SYSTEM

Fig. 4 shows the 95% envelope (that is to say the confidence region constructed with a probability level of 0.95) for the rate-of-penetration and the rotation speed of the bit for a standard deviation $\sigma = 1000$ N, which means $\delta = \sigma/\underline{woh} \sim 1^{-3}$. The envelopes (the upper and lower envelopes of the confidence region) are calculated using the method of quantiles (Serfling, 1980).

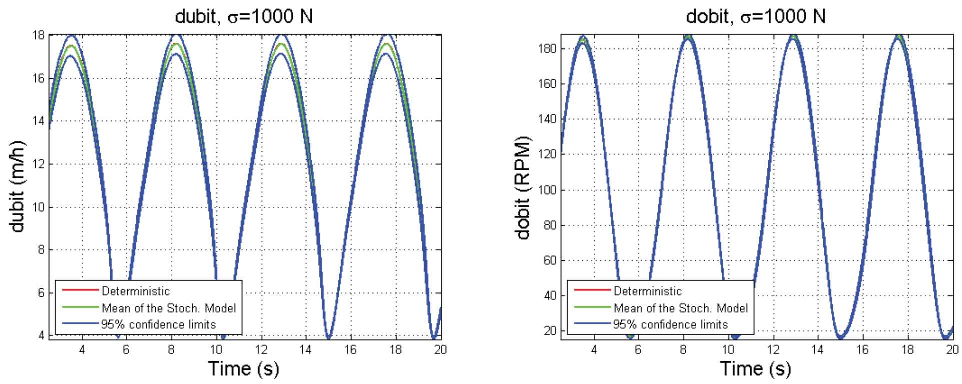

Figure 4 – 95% envelope for $\sigma = 1000$ N. Left: rate-of-penetration, ROP. Right: rotation speed of the bit.

We are plotting two important variables: the rate-of-penetration (ROP) and the rotation speed at the bit (ω_{bit}). So we analyze the influence of the random weight-on-hook in the system response. Fig. 5 shows the stochastic response of the torque and force on the bit.

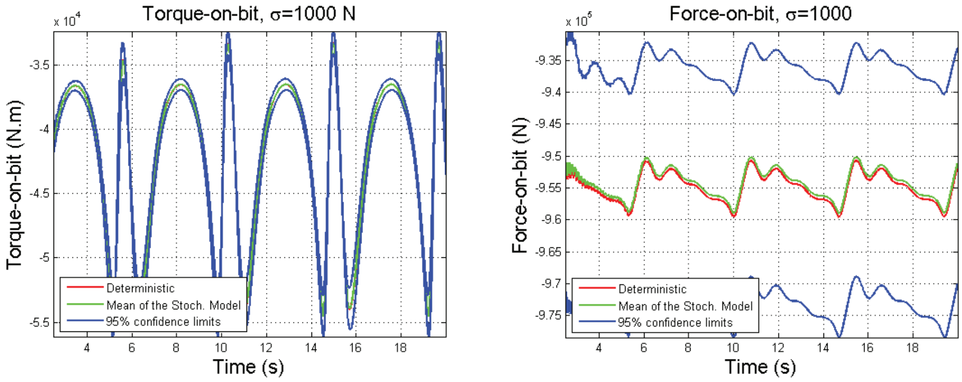

Figure 5 – 95% **envelope for** $\delta \sim 0.01$**. Left: torque-on-bit. Right: force-on-bit.**

It is noted that for $\sigma = 1000$ N, the response changes just a little, therefore σ will be increased in the next analysis. In our analysis we can not increase σ too much because the model used for the bit-rock interaction assumes a weight-on-bit $wob \sim 100$ kN, so the standard deviation σ of the WOH is increased in a way that the wob has a maximum variation around 5%, that is to say that the $\sigma_{max} = 5000$N and therefore $\delta_{max} \sim 0.005$ (0.5% variation), which is a constraint to our analysis. But, as it will be seen, a small variation on the WOH may cause a big variation in the system response. Fig. 6 shows the system response for $\sigma = 3000$ N ($\delta \sim 0.003$).

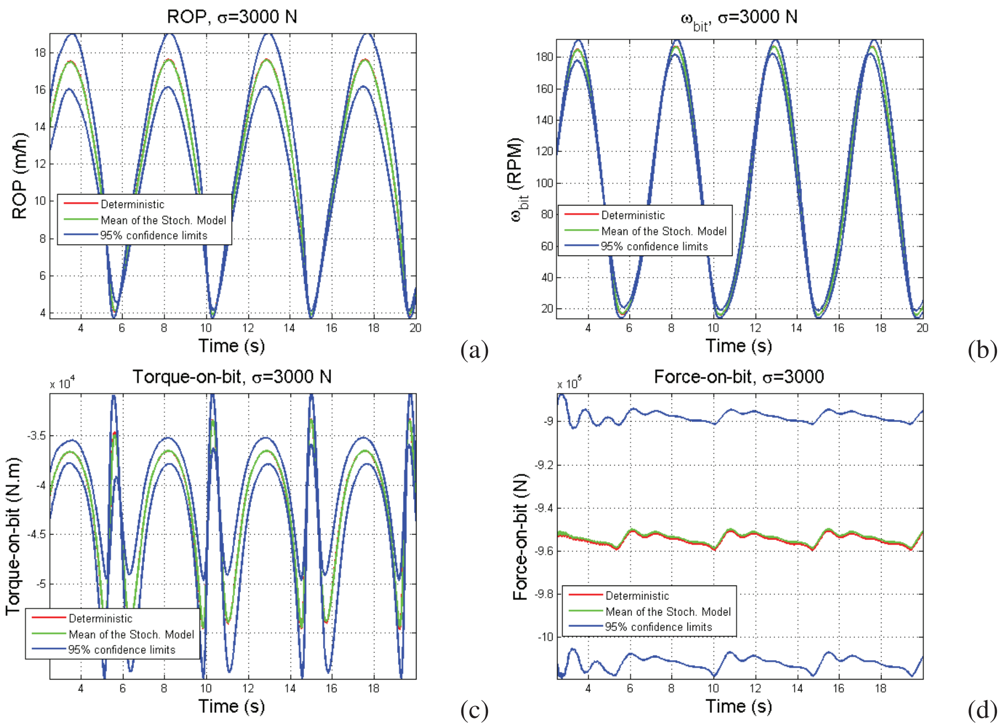

Figure 6 – 95% **envelope for** $\delta \sim 0.03$**. (a) rate-of-penetration, ROP; (b) rotation speed of the bit; (c) torque-on-bit; and (d) force-on-bit.**

We want too see how a variation on the WOH affects the performance of the system, so, in Fig. 7 it is shown the evolution of the dispersion of the response for the: ROP, rotation speed of the bit, torque-on-bit, and force-on-bit. The dispersion of the response is calculated by taken the square root of the variance divided by the value of the mean response for each time instant.

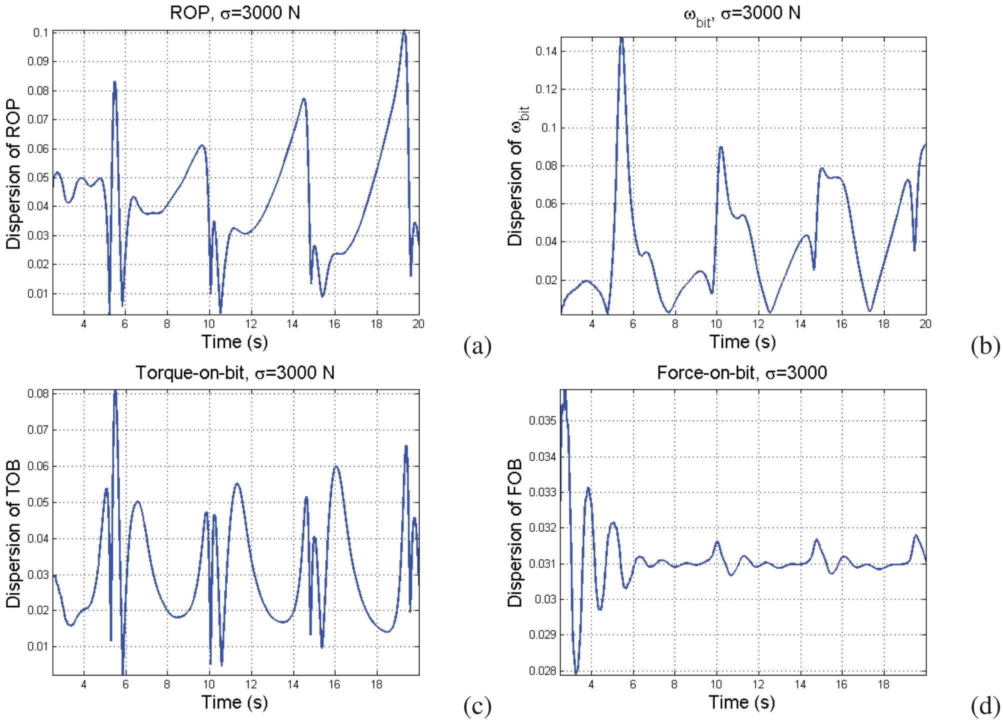

Figure 7 – Dispersion of the response for $\sigma = 3000$ N. (a) rate-of-penetration, ROP; (b) rotation speed of the bit; (c) torque-on-bit; and (d) force-on-bit.

Fig. 8 shows the system response for $\sigma = 5000$ N ($\delta \sim 0.005$).

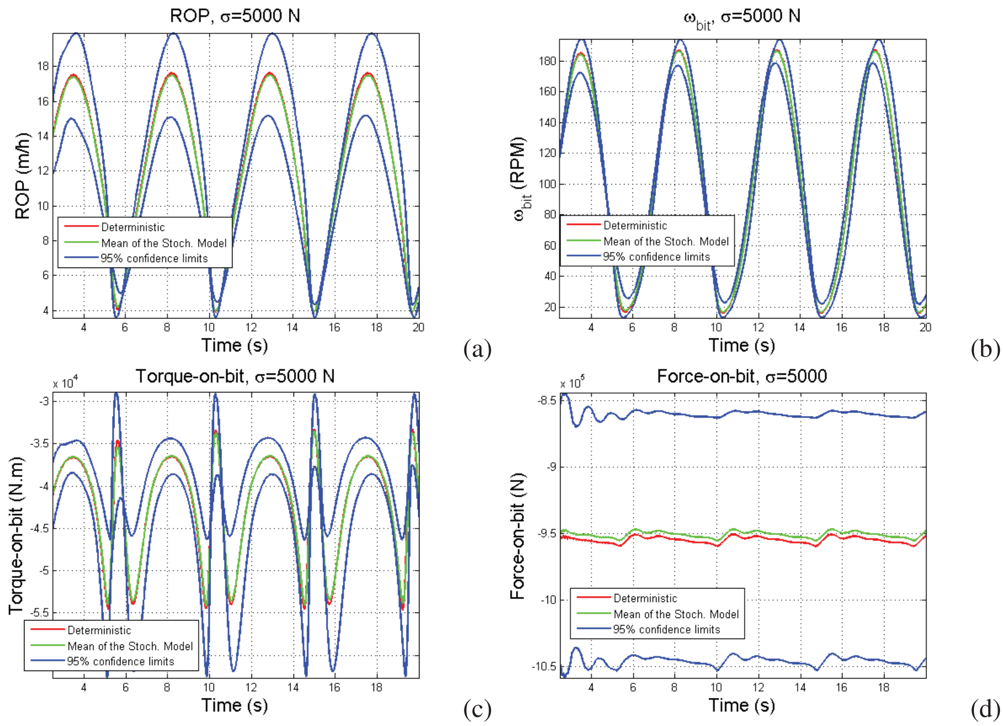

Figure 8 – 95% envelope for $\delta \sim 0.05$. (a) rate-of-penetration, ROP; (b) rotation speed of the bit; (c) torque-on-bit; and (d) force-on-bit.

As expected, as δ increases the envelope of the response gets wider. Fig. 9 shows the dispersion of the response for $\delta \sim 0.05$ of the WOH.

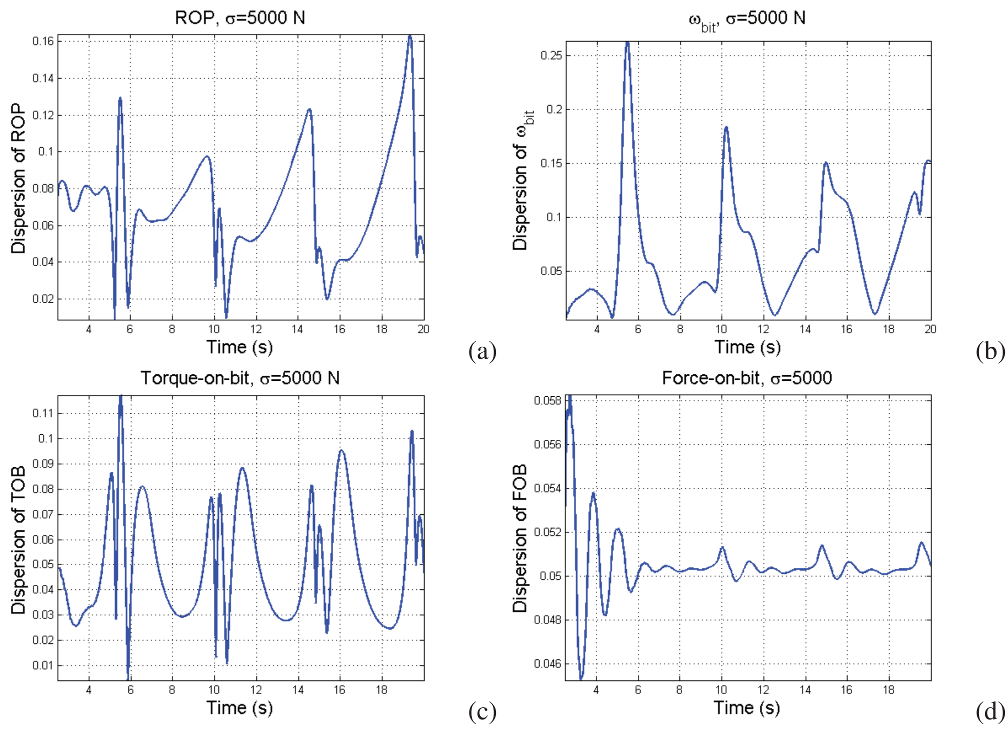

Figure 9 – Dispersion of the response for $\sigma = 5000$ N. (a) rate-of-penetration, ROP; (b) rotation speed of the bit; (c) torque-on-bit; and (d) force-on-bit.

It is noted that, even for a small variation of the WOH ($\sim 0.5\%$), there is a big dispersion in the response. See for instance the rate-of-penetration: the mean dispersion is 4.3%, which is more than eight times greater than the dispersion of the WOH. It gets worse if we take the maximum dispersion, which is 16%. It means that if the WOH has a dispersion of half percent, the variation in the ROP may achieve sixteen percent and the variation of the rotation speed of the bit may achieve twenty six percent!

CONCLUDING REMARKS

In this paper a stochastic model of the drill-string dynamics was analyzed. The main efforts that the column is subjected to are considered: rotation at the top; hanging force at the top; bit-rock interaction; fluid-structure interaction that takes into account the drilling fluid that flows downwards the column then goes upwards in the annulus; shock and rubbing between the column and the borehole; and the own weight of the column.

The weight-on-hook was modeled as a random variable with a normal probability density function. This distribution was constructed by means of the Maximum Entropy Principle. As the bit-rock interaction model used was derived for a weight-on-bit $\sim$ 100 kN, the dispersion of the weight-on-hook was constrained to $\sigma_{max} = 5000$ N, which represents 0.5 percent of variation for the WOH. What is interesting is that a small variation of the WOH (half percent) induces a considerable variation on the system response, for example: 6% on the force-on-bit, 12% on the torque-on-bit, 16% on the ROP, and 26% on the ω_{bit} (these are the maximum dispersions observed).

There are many sources of uncertainties in this problem, so more stochastic analysis should be done to identify the uncertainties that affect the most the performance of the system, but the results of this paper showed that the system response is very sensible to a dispersion of the weight-on-hook.

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RESPONSIBILITY NOTES

The authors are the only responsible for the printed material included in this paper.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support of CNPQ, CAPES, and FAPERJ.