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Abstract: A drill-string is a slender structure that turns and drills into the rock in search of oil. A numerical model is developed using the Timoshenko beam theory and approximated by means of the Finite Element Method. The aim this work is to investigate the influence of the drilling fluid in the dynamics of the drill-string. So, a model for the fluid-structure interaction that takes into account the drilling fluid that flows downwards the column then goes upwards in the annulus is used. In addition, other efforts that the column is subjected to are considered: rotation at the top; hanging force at the top; bit-rock interaction; shock and rubbing between the column and the borehole; finite strains (what couples axial, torsional, and lateral vibrations); and the own weight of the column.

Keywords: drill-string dynamics, nonlinear dynamics, fluid-structure interaction, bit-rock interaction

NOMENCLATURE

A = area of the transversal section, [m^2]
$[C]$ = damping matrix, [N.s/m] or [N.s]
D = diameter, [m]
E = Young Modulus, [Pa]
$\mathbf{f}$ = force vector, [N] or [N.m]
G = shear coefficient, [Pa]
h = head loss, [m]
I = inertia moment of the transversal section, [m^4]
$[K]$ = stiffness matrix, [N/m] or [N]
L = length, [m]
M = mass per unit length, [kg/m]
$[M]$ = mass matrix, [kg] or [kg.m]
$\mathbf{N}$ = shape functions, dimensionless
p = pressure, [Pa]
$\mathbf{q}$ = displacement vector, [m] or [rd]
t = time, [s]
T = kinetic energy, [N.m]
U = potential energy of deformation, [N.m]; or fluid velocity, [m/s]
u = displacement in x-direction, [m]
v = displacement in y-direction, [m]
w = displacement in z-direction, [m]
W = work done by the external forces and work not considered in U or T, [N.m]
Z = regularizing function, dimensionless

Greek Symbols

δ = symbol of variation, dimensionless
ε and γ = deformation, dimensionless
ω = angular velocity vector, [rd/s]
ρ = density, [kg/m^3]
$[\Phi]$ = modal basis, dimensionless
Π = total potential of the system, [N.m.t]
θ_x = rotation about x-axis, [rd]
θ_y = rotation about y-axis, [rd]
θ_z = rotation about z-axis, [rd]
ξ = damping factor, dimensionless

Subscripts

br = bit-rock
ch = channel (or borehole)
r = reduced
e = element
f = fluid
g = geometric (for $[K]$) and gravity (for $\mathbf{f}$)
i = inside
ke = kinetic energy
o = outside
p = polar
se = strain energy
NL = nonlinear
S = static response
u = displacement in x-direction
v = displacement in y-direction
w = displacement in z-direction
θ_x = rotation about x-axis
θ_y = rotation about y-axis
θ_z = rotation about z-axis

INTRODUCTION

There are some papers in which a nonlinear dynamic model is developed inspired in the drill-string dynamics, e.g. Christoforou and Yigit (1997 and 2003), Tucker and Wang (1999 and 2003), Khulief et al. (2005 and 2007), Berlioz (1996), Trindade et al. (2005), Sampaio et al. (2006 and 2007). These models are able to quantify some effects that occur in a drilling operation, as for instance stick-slip oscillations, but they cannot predict correctly the dynamic response of a real system. This is explained, first, because the above models are too simple compared to the real system and, second, because the uncertainties are not taken into account. Each author uses a different approach to the problem: Yigit and Christoforou (1996 and 1997) use a one-mode approximation to analyze the problem; Khulief et al. (2005 and 2007), Sampaio et al. (2006 and 2007) use the Euler-Bernoulli beam model with the Finite Element Method; while Tucker and Wang (1999 and 2003) use the Cosserat theory. A fluid-structure interaction that takes into account the drilling fluid that flows inside and outside the column is not considered in any of the above works. This kind of fluid-structure interaction model was proposed in Paidoussis et al. (2007) for a plane problem in another context, and it will be extended here for our problem. In this work the Timoshenko beam model is employed and the Finite Element Method is used to find an approximation to the system. Besides, it is considered: finite strain with no simplifications (higher order terms are not neglected); quadratic terms derived from the kinetic energy; shock and rubbing between the column and the borehole; stabilizers; fluid-structure interaction; and a bit-rock interaction (Tucker and Wang, 2003) that models how the bit penetrates the rock. To derive the equations of motion, the Total Lagrangian (TL) formulation is used (the equations are written in the undeformed configuration and in the inertial frame), six degrees of freedom are considered in the points of discretization (three translations u, v, and w, and three rotations θ_x, θ_y, and θ_z), the stress tensor is the second Piola-Kirchhoff tensor, and finite strains are considered (Green-Lagrange strain tensor). The strategy used in this work is, in some respects, similar to the one used in Khulief et al. (2007), but there are several important additional features, such as: shock and rubbing between the column and the borehole, shear (Timoshenko beam model), finite θ_x, fluid-structure interaction that takes into account the flow downwards inside the column and upwards in the annulus, all the terms of the strain energy are used in the analysis, a bit-rock interaction model that allows the simulation of the bit penetration is used, constant force at the top (hanging force or weight-on-hook, WOH).

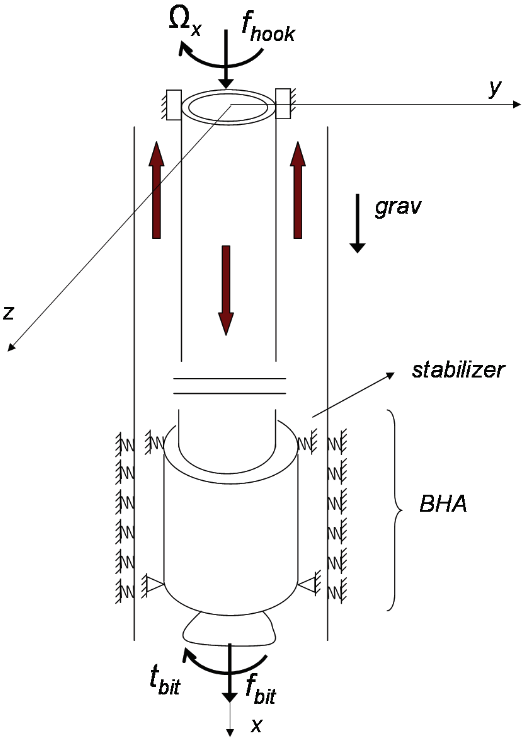

Figure 1 – General scheme.

A drill-string is a slender structure composed by thin tubes called drill-pipes that together have some kilometers (in our simulations 1.4 km) and some thicker tubes called drill-collars that together have some hundred meters (in our simulations 200 m). The region composed by the thicker tubes, called Bottom-Hole-Assembly (BHA), is under compression and subjected to shocks, see Fig. 1. That is the reason the tubes are stiffer in the bottom. The imposed forces are: the motor torque (as a constant rotation speed at the top Ω_x); a constant hook force f_{hook}; the torque t_{bit} and force f_{bit} at the bit; the weight of the column; the fluid forces; the shock and rubbing between the column and the borehole; the forces due to the stabilizer; plus the elastic and kinetic forces due to the deformation and the motion of the structure.

FINITE ELEMENT DISCRETIZATION

To derive the dynamic equations the extended Hamilton Principle is used (Rosenberg, 1980, Papastravridis, 2002). The first variation of Π must vanish:

$$\delta\Pi = \int_{t_1}^{t_2} (\delta U - \delta T - \delta W)dt = 0. \tag{1}$$

where U is the potential strain energy, T is the kinetic energy, and W is the work done by the nonconservative forces and any other force not accounted in the potential energy. To project the equation by the Finite Element Method (Hughes, 1987, Bathe, 1996, Reddy, 2005), a two-node approximation with six degrees of freedom per node is chosen. The nodal displacement is written as: $u_e = \mathbf{N}_u\mathbf{q}_e,\ v_e = \mathbf{N}_v\mathbf{q}_e,\ w_e = \mathbf{N}_w\mathbf{q}_e,\ \theta_{xe} = \mathbf{N}_{\theta_x}\mathbf{q}_e,\ \theta_{y_e} = \mathbf{N}_{\theta_y}\mathbf{q}_e,\ \theta_{ze} = \mathbf{N}_{\theta_z}\mathbf{q}_e$, where $\mathbf{N}$ are the shape functions (see Nelson, 1980, Bazoune and Khulief, 2002) ; u_e, v_e, and w_e are the displacements in x, y, and z directions; θ_{xe}, θ_{ye}, and θ_{ze} are the rotations in x, y, and z axis.$\mathbf{q}_e = \begin{pmatrix} u_1 & v_1 & \theta_{z1} & w_1 & \theta_{y_1} & \theta_{x1} & u_2 & v_2 & \theta_{z2} & w_2 & \theta_{y_2} & \theta_{x2}\end{pmatrix}^T$, where $(\cdot)^T$ means transpose. After assemblage, the projected equation is written as

$$([M]+[M_f])\ddot{\mathbf{q}}+([C]+[C_f])\dot{\mathbf{q}}+([K]+[K_f]+[K_g(\mathbf{q})])\mathbf{q}=\mathbf{f}_{NL}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})+\mathbf{f}_c+\mathbf{f}_g+\mathbf{f}_f\,. \tag{2}$$

The response $\mathbf{q}$ is represented in a subspace $V_m \subset \mathbb{R}^m$, where m equals the number of degrees of freedom of the system. $[M]$, $[C]$, and $[K]$ are the classical mass, damping and stiffness matrices; $[M_f]$, $[C_f]$, $[K_f]$ are the fluid mass, damping and stiffness matrices, and $\mathbf{f}_f$ is the fluid force vector; $[K_g(\mathbf{q})]$ is the geometric stiffness matrix; $\mathbf{f}_g$ is the gravity force; $\mathbf{f}_c$ is a concentrated reaction force at the bit; $\mathbf{f}_{NL}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ is the nonlinear force vector that is decomposed in four parts as follows,

$$\mathbf{f}_{NL}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=\mathbf{f}_{ke}(\ddot{\mathbf{q}},\dot{\mathbf{q}},\mathbf{q})+\mathbf{f}_{se}(\mathbf{q})+\mathbf{f}_{sh}(\mathbf{q})+\mathbf{f}_{br}(\dot{\mathbf{q}})\,. \tag{3}$$

where $\mathbf{f}_{ke}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ is composed by the quadratic terms of the kinetic energy; $\mathbf{f}_{se}(\mathbf{q})$ is composed by the quadratic and higher order terms of the strain energy; $\mathbf{f}_{sh}(\mathbf{q})$ are the forces due to the shock and rubbing between the column and the borehole; $\mathbf{f}_{br}(\dot{\mathbf{q}})$ are the forces due to the bit-rock interactions.

INITIAL PRESTRESSED CONFIGURATION

Before starting the rotation about x-axis, the column is lie down through the hole until it reaches the soil. As is reaches an equilibrium there are three forces acting: the reaction force at the bit, the weight of the drill-string and the hook force that supports the system. In this equilibrium configuration, the system is prestressed. There is a neutral point: above it the column is under tension and below the column is under compression.

We consider variations from the initial stressed state which is calculated using $\mathbf{f}_g$ (gravity force) and $\mathbf{f}_c$ (concentrated force at the bit that depends on the weight-on-bit, *wob*):

$$\mathbf{q}_S=[K]^{-1}(\mathbf{f}_g+\mathbf{f}_c+\mathbf{f}_f)\,. \tag{4}$$

Substituting the above expression in Eq. (2) yields

$$([M]+[M_f])\ddot{\bar{\mathbf{q}}}+([C]+[C_f])\dot{\bar{\mathbf{q}}}+([K]+[K_f]+[K_g(\mathbf{q}_S)])\bar{\mathbf{q}}=\mathbf{f}_{NL}(\bar{\mathbf{q}},\dot{\bar{\mathbf{q}}},\ddot{\bar{\mathbf{q}}})\,, \tag{5}$$

in which $\bar{\mathbf{q}}=\mathbf{q}-\mathbf{q}_S$.

KINETIC ENERGY

The first variation of the kinetic energy is written as

$$\begin{aligned}\delta T = -\int_0^L \big[&\rho A\ddot{u}\delta u+\rho A\ddot{v}\delta v+\rho A\ddot{w}\delta w+\rho I\ddot{\theta}_y\delta\theta_y+\rho I\ddot{\theta}_z\delta\theta_z+\rho I_p\ddot{\theta}_x\delta\theta_x+\\ &+(\rho I_p(-\theta_y\ddot{\theta}_z\delta\theta_x-\dot{\theta}_y\dot{\theta}_z\delta\theta_x+\dot{\theta}_x\dot{\theta}_z\delta\theta_y-\dot{\theta}_x\dot{\theta}_y\delta\theta_z-\theta_y\ddot{\theta}_x\delta\theta_z))\big]\,dx\,.\end{aligned} \tag{6}$$

where ρ is the mass density, A is the cross sectional area, L is the length of the column, $[I_t]$ is the cross sectional inertial matrix, $\mathbf{v}$ is the velocity vector, and ω the angular velocity vector. The three quantities $\mathbf{v}$, $[I_t]$, and ω are written as

$$\mathbf{v}=\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix}\quad,\quad [I_t]=\begin{pmatrix}I_p & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{pmatrix}\quad,\quad \omega=\begin{pmatrix}\dot{\theta}_x+\theta_y\dot{\theta}_z\\ cos(\theta_x)\dot{\theta}_y-sin(\theta_x)\dot{\theta}_z\\ sin(\theta_x)\dot{\theta}_y+cos(\theta_x)\dot{\theta}_z\end{pmatrix}. \tag{7}$$

The time derivative is denoted by a superposed dot, $da/dt=\dot{a}$. The angular velocity ω is written in the material configuration assuming small angles (θ_y and θ_z) and it is derived by first rotating the inertial frame about the x-axis, θ_x, then rotating the resulting frame about the y-axis, θ_y, and finally, rotating the resulting frame about the z-axis, θ_z.

STRAIN ENERGY

The first variation of the strain energy is given by

$$\delta U=\int_V(E\delta\varepsilon_{xx}\varepsilon_{xx}+4k_sG\delta\gamma_{xy}\gamma_{xy}+4k_sG\delta\gamma_{xz}\gamma_{xz})dV\,. \tag{8}$$

where V is the region of integration, $\varepsilon=(\varepsilon_{xx}\;\;2\gamma_{xy}\;\;2\gamma_{xz})^T$ is the Green-Lagrange strain tensor.

The Timoshenko beam model is used because the impacts at the bottom and the complex nonlinear dynamics may induce shear. Of course, the Timoshenko beam model generalizes Euler-Bernoulli model, so nothing is lost. The displacements written in the undeformed configuration are

$$u_x = u - y\theta_z + z\theta_y \quad , \quad u_y = v + y(\cos(\theta_x) - 1) - z\sin(\theta_x) \quad , \quad u_z = w + z(\cos(\theta_x) - 1) + y\sin(\theta_x)\,, \tag{9}$$

in which u, v, and w are the displacements of a point in the neutral line. The space derivative is denoted as $da/dx = a'$. Note that θ_x has not been simplified in the above expression. Note also that due to the finite strain formulation, the axial, torsional, and lateral vibrations are coupled.

SHOCK AND RUBBING

The forces due to the shocks between the column and the borehole are modeled by concentrated forces and torques at the nodes in the region of the drill-collar. The lateral forces are modeled as elastic forces governed by the stiffness parameter k_{sh} [N/m]. The rubbing between the column and the borehole is simply modeled as a friction torque governed by the friction coefficient μ_{sh}.

BIT-ROCK INTERACTION MODEL

In this work, the model used is the one developed by Tucker and Wang (2003), which is rewritten as

$$f_{xbit} = -\frac{\dot{u}_{bit}}{a_2 Z(\dot{\theta}_{bit})^2} + \frac{a_3 \dot{\theta}_{bit}}{a_2 Z(\dot{\theta}_{bit})} - \frac{a_1}{a_2} \quad , \quad t_{xbit} = -\frac{\dot{u}_{bit} a_4 Z(\dot{\theta}_{bit})^2}{\dot{\theta}_{bit}} - a_5 Z(\dot{\theta}_{bit}) \tag{10}$$

in which f_{xbit} is the axial force, where t_{xbit} is the torque about the x-axis and where $Z(\dot{\theta}_{bit})$ is a regularizing function.

In the above equation, $a_1, \ldots, a_5$ are positive constants that depend on the bit and rock characteristics as well as on the weight-on-bit (*wob*). This equation was derived for a stable operation with $\dot{\theta}_{bit} \sim 100$ RPM and with $wob \sim 100$ kN.

In this model, the bit exerts only an axial force (f_{xbit}) and a torque (t_{xbit}) about the x-axis. These force and torque exerted by the rock at the bit depend on the axial speed ($\dot{u}_{bit}$) and on the rotation speed ($\dot{\theta}_{bit}$) of the bit. Note that these forces at the bit couple axial and torsional vibrations.

FLUID

The drilling fluid (mud) is responsible to transport the cuttings (drilled solids) from the bottom to the top to avoid clogging of the hole. It also plays an important role in cooling and stabilizing the system (ASME Handbook, 2005). The rheological properties of the mud are complexes, see for instance Coussot et al. (2004) There is no doubt that the drilling fluid influences the dynamics of a drill-string, but to solve the complete problem would be computationally too expensive. There are some works that study only the drilling fluid flow, as, for example, Esculier (2000 and 2002) and Pina and Carvalho (2006). We decided to use a linear fluid-structure coupling model similar to Paidoussis et al. (1998 and 2007). In this simplified model there are the following hypotheses: in the inside flow the fluid is taken as inviscid, and in the outside as viscous, the flow induced by the rotation speed about x-axis is not considered in this analysis, the pressure varies linearly with x, the fluid is added in the formulation as a constant mass matrix $[M_f]$, a constant stiffness matrix, $[K_f]$, a constant damping matrix $[C_f]$, and a constant force $\mathbf{f}_f$ (see Eq. (11)).

For short we will write directly the equations for an element. These equations are an extension and an adaptation of the model developed in Paidoussis et al. (2007). We have,

$$\begin{aligned}
[M_f]^{(e)} &= \int_0^1 \left(M_f + \chi\rho_f A_o\right)(\mathbf{N}_w^T\mathbf{N}_w + \mathbf{N}_v^T\mathbf{N}_v) l_e d\xi \text{ , (added mass)}\,, \\
[K_f]^{(e)} &= \int_0^1 \left(-M_f U_i^2 - A_i p_i + A_o p_o - \chi\rho_f A_o U_o^2\right)(\mathbf{N'}_w^T\mathbf{N'}_w + \mathbf{N'}_v^T\mathbf{N'}_v)\frac{1}{l_e} d\xi + \\
&\quad + \int_0^1 \left(-A_i\frac{\partial p_i}{\partial x} + A_o\frac{\partial p_o}{\partial x}\right)(\mathbf{N}_{\theta_y}^T\mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T\mathbf{N}_{\theta_z}) l_e d\xi \text{ , (added stiffness)}\,, \\
[C_f]^{(e)} &= \int_0^1 \left(-2M_f U_i + 2\chi\rho_f A_o U_o\right)(\mathbf{N}_{\theta_y}^T\mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T\mathbf{N}_{\theta_z}) l_e d\xi + \\
&\quad + \int_0^1 \left(\frac{1}{2}C_f\rho_f D_o U_o + k\right)(\mathbf{N}_w^T\mathbf{N}_w + \mathbf{N}_v^T\mathbf{N}_v) l_e d\xi \text{ , (added damping)}\,, \\
\mathbf{f}_f^{(e)} &= \int_0^1 \left(M_f g - A_i\frac{\partial p_i}{\partial x} - \frac{1}{2}C_f\rho_f D_o U_o^2\right)\mathbf{N}_u^T l_e d\xi \text{ . (added axial force)}\,,
\end{aligned} \tag{11}$$

in which,

M_f is the fluid mass per unit length,
ρ_f is the density of the fluid,
$\chi = \dfrac{(D_{ch}/D_o)^2+1}{(D_{ch}/D_o)^2-1} \quad (>1)$,
D_{ch} is the borehole (channel) diameter,
D_i, D_o are the inside and outside diameters of the column,
U_i, U_o are the inlet and outlet flow velocities,
p_i, p_o are the pressures inside and outside the drill-string,
A_i, A_o are the inside and outside cross sectional area of the column,
C_f, k are the fluid viscous damping coefficients.

It is assumed that the inner and the outer pressures (p_i and p_o) vary linearly with x and are then written as

$$p_i = \left(\rho_f g\right) x + p_{cte}\,, \tag{12}$$

$$p_o = \left(\rho_f g + \frac{F_{fo}}{A_o}\right) x\,, \tag{13}$$

where p_{cte} is a constant pressure and where F_{fo} is the friction force due to outside flow given by

$$F_{fo} = \frac{1}{2} C_f \rho_f \frac{D_o^2 U_o^2}{D_h}\,. \tag{14}$$

In the above equation, D_h is the hydraulic diameter (=$4A_{ch}/S_{tot}$) and S_{tot} is the total wetted area per unit length ($\pi D_{ch} + \pi D_o$). Note that the reference pressure is $p_o(x=0)=0$. Another assumption is that there is no head loss when the fluid passes from the drill-pipe to the drill-collar (and vice-versa). The head loss due to the change in velocity of the fluid at the bottom (it was going down, then it goes up) is given by

$$h = \frac{1}{2g}(U_i - U_o)^2\,. \tag{15}$$

In Paidoussis et al. (2007) this expression is derived for a column free at the bottom, which is not the case here. Nevertheless in this work we will not change this expression because the simulations showed that, even if h is multiplied by ten, the results do not change significantly.

Note that if the geometry and the fluid characteristics are given, we can only control the inlet flow velocity $U_i(x=0)$ because the outlet velocity is calculated using the continuity equation and the pressures are calculated using the Bernoulli equation. Examining Eq. (11), it can be seen that the mass matrix due to the fluid is the usual added mass that, in our case, represents a significative contribution. For example, using representative values (used in our simulations), the added mass is around 50%, what changes the natural frequencies in about 20%.

The stiffness matrix due to the fluid depends on the speed of the inside and outside flow, on the pressure and on the pressure derivatives. Analyzing the signs in the equation (Eq. 11) we see that the outside pressure tends to stabilize the system while the inside pressure and the flow tends to destabilize the system. The term $(-p_i A_i + p_o A_0)$ plays a major role on the stiffness of the system because, even though p_i is close to p_o, in the drill collar region (in the bottom) A_0 is around ten times A_i what turns the system stiffer at the bottom.

The damping matrix due to the fluid depends on the flow velocity as well as in the viscous parameter of the fluid, which are not well established values. There are uncertainties to determine the damping characteristics and a stochastic model should be developed to the damping, but in this work a detailed analysis will not be addressed. Finally, the force vector ($\mathbf{f}_f$) represents the buoyancy induced by the fluid and it is the only force in the axial direction (x-direction).

BOUNDARY AND INITIAL CONDITIONS

As boundary conditions it is considered that at the top ($x=0$) the transversal displacements are zero ($v=w=0$), the rotations about y and z-axis are zero ($\theta_y=\theta_z=0$), and a constant rotation speed about x-axis is imposed: $\theta_x(x=0)=\Omega_x$. At the bit ($x=L$) the transversal displacements are zero ($v=w=0$).

In drilling operations there are stabilizers in the BHA region that turn the system stiffer and help to diminish the amplitude of the lateral vibrations. Stabilizers are considered as an elastic element at the point separating the drill-pipes from the drill-collars: $F_y(x=stab)=k_{stab}\,v(x=stab)$ and $F_z(x=stab)=k_{stab}w(x=stab)$, where $stab$ means stabilizer.

To apply the boundary conditions above, the lines and rows corresponding to $v|_{x=0}$, $v|_{x=L}$, $w|_{x=0}$, $w|_{x=L}$, $\theta_y|_{x=0}$, $\theta_z|_{x=0}$, and $\theta_x|_{x=0}$ of the full matrices ($[M]$, $[C]$, and $[K]$) are eliminated and the corresponding forces of the imposed velocity at the top (Ω_x) are considered in the right side of the equation.

For the initial condition in time, it is assumed that $\theta_x(t=0) = 100$ RPM and that $u(t=0)$=15 m/h, so the system starts with two rigid body motions.

REDUCED MODEL: CHOICE OF THE REDUCTION BASIS

Usually the FE projected system has big matrices (dimension $m \times m$) and the dynamic analysis may be time consuming, which is the case of the analysis presented herein. A possible method for nonlinear dynamical systems (see for instance Crolet and Ohayon (1994), Soize (2000), and Sampaio and Soize (2007)) is to project the nonlinear dynamical equation on a subspace $V_n \in \mathbb{R}^m$, with $n << m$, spanned by a basis related to the dynamics that is able to represent the system in this subspace. There are several possibilities for this choice, POD/Karhunen-Loève could be one of them, Sampaio et al. (2005 and 2006).

In this paper, the basis used for the reduction corresponds to the normal modes. The normal modes are calculated with $[M]$ and $([K]+[K_g(\mathbf{q}_S)])$ solving the following generalized eigenvalue problem,

$$(-[M]\omega^2 + ([K]+[K_g(\mathbf{q}_S)]))\phi = \mathbf{0}\,, \tag{16}$$

where ϕ_i is the i-th normal mode and ω_i is the i-th natural frequency. Note: if the fluid is taken into account the normal modes are calculated with $([M]+[M_f])$ and $([K]+[K_f]+[K_g(\mathbf{q}_S)])$.

Using the representation

$$\bar{\mathbf{q}}(t) = [\Phi]\,\mathbf{a}(t)\,, \tag{17}$$

and substituting it in the equation of motion yield

$$[M][\Phi]\ddot{\mathbf{a}} + [C][\Phi]\dot{\mathbf{a}} + ([K]+[K_g(\mathbf{q}_S)])[\Phi]\mathbf{a} = \mathbf{f}_{NL}(\bar{\mathbf{q}}, \dot{\bar{\mathbf{q}}}, \ddot{\bar{\mathbf{q}}})\,. \tag{18}$$

where $[\Phi]$ is a $(m \times n)$ real matrix composed by n normal modes. Projecting the equation on the subspace spanned by these normal modes yields

$$[\Phi]^T[M][\Phi]\ddot{\mathbf{a}} + [\Phi]^T[C][\Phi]\dot{\mathbf{a}} + [\Phi]^T([K]+[K_g(\mathbf{q}_S)])[\Phi]\mathbf{a} = [\Phi]^T\mathbf{f}_{NL}(\bar{\mathbf{q}}, \dot{\bar{\mathbf{q}}}, \ddot{\bar{\mathbf{q}}})\,, \tag{19}$$

which can be rewritten as

$$[M_r]\,\ddot{\mathbf{a}}(t) + [C_r]\,\dot{\mathbf{a}}(t) + [K_r]\,\mathbf{a}(t) = [\Phi]^T\mathbf{f}_{NL}(\bar{\mathbf{q}}, \dot{\bar{\mathbf{q}}}, \ddot{\bar{\mathbf{q}}})\,, \tag{20}$$

in which

$$[M_r] = [\Phi]^T[M][\Phi], \quad [C_r] = [\Phi]^T[C][\Phi]$$

$$[K_r] = [\Phi]^T([K]+[K_g(\mathbf{q}_S)])[\Phi] \tag{21}$$

are the reduced matrices. In this application we use 56 finite elements, so the number of d.o.f $= 57 \times 6 = 342$. For the dynamic response, 32 normal modes were necessary to reach convergence, i.e. 10 % of the DOF.

DAMPING

The damping of the system is modeled as a viscous damping with matrix $[C] = \alpha[M] + \beta([K]+[K_g(\mathbf{q}_S)])$ (where α and β are constants). Since we need to construct the reduced damping matrix, we will not effectively construct the damping matrix $[C]$. Instead we will select a damping factor ξ_i for each i-th normal mode normalized with respect to the mass matrix. The reduced damping matrix $[C_r]$ can then be written as

$$[C_r] = \begin{pmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & 2\xi_3\omega_3 & 0 \\ 0 & 0 & 0 & ... \end{pmatrix}. \tag{22}$$

where ω_i and ξ_i are the natural frequency and the damping rate related to the i-th normal mode. Note that in our analysis, the first two natural frequencies are zero due to the rigid body motion, so, these two parameters c_1 and c_2 will control how the rigid body motions loose energy due to friction in time.

NUMERICAL RESULTS

The drill-string was discretized using 56 finite elements. For the dynamics analysis it was used 10 lateral modes, 10 torsional modes, 10 axial modes, and also the two rigid body modes of the structure (axial and torsional). So the matrix $[\Phi]$ is composed by 32 modes. This number was chosen after several experiments. Using more modes the response of the system does not change significantly, for the precision considered.

For the time integration procedure, a routine that uses the implicit Newmark integration scheme (Bathe, 1996) was implemented with a predictor and a fix point procedure to equilibrate the system response at each time step. The results were the same as the ones achieved with an explicit scheme using the Runge-Kutta method of 4th and 5th order with time step controller to maintain the error within a given precision. Because the simulation time is smaller in the latter strategy, the explicit scheme is used in the next simulations. The system parameters used are representative values that are found in the literature (Christoforou and Yigit, 2003, Tucker and Wang, 2003, Khulief et al., 2007, and Piovan and Sampaio, 2006).

Influence of the fluid

Fig. 2 shows a comparison of the dynamic response with and without the fluid-structure interaction mode.

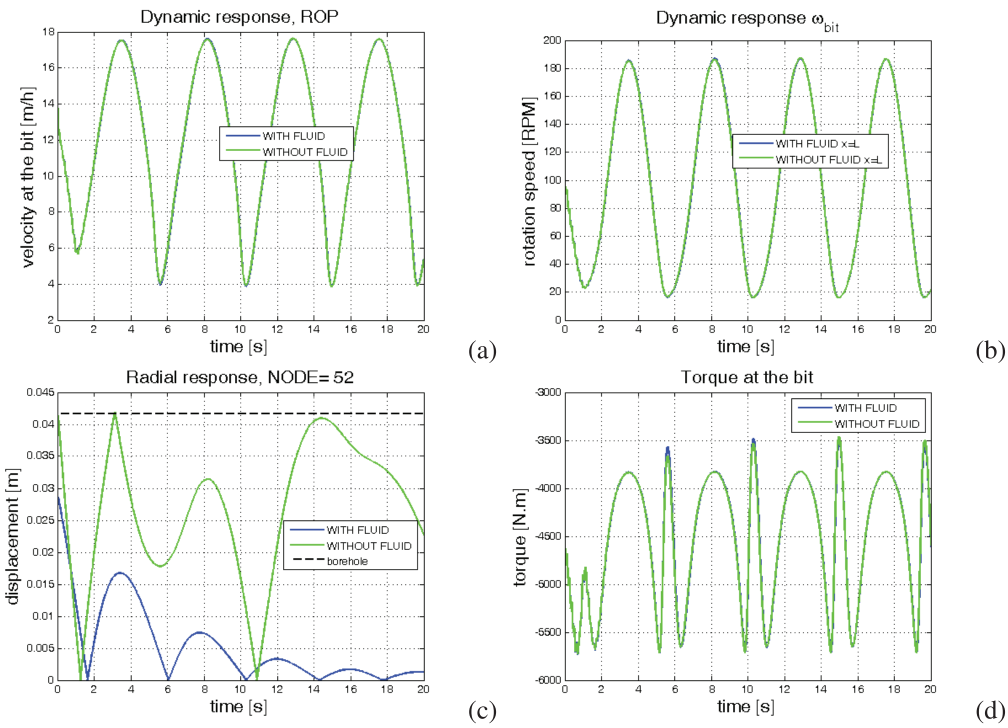

Figure 2 – Response with fluid $\times$ without fluid. (a) axial speed at $x = L$, or rate of penetration (ROP); (b) rotation speed at $x = L$ (ω_{bit}); (c) radial displacement at $x = 1560$ m; and (d) torque at the bit.

See in Fig. 2 that the main difference in the dynamic response with and without the fluid-structure interaction model is in the lateral dynamic response, Fig. 2(c). For the model used, the fluid has a major influence in the lateral frequencies and lateral mode shapes, see Table 1, but the axial and torsional frequencies are unaffected, which is not a surprise since in the formulation used the axial movement is only affected by a constant force $\mathbf{f}_f$, Eq. (11).

Frequency number	Lateral natural freq. (no fluid) (Hz)	Lateral natural freq. (with fluid) (Hz)	Difference (%)
1	0.0287	0.0372	22.8495
2	0.0287	0.0372	22.8495
3	0.0464	0.0744	37.6344
4	0.0464	0.0744	37.6344
5	0.0928	0.1065	12.8638
6	0.0928	0.1065	12.8638
7	0.1098	0.1117	1.7010
8	0.1098	0.1117	1.7010
9	0.1394	0.1488	6.3172
10	0.1394	0.1488	6.3172

Table 1 – Natural frequencies with and without fluid

Fig. 3 shows the first lateral mode shapes with and without fluid. The normal modes are calculated in the prestressed configuration, i.e., with

$$([M]+[M_f]) \quad \text{and} \quad ([K]+[K_f]+[K_g(\mathbf{q}_S)]) \quad , \quad \text{where} \quad \mathbf{q}_S = [K]^{-1}(\mathbf{f}_g+\mathbf{f}_c+\mathbf{f}_f)\,. \tag{23}$$

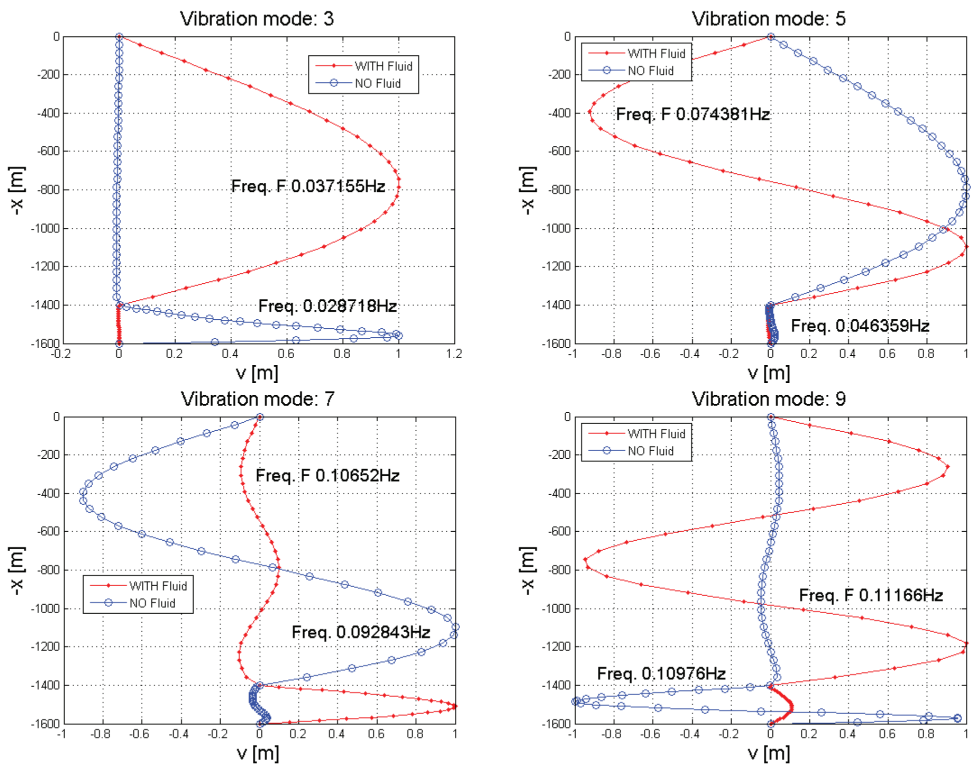

Figure 3 – First lateral mode shapes with and without fluid.

Note that because of the presence of the fluid the column is stiffer in the bottom (see the first lateral mode, for instance). This effects occurs mainly due to the term $(p_iA_i + p_oA_o)$ where A_o is around ten times A_i.

CONCLUDING REMARKS

A dynamical model was developed to simulate the drill-string dynamics and it showed to be well suited. The results are coherent with the ones found in the literature, but a different model is proposed where the Timoshenko beam model is used and the main forces that influence the dynamics are considered: motor torque (as a constant rotation speed at the top), hanging force, stabilizers, bit-rock interaction that describes the rate-of-penetration, shock and rubbing between the column and the borehole, fluid-structure interaction (that flows downwards then goes upwards). It was also considered a nonlinear 3D beam with finite deformations without neglecting the higher order terms and the vibration was computed about a prestressed configuration.

The fluid-structure model used have a major influence in the lateral vibration of the system. For example, if we consider only the added mass ($[M_f]$) the lateral natural frequencies decrease around 20%. If only the added stiffness ($[K_f]$) is considered, the lateral frequencies increase around 50%. In addition, the added stiffness changes the modes of the system and consequently changes the reduction basis, so the projection basis generates a different subspace in which the dynamics is represented (see a comparison of the modes with and without fluid in Fig. 3). Nevertheless the axial and torsional vibration are little affected by the fluid flow.

A reduced model is proposed where the lateral, axial, and torsional modes are chosen to compose the reduced basis. For the system analyzed, if we order the modes there are two rigid body modes (axial and torsional) and then more than 100 lateral modes. The axial and torsional modes must be identified and used in the reduced basis so that the dynamics can be computed properly.

There are many sources of uncertainties in this problem. In Ritto et al. (2009) the stochastic dynamics is analyzed for a random force at the hook, also called weight-on-hook.

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RESPONSIBILITY NOTES

The authors are the only responsible for the printed material included in this paper.

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DATA USED IN THE SIMULATIONS

$L_{dp} = 1400$ [m] (length of the drill pipe), $L_{dc} = 200$ [m] (length of the drill collar), $D_{odp} = .127$ [m] (outside diameter of the drill pipe), $D_{odc} = .2286$ [m] (outside diameter of the drill collar), $D_{idp} = .095$ [m] (inside diameter of the drill pipe), $D_{idc} = 0.0762$ [m] (inside diameter of the drill collar), $D_{ch} = 0.3$ [m] (diameter of the borehole (channel)), $x_{stab} = 1400$ [m] (location of the stabilizer), $k_{stab} = 17.5$ [MN/m] (stiffness of the stabilizer per meter), $E = 210$ [GPa] (elasticity modulus of the drill string material), $\rho = 7850$ [kg/m^3] (density of the drill string material), $\nu = .29$ [-] (poisson coefficient of the drill string material), $k_s = 6/7$ [-] (shearing correcting factor), $c_1 = 0.05$ [N.s/m] (friction coefficient for the axial rigid body motion), $c_2 = 0.05$ [N.s/m] (friction coefficient for the rotation rigid body motion), $\xi_i = 0.3$ [-] i-th (damping factor), $k_{sh} = 1e8$ [N/m] (stiffness per meter used for the shocks), $\mu_{sh} = 0.0005$ [-] (friction coefficient between the string and the borehole), $\Omega_x = 100$ [RPM] (constant speed at the top), $U_i = 1.5$ [m/s] (flow velocity in the inlet), $\rho_f = 1200$ [kg/m^3] (density of the fluid), $C_f = .0125$ [-] (fluid viscous damping coefficient), $k = 0$ [-] (fluid viscous damping coefficient), $wob = 100$ [kN] (initial weight on the bit), $g = 9.81$ [m/s^2] (gravity acceleration), $a_1 = 3.429e-3$ [m/s] (constant of the bit-rock interaction model), $a_2 = 5.672e-8$ [m/(N.s)] (constant of the bit-rock interaction model), $a_3 = 1.374e-4$ [m/rd] (constant of the bit-rock interaction model), $a_4 = 9.537e6$ [N.rd] (constant of the bit-rock interaction model), $a_5 = 1.475e3$ [N.m] (constant of the bit-rock interaction model), $e = 2$ [rd/s] (regularization parameter).