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A Reduced-order Model of Mistuned Cyclic Dynamical Systems With Finite Geometric Perturbations Using a Basis of Cyclic Modes

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Abstract: A new method for vibration analysis of mistuned bladed disks is presented. The method is built for solving the dynamic problem of cyclic structures with finite geometric perturbations. It is based on the use of the cyclic modes of the different sectors which can be obtained from a usual cyclic symmetry modal analysis. Hence the projection basis is constituted as well as on the whole bladed disk, each sector operator matrix is reduced by its own cyclic modes. The method is validated numerically on an academic bladed disk model, by comparing free responses of a full model finite element analysis to those of a reduced-order model using the above reduction method.

Keywords: Reduced-order model, Mistuning, Cyclic dynamical systems, Geometric perturbations, Cyclic modes

NOMENCLATURE

$\mathbb{B}$ = frequency band
$[B]$ = continuity matrix
$[\mathcal{B}]$ = reduced continuity matrix
$[D]$ = damping matrix
$[E]$ = dynamic matrix
$[\hat{E}]$ = cyclic dynamic matrix
$[\mathcal{E}]$ = reduced dynamic matrix
f = force vector
g = generalized forces vector
$[K]$ = stiffness matrix
$[K_0]$ = reference sector stiffness matrix
$[M]$ = mass matrix
$[M_0]$ = reference sector mass matrix
MSF = modal scale factor
n = circumferential wave number
$\overline{n}$ = sector degrees-of-freedom number
n_{ddl} = bladed disk degrees-of-freedom number
N = number of blades
q = generalized coordinates vector
u = displacement vector
$\overline{u}$ = cyclic displacement vector
$[W]$ = Fourier matrix

Greek Symbols
α and β = mode number
δ = Kronecker symbol
ϕ = mode
$\hat{\phi}$ = cyclic mode
λ = eigenvalue
μ = generalized mass
ω = frequency [Hz]
Ω_p = sector p
$[\hat{\Psi}]$ = cyclic modal basis

Subscripts
c = constrained
i = inside
ini = initial
l = left
max = maximum
min = minimum
nom = nominal
r = right
t = tuned

INTRODUCTION

In the context of turbomachine design, small variations in the blade characteristics due to manufacturing tolerances affect the structural cyclic symmetry creating mistuning and can then increase the forced response amplitudes (see Whitehead (1966), El-Bayoumi and Srinivasan (1975), Dye and Henry (1969), Ewins (1976), Sogliero and Srinivasan (1980), Capiez et Soize (2004)). However, it is possible (see Ewins (1969), Choi et al. (2003), Castanier and Pierre (1997, 2002), Mignolet et al. (2000)). to voluntary detune the mistuned system in order to reduce the forced response amplification. The main technologic solutions to introduce detuning are based on modifying blade material properties, the interface between blade and disk, or the blade shape by introducing several types of blades with different geometries corresponding to finite geometric perturbations of the nominal blades.

To reduce numerical computational costs, many reduced-order methods (see Castanier et al. (1997), Moyroud et al. (2002), Yang and Griffin (2001)) have been introduced, among which reduction techniques Yang and Griffin (2001) employing a modal reduction approach with a modal basis consisting in using cyclic modes of the tuned bladed disk sector. These techniques are very efficient in the case of cyclic structures with material properties perturbations but do not solve the case of finite geometric perturbations. In fact, in the latter case, the tuned bladed disk sector cyclic modes are computed on a finite element mesh related to the nominal geometry of the cyclic bladed disk. The finite perturbation mass and stiffness matrices relative to the finite geometric perturbation of certain blades are constructed with another finite element mesh which is not compatible with the nominal finite element mesh. Consequently, this non compatible finite element meshes induce a difficulty for constructing the projection of the finite perturbation mass and stiffness matrices using the tuned bladed disk sector cyclic modes. That's why in Yang et Griffin (2001), they only simulated proportional mistuning by perturbing the Young's moduli of individual blades.

There are some methods known as "shape design sensitivity methods"(see for example Irwanto et al. (2003), Dailey (1989), Ojalvo (1988)) which can solve the problem of little geometric perturbations. However, these methods need nominal and perturbed finite element meshes, as well as matrices and modal shapes of the tuned system in order to perform a sensitivity analysis by using eigenvalues and eigenvectors derivatives.

In this paper, it is assumed that a commercial software (black box) is used to compute the cyclic modes of a bladed disk sector and is also used, with another finite element mesh to construct the finite perturbation mass and stiffness matrices. Consequently, the usual methods to take into account non compatible finite element meshes cannot be used, because an intrusive development should be carried out in the software. In this particular context, this paper presents a new reduction method for the dynamic problem of cyclic structures with finite geometric perturbations for the mass and stiffness matrices, based on the use of the cyclic modes of the different sectors which can be obtained from a usual cyclic symmetry modal analysis. The projection basis is constituted as well as on the whole bladed disk, each sector operator matrix is reduced by its own cyclic modes. Linear constraints are also applied on common boundaries between sectors to make the displacement field admissible. This reduction method is suitable for constructing the reduced computational model of a detuned bladed disk in unsteady aeroelasticity with different types of blades distributed on the disk circumference. An application is done on a bladed disk model by comparing its dynamic characteristics using this reduction method to those obtained on a full model. This method is an alternative on to the Static Mode Compensation developed in [Lim et al. 2004] and used in [Ganine et al. 2008].

DYNAMIC EQUATION OF THE TUNED SYSTEM

Let Ω be a fixed structure with an N-order cyclic symmetry, submitted to external forces .The problem under consideration is related to rotating structures, and the vibration analysis is performed in the frequency band $\mathbb{B}$ defined by $\mathbb{B} = [\omega_{min}, \omega_{max}], 0 < \omega_{min} < \omega_{max}$. Then, the dynamic equation of the tuned system can be written in this frequency band

$$(-\omega^2[M_t] + j\omega[D_t] + [K_t])u_t(\omega) = f(\omega) \quad , \tag{1}$$

where f denotes the n_{ddl} vector of external structural forces and fluid-structure coupling forces, u_t is the n_{ddl} displacement field of the complete tuned structure, matrices $[M_t]$, $[D_t]$, $[K_t]$ represent real mass, damping and stiffness matrices, and n_{ddl} is the degrees-of-freedom number of the tuned structure and $j^2 = -1$. Let's introduce the dynamic stiffness matrix

$$[E_t](\omega) = -\omega^2[M_t] + j\omega[D_t] + [K_t] \quad , \tag{2}$$

Then the dynamic equation (1) becomes

$$[E_t](\omega)u_t(\omega) = f(\omega) \quad . \tag{3}$$

It results from the cyclic symmetry of the structure that the tuned structure is made of N identical sectors compatible on their coupling interfaces. Consequently, the dynamic stiffness matrix is formed by $N \times N$ sub-matrices, each one having $\overline{n} \times \overline{n}$ components and displacement and forces vectors are formed by N sub-vectors.

$$[E_t] = \begin{pmatrix} [E]^{0,0} & . & . & . & [E]^{0,N-1} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ [E]^{N-1,0} & . & . & . & [E]^{N-1,N-1} \end{pmatrix} \quad , \quad u_t = \begin{pmatrix} {u_t}^0 \\ . \\ . \\ . \\ {u_t}^{N-1} \end{pmatrix} \quad , \quad f = \begin{pmatrix} f^0 \\ . \\ . \\ . \\ f \end{pmatrix} \quad . \tag{4}$$

Since each sector Ω_p is adjacent to sectors Ω_{p-1} and Ω_{p+1}, the dynamic stiffness matrix yields

$$[E_t] = \begin{pmatrix} [E_i] & [E_r] & [0] & . & . & . & [0] & [E_l] \\ [E_l] & [E_i] & [E_r] & [0] & . & . & . & .[0] \\ [0] & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & [0] \\ [0] & . & . & . & . & . & . & .[E_r] \\ [E_r] & [0] & . & . & . & [0] & [E_l] & [E_i] \end{pmatrix} \quad . \tag{5}$$

Where $[E_i]$ is the block matrix related to inner degrees-of-freedom of sector Ω_p, $[E_r]$ and $[E_l]$ are bloc matrices related to coupling right and left side degrees-of-freedom of sector Ω_p. At this step, the dynamic system operators and displacement field are expressed in local coordinates systems associated to each sector Ω_p, $\forall p \in \{0, N-1\}$. A Discrete Fourier transform of the displacement field yields

$$u(\omega) = [W]\hat{u}(\omega) \quad , \quad \hat{u} = [\hat{u}_0, \ldots, \hat{u}_{N-1}]^T \quad , \tag{6}$$

where $u(\omega)$ is the displacement field in local coordinates system, $\hat{u}(\omega)$ is the displacement field in global cyclic coordinates system, and $[W]$ is the Fourier matrix defined by

$$[W] = \begin{pmatrix} w_{0,0}[I_{\bar{n}}] & w_{0,1}[I_{\bar{n}}] & . & . & . & w_{0,N-1}[I_{\bar{n}}] \\ w_{1,0}[I_{\bar{n}}] & w_{1,1}[I_{\bar{n}}] & . & . & . & w_{1,N-1}[I_{\bar{n}}] \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ w_{N-1,0}[I_{\bar{n}}] & w_{N-1,1}[I_{\bar{n}}] & . & . & . & w_{N-1,N-1}[I_{\bar{n}}] \end{pmatrix} \quad , \tag{7}$$

where $[I_{\bar{n}}]$ is the identity matrix

$$w_{n,m} = \exp\left(\frac{2jmn\pi}{N}\right) \quad , \quad [W]^{-1} = \frac{1}{N}[W]^{\star} = \frac{1}{N}[\overline{W}]^{T} \quad , \quad j^2 = -1 \quad . \tag{8}$$

The Fourier matrix is invertible and (2) yields

$$[W]^{-1}[E(\omega)][W]\hat{u}(\omega) = [W]^{-1}f(\omega) \quad , \tag{9}$$

which can be written as

$$[\hat{E}_t(\omega)]\hat{u}(\omega) = \hat{f}(\omega) \quad , \tag{10}$$

This new dynamic stiffness matrix expressed in global cyclic coordinates system is a block-diagonal matrix. Thus, by solving the forced or free response of the system on one sector, the dynamic behavior of the entire structure can be obtained using the interblade phase angles.

GENERALIZED EIGENVALUE PROBLEM RELATED TO THE TUNED SYSTEM

This problem is considered without gyroscopic coupling forces and is formulated using the cyclic symmetry properties of the structure. In this case, the generalized eigenvalue problem $([K_t] - \lambda[M_t])\phi = 0$, related to the entire structure Ω is equivalent to the generalized eigenvalue problem $([K_0] - \lambda_n[M_0])\hat{\phi}_n = 0$, formulated on a reference sector Ω_0. If the structure is fixed, then $[K_0]$ and $[M_0]$ are positive definite matrices and eigenvalues $0 < \lambda_{n,1} \leq \lambda_{n,2} \leq \ldots$ and associated eigenvectors $\{\hat{\phi}_{n,1}, \hat{\phi}_{n,2}, \ldots\}$ corresponding to the circumferential wave number n are such that

$$< [M_0]\hat{\phi}_{n,\alpha}, \hat{\phi}_{n,\beta} >= \delta_{\alpha\beta}, \quad < [K_0]\hat{\phi}_{n,\alpha}, \hat{\phi}_{n,\beta} >= \omega_{n,\alpha}^2 \delta_{\alpha\beta} \quad , \tag{11}$$

in which $\omega_{n,\alpha}^2 = \sqrt{\lambda_{n,\alpha}}$ is the eigenfrequency of the structural mode $\phi_{n,\alpha}$ whose normalization is defined by generalized mass $\mu_{n,\alpha} = 1$ and where $< X_{n,\alpha}, Y_{n,\beta} >= X_{n,1}Y_{n,1} + \ldots + X_{n,m}Y_{n,m}$. If the structure is free, then $[K_0]$ and $[M_0]$ are positive semidefinite matrices. The dimension of the null space of $[K_0]$ is denoted as m_{rig} and is assumed to be such that $0 < m_{rig} \leq 6$. The rigid body modes are denoted as $\{\hat{\phi}_{n,-m_{rig}}, \ldots, \hat{\phi}_{n,-1}\}$ and verify $[K_0]\hat{\phi}_{n,\alpha} = 0$, for α in $\{-m_{rig}, \ldots, -1\}$. Then the structural modes $\{\hat{\phi}_{n,1}, \hat{\phi}_{n,2}, \ldots\}$ are associated with positive eigenvalues $0 < \lambda_{n,1} \leq \lambda_{n,2} \leq \ldots$.

DYNAMIC EQUATION OF THE MISTUNED SYSTEM

Let's consider the finite element model of a mistuned structure, which mistuning result on finite geometric perturbations of some blades. in the frequency band $\mathbb{B}$ defined by $\mathbb{B} = [\omega_{min}, \omega_{max}], 0 < \omega_{min} < \omega_{max}$. Then, the dynamic equation of the mistuned system can be written in the frequency band B

$$(-\omega^2[M] + j\omega[D] + [K])u(\omega) = f(\omega) \quad , \tag{12}$$

where f denotes the n_{ddl} vector of external structural forces and fluid-structure coupling forces, u is the n_{ddl} displacement field of the complete tuned structure, matrices $[M]$, $[D]$, $[K]$ represent real mass, damping and stiffness matrices, and n_{ddl} is the degrees-of-freedom number of the mistuned structure and $j^2 = -1$. Note that the degrees-of-freedom number of the mistuned entire bladed disk is equal to the degrees-of-freedom number of the tuned entire bladed disk. By introducing the dynamic stiffness matrix

$$[E(\omega)] = -\omega^2[M] + j\omega[D] + [K] \quad , \tag{13}$$

the dynamic equation of the mistuned system becomes

$$[E](\omega)u(\omega) = f(\omega) \quad . \tag{14}$$

REDUCED-ORDER MODEL OF THE MISTUNED BLADED DISK

The mistuned entire model is reduced by projecting its matrices in cyclic coordinates on a modal basis made of cyclic modes of the different types of sectors of the mistuned system. This basis is chosen as well as, each sub-matrix related to a certain type of sector is reduced by its own cyclic modes.

Choice of the projection basis

let $\hat{\phi}^{ini}_{\alpha,n}$ be a complex cyclic mode α associated to the circumferential wave number n of one type of sector. We introduce a modal scale factor MSF to make sure that cyclic modes of different sector kinds computed apart are taken in the same phase reference system. Then new mode $\hat{\phi}_{\alpha,n}$ given the same phase referential system are given by

$$\hat{\phi}_{\alpha,n} = \frac{MSF(\hat{\phi}^{nom}_{\alpha,n}, \hat{\phi}^{ini}_{\alpha,n})}{|MSF(\hat{\phi}^{nom}_{\alpha,n}, \hat{\phi}^{ini}_{\alpha,n})|} \hat{\phi}^{ini}_{\alpha,n} \quad , \tag{15}$$

where

$$MSF(x,y) = \frac{\{x\}^T \{\overline{y}\}}{\{x\}^T \{\overline{x}\}} \quad , \quad \hat{\phi}_{\alpha,n} = \hat{\phi}^{'}_{\alpha,n} + j\hat{\phi}^{''}_{\alpha,n} \quad , \quad j^2 = -1 \quad , \tag{16}$$

and $\hat{\phi}^{nom}_{\alpha,n}$ is the corresponding cyclic mode computed on the nominal sector. let $\hat{\psi}^{p}_{\alpha,n}$ be a cyclic real mode α associated to sector p given by

$$\hat{\psi}^{p}_{\alpha,n} = \hat{\phi}^{'}_{\alpha,n} cos(\frac{2np\pi}{N}) - \hat{\phi}^{''}_{\alpha,n} sin(\frac{2np\pi}{N}) \quad . \tag{17}$$

This mode is present in the projection basis and is a $(\overline{n},1)$ vector, where $\overline{n} = \frac{n_{ddl}}{N}$. On the whole structure, the $(N \times \overline{n}, 1)$ vector of modes is written

$$\hat{\psi}_{\alpha} = \begin{pmatrix} \hat{\psi}^{\Omega_0}_{\alpha} \\ . \\ . \\ . \\ \hat{\psi}^{\Omega_{(N-1)}}_{\alpha} \end{pmatrix} \quad . \tag{18}$$

Then, on the whole structure and for the $\alpha^{\star}$ modes selected, the $(N \times \overline{n}, \alpha^{\star})$ matrix of modes is written

$$[\hat{\psi}] = \begin{pmatrix} \hat{\psi}_1 & . & . & . & \hat{\psi}_{\alpha} \end{pmatrix} \quad . \tag{19}$$

Which can also be written as

$$[\hat{\Psi}] = \begin{pmatrix} \hat{\psi}^{\Omega_0}_{1} & . & . & \hat{\psi}^{\Omega_0}_{\alpha^{\star}} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \hat{\psi}^{\Omega_{N-1}}_{1} & . & . & \hat{\psi}^{\Omega_{N-1}}_{\alpha^{\star}} \end{pmatrix} \quad . \tag{20}$$

Generalized dynamic equation of the mistuned system

Let's introduce generalized participation coefficients of selected modes on the displacement field. Then, the displacement field can be written as

$$\hat{u}(\omega) = [\hat{\Psi}] q(\omega) \quad , \tag{21}$$

where $q = [q_0, \ldots, q_{N-1}]^T$ is the complex-valued generalized coordinates vector associated to the system. Then, the generalized problem in global coordinates system becomes

$$[\mathcal{E}(\omega)] q(\omega) = [\hat{\Psi}]^T \hat{f}(\omega) \quad , \tag{22}$$

where the reduced dynamic $(\alpha^{\star}, \alpha^{\star})$ operator is a bloc diagonal matrix defined by

$$[\mathcal{E}(\omega)] = [\hat{\Psi}]^T [\hat{E}(\omega)] [\hat{\Psi}] \quad , \quad [\mathcal{E}(\omega)]_{\beta,\alpha} = \sum_{p=0}^{N-1} \left(\hat{\psi}^{\Omega_p}_{\beta}\right)^T [\hat{E}(\omega)]^{pp} \left(\hat{\psi}^{\Omega_p}_{\alpha}\right) \quad , \tag{23}$$

and the generalized forces are defined by

$$g(\omega) = [\hat{\Psi}]^T \hat{f}(\omega) \quad , \tag{24}$$

Clearly, the projection basis is constructed with respect to the distribution of the different sector's types and by keeping orthogonality properties between modes.

DISPLACEMENT FIELD AT COUPLING SECTOR INTERFACES

To solve the dynamic equation of motion of the mistuned bladed disk, by using a projection basis made of different sector types cyclic modes, we need to insure an admissible displacement field on the coupling sector interfaces. This admissibility is defined by two conditions on interfaces: the compatibility of meshes which is naturally insured by the fact geometric modifications are only done on blades, and the continuity of displacement field between two adjacent sectors which can be insured by imposing linear constraints on the restricted displacement on interfaces. These constraints can be introduced by using a Lagrange multiplier field (see Ohayon et al. (1997)) or by expressing constrained generalized coordinates as a function of non constrained ones. The latter formulation is made here. Let's consider two sectors Ω^p and Ω^{p+1}, with $p \in [0, N-1]$, interacting through a common boundary Γ^p. The linear coupling condition on Γ^i is written as

$$u^p = u^{p+1} \quad on \quad \Gamma^p \quad . \tag{25}$$

Then, on the entire structure, the physical displacement field u_c can be written as a function of the free (non-constrained) displacement field u by

$$u_c = [B]u \quad , \tag{26}$$

where $[B]$ is an (n_{ddl}, n_{ddl}) integer continuity matrix in physical coordinates system. Thus, with generalized coordinates, we have

$$q_c = [\mathcal{B}]q \quad , \quad [\mathcal{B}] = ([\hat{\Psi}]^T[\hat{\Psi}])^{-1}[\hat{\Psi}][B][\hat{\Psi}] \quad , \tag{27}$$

where $[\mathcal{B}]$ is a $(\alpha^\star \times \alpha^\star)$ real continuity matrix in generalized coordinates system. For solving the problem defined by (22), the variational formulation yields

$$< [\mathcal{E}]q_c, \delta q_c > = < g, \delta q_c > \quad , \tag{28}$$

which becomes on free-generalized coordinates system

$$< [\mathcal{B}]^T[\mathcal{E}][\mathcal{B}]q, \delta q > = < [\mathcal{B}]^T g, \delta q > \quad , \quad \forall \quad \delta q \quad . \tag{29}$$

Consequently, the problem, formulated on free-generalized coordinates system and including linear constraints that make the displacement field admissible over the entire structure is such that

$$[\mathcal{B}]^T[\mathcal{E}][\mathcal{B}]q = [\mathcal{B}]^T g \quad . \tag{30}$$

Note that due to the fact that we have to solve a reduced size problem, $\alpha^\star$ is small and the $(\alpha^\star \times \alpha^\star)$ matrix $[\hat{\Psi}]^T[\hat{\Psi}]$ can easily be inverted.

NUMERICAL EXAMPLE FOR A BLADED DISK

A simple example is presented in order to illustrate the application of the proposed reduction method.

Presentation of the studied case

The tuned bladed disk considered is constituted of a disk and 24 blades. The disk is made of a homogeneous and isotropic material with constant thickness 0.012 m, inner radius 0.05 m, outer radius 0.25 m, mass density 7860 kg/m3, Poisson ratio 0.25 and Young modulus 2.0 N/m2. A Dirichlet condition is applied along the internal boundary defined by the inner radius. Each blade is made of the same homogeneous and isotropic material as the disk one with length 0.045 m, width 0.013 m, thickness 0.004 m. In order to provide a simple test case the bladed disk was mistuned by significantly changing geometry of two arbitrarily selected blades with the assumption that mass density and Young's modulus are unchanged. The model is considered without damping. The purpose here is to obtain and compare natural frequencies of the reduced-order model to those of the full finite element model model. For the reduced-order model (ROM), matrices and tuned modes are computed for each type of sector with a finite element sector model using the commercially available ANSYS FE code. For the full finite element model mistuned modes are also computed using ANSYS FE code. The full mistuned finite element model shown in Fig. 1 is build with quadratic eight node brick elements totaling 39366 degrees-of-freedom (DOF). The finite perturbations that make the blades different are introduced by locally increasing or decreasing the blade thickness.

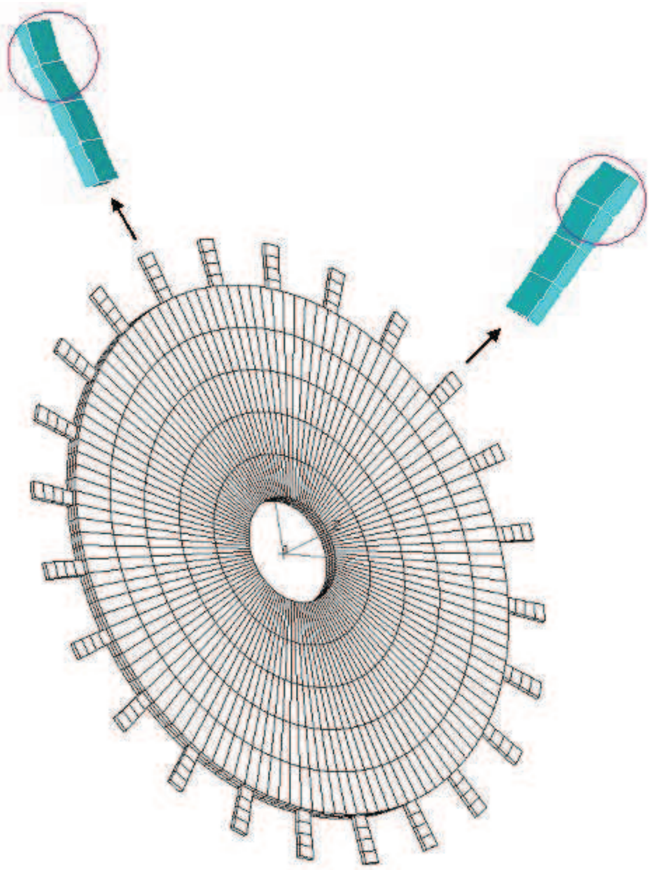

Figura 1 – Finite element model of the mistuned bladed disk.

Validation of the reduction method

Figure 2 displays the eigenfrequencies of the generalized eigenproblem associated with the tuned bladed disk in function of the circumferential wave number. To validate our method, we are going to approximate the 100 first modes of the muistuned bladed disk. Note that the 100 first natural frequencies of the tuned bladed disk are above 7000Hz.

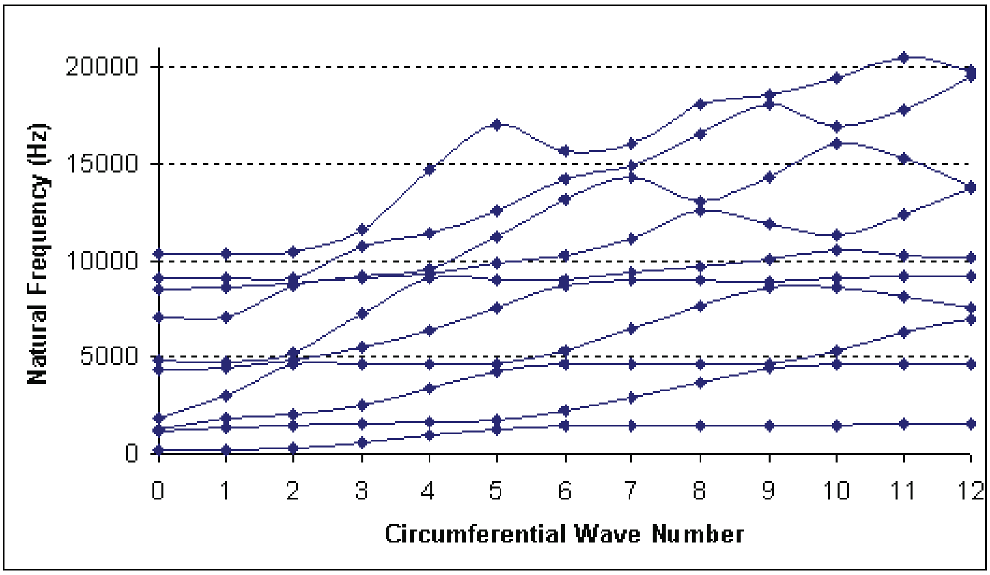

Figura 2 – Natural frequencies versus circumferential wave numbers of the tuned bladed disk.

The different modified blades level of mistuning on the three first frequencies estimated in clamped blade alone configuration by the normalized difference between their frequencies and the nominal blade frequency is presented in table 1. These values show a large mistuning level characterized by frequency deviation of about 9%.

Tabela 1 – Natural and relative clamped blades frequencies.

Mode number	First mode	second mode	third mode
Nominal blade frequency (Hz)	1572.6	4971.2	9196.3
Increased thickness blade frequency (Hz)	1695.5	5415.4	8984.5
Increased thickness blade frequency deviation (%)	-7.8	-8.9	2.3
Decreased thickness blade frequency (Hz)	1464.5	4610.5	8781
Decreased thickness blade frequency deviation (%)	6.9	7.3	4.5

Figure 4 and 5 displays the 100 first eigenfrequencies of the generalized eigenproblem associated with the mistuned bladed disk and the corresponding mistuned frequencies errors. In this test, a 130 ROM model obtained by taking all the tuned modes with frequencies above 9000Hz. the natural frequency errors obtained for all approximated mistuned modes are below 0.6%, which demonstrates a sufficient accuracy in capturing the dynamics of the simple mistuned system.

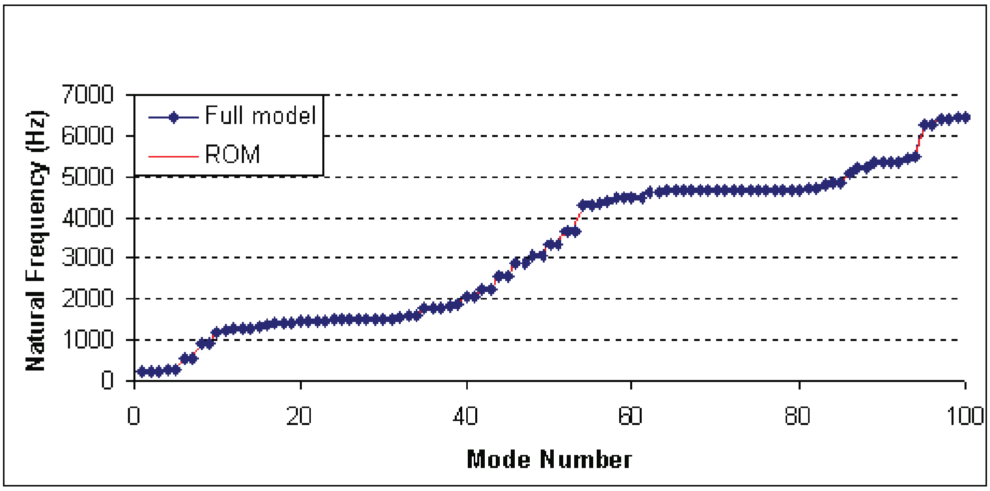

Figura 3 – Comparison of the 100 first mistuned natural frequencies between the full model and the ROM.

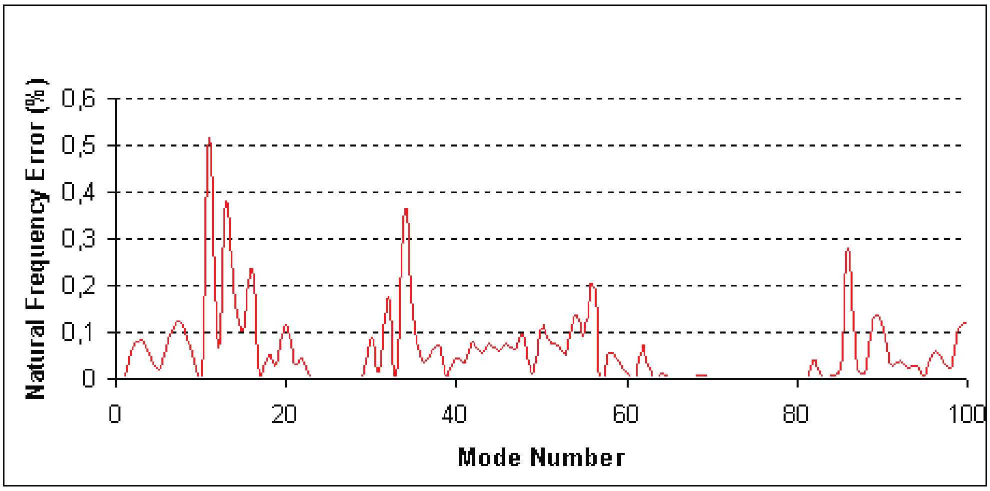

Figura 4 – Comparison of the 100 first mistuned natural frequencies errors between the full model and the ROM.

Figure 6 shows a convergence analysis of the reduced-order model by displaying the 100 first mistuned frequencies errors for different mode numbers of the projection basis. Note that while taking a 100 DOF system alone, the error level is low. The models of greater size are shown to demonstrate the convergence towards the full model results.

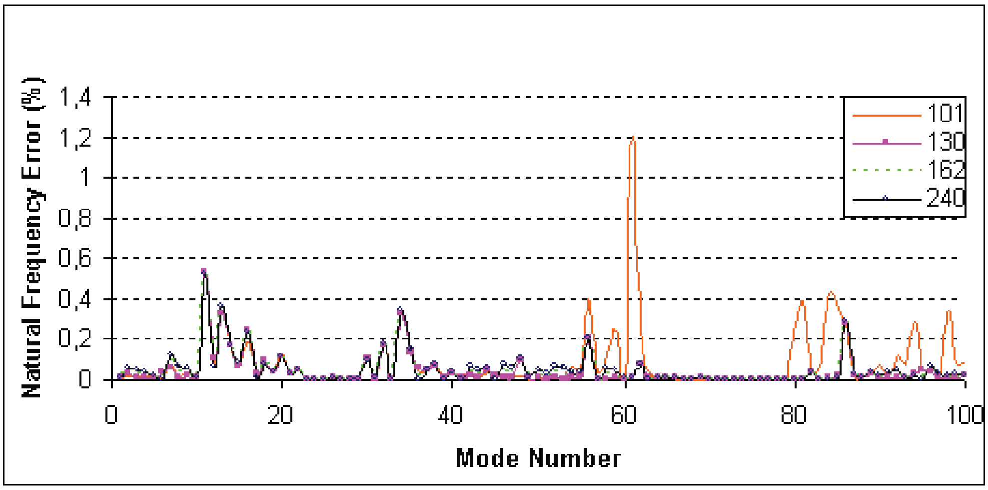

Figura 5 – 100 first mistuned natural frequencies errors for several ROM sizes.

Note that, there is a residual error, although convergence is reached. This is probably due to the simple test studied case used here which does not have enough elements to correctly represent cyclic mode shapes. By this way, we think that better results will be obtained by refining the mesh.

CONCLUDING REMARKS

In this paper a new reduction method for the dynamic problem of cyclic structures with finite geometric perturbations for the mass and stiffness matrices, based on the use of the cyclic modes of the different sectors is presented.The modal projection basis is constituted as well as on the whole bladed disk, each sector operator matrix is reduced by its own cyclic modes and linear constraints are applied on common boundaries between sectors to make the displacement field admissible. This method is used on a simple example of bladed disk to show its efficiency. This method allows very compact reduced-order models to be obtained and accurately capture the mistuned system dynamics. Thus, it was demonstrated with numerical examples that this method can accurately predict mistuned frequencies of a bladed disk with non symmetric finite geometric mistuning.

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RESPONSIBILITY NOTES

The authors are the only responsible for the printed material included in this paper.

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