Stochastic computational dynamical model of uncertain structure coupled with an insulation layer modelled by a fuzzy structure - Theory and experimental validation

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STOCHASTIC COMPUTATIONAL DYNAMICAL MODEL OF UNCERTAIN STRUCTURE COUPLED WITH AN INSULATION LAYER MODELLED BY A FUZZY STRUCTURE - THEORY AND EXPERIMENTAL VALIDATION

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ABSTRACT

In order to take into account the effect of insulation layers in complex dynamical systems for low- and medium-frequency ranges such as car booming noise analysis, one introduces a simplified stochastic model of insulation layers based on the use of the fuzzy structure theory and of a probabilistic approach of model uncertainties and data uncertainties. Such a stochastic model improves the numerical prediction robustness of the vehicle models without increasing the number of generalized degrees of freedom in the stochastic reduced computational model. An experimental validation of the proposed theory using the above simplified stochastic model is presented for an uncertain structure (plate connected to an elastic framework on its edges) coupled with an insulation layer.

1. INTRODUCTION

This paper deals with complex dynamical master systems coupled with an insulation layer in low- ($[15, 200]$ Hz) and medium-frequency ($[200, 450]$ Hz) ranges. A usual approach consists of a finite element model for both the master system and the insulation layer (see [1, 2]) yielding a high number of physical and generalized degrees of freedom (DOF). For example, a car booming noise analysis ($[30, 250]$ Hz) model involves about two millions physical DOF and one thousand elastic modes. A finite element model for the insulation layer would add up to five millions physical DOF and twenty thousand elastic modes would appear in the frequency band of analysis. Other kinds of insulation layer modelling have been proposed in the litterature (see [3–7]). These models are based on analytical representations and mixed methods and do not increase systematically the number of DOF. In this paper, the insulation layers considered have a rather simple dynamic behaviour in the frequency band of analysis and do not require an advanced material modelling like the Biot equations. Due to the actual variability of thicknesses, curvature and material properties, the insulation layer is considered complex and therefore, a statistical description of its internal DOF is proposed. We are naturally inclined to use the fuzzy structure theory (introduced in [8]) which fits this framework and has already been validated (see [8–12]). The representation of the insulation layer using the fuzzy structure theory is simply characterized by a few physical parameters (see [13, 14]): the participating mass, the modal density and the internal damping rate. In the original construction of the fuzzy structure theory, a parametric probabilistic model of uncertainties was used to take into account the data uncertainties. Today, it is well understood that such an approach cannot address model uncertainties. Recently, a non-parametric probabilistic approach has been introduced (see [15–17]), encompassing both data and model uncertainties. The use of such a non-parametric probabilistic approach allows to take into account both data and model uncertainties in the insulation layer simplified model. That constitutes a new extension of the fuzzy structure theory with respect to the model uncertainties problems.

This paper is divided as follows. Section 2. deals with the construction of the mean computational model of the structure including the mean simplified model of the insulation layer. Some details of the construction of the simplified model using the fuzzy structure theory are reminded in this section. In Section 3., the construction of the stochastic computational model including the model of uncertainties is presented. Section 4. is devoted to the experimental identification of the dispersion parameters of the stochastic computational model.

2. REDUCED MEAN COMPUTATIONAL MODEL WITH THE SIMPLIFIED MEAN MODEL OF INSULATION LAYER

2.1 Mean finite element model with fuzzy structure modelling

The finite element method [5, 18] is used to solve numerically the classical equations of a structural boundary value problem. The structure of the car is denoted by the superscript s. The insulation layer is modelled using the fuzzy structure theory (see [13, 14]). We remind here the steps of such a construction. The insulation layer is assumed to be equivalent to a surfacic distribution of single-DOF oscillators on the interface Γ_s shared by the structure and the insulation layer. These DOF move in the normal direction to the interface. We perform a statistical average of the eigenfrequency of these DOF that allows the construction of the fuzzy coefficient $\underline{a}^s(\omega;\underline{n}(\omega),\underline{\mu}(\omega),\underline{\xi}(\omega)) = -\omega^2\underline{a}^s_R(\omega;\underline{n}(\omega),\underline{\mu}(\omega),\underline{\xi}(\omega)) + i\omega\underline{a}^s_I(\omega;\underline{n}(\omega),\underline{\mu}(\omega),\underline{\xi}(\omega))$, where $\underline{n}(\omega)$ is the mean modal density, $\underline{\mu}(\omega)$ the mean participating mass and $\underline{\xi}(\omega)$ is the mean damping rate of the insulation layer. The dependency in $\underline{n}(\omega),\underline{\mu}(\omega),\underline{\xi}(\omega)$ of coefficients $\underline{a}^s$, $\underline{a}^s_R$ and $\underline{a}^s_I$ will be skipped from now on for the sake of brevity. The coefficients $\underline{a}^s_R(\omega)$ and $\underline{a}^s_I(\omega)$ are defined by

$$\underline{a}^s_R(\omega) = \underline{\mu}(\omega)\underline{n}(\omega)\ \left[\frac{1}{\underline{n}(\omega)} - \omega\,\underline{\lambda}(\omega)\,\Theta_R(\omega)\right], \tag{1}$$

$$\underline{a}^s_I(\omega) = \underline{\mu}(\omega)\underline{n}(\omega)\ \omega^2\underline{\lambda}(\omega)\ \Theta_I(\omega)\,, \tag{2}$$

where the functions Θ_R, Θ_I and $\underline{\lambda}$ are defined in Appendix B. One then considers a finite element mesh of the structure Ω_s. Let $\underline{\boldsymbol{u}}^s$ be the complex vector of the m_s DOF of the structure corresponding to the finite element discretization of the displacement field $\boldsymbol{u}$. The mean value of a variable M is denoted by $\underline{M}$. The mean computational model for the master structure coupled with the insulation layer can be written as,

$$[\,[\underline{\mathbb{A}}^s(\omega)]+\underline{a}^s(\omega)[\underline{\mathbb{B}}^s]] \times \underline{\boldsymbol{u}}^s(\omega) = \underline{\mathbf{f}}^s(\omega) \quad , \tag{3}$$

where $[\underline{\mathbb{A}}^s(\omega)]$ is a complex $(m_s \times m_s)$ matrix such that $[\underline{\mathbb{A}}^s(\omega)] = -\omega^2[\underline{\mathbb{M}}^s] + i\omega[\underline{\mathbb{D}}^s(\omega)] + [\underline{\mathbb{K}}^s(\omega)]$ in which $[\underline{\mathbb{M}}^s]$, $[\underline{\mathbb{D}}^s(\omega)]$ and $[\underline{\mathbb{K}}^s(\omega)]$ are the mass, damping and stiffness matrices of the structure *in vacuo*. The matrix $\underline{a}^s(\omega)[\underline{\mathbb{B}}^s]$ is the matrix associated with the model of insulation layer using the fuzzy structure theory where $(m_s \times m_s)$ real matrix $[\underline{\mathbb{B}}^s]$ is the matrix associated to the bilinear form $b^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s) = \int_{\Gamma_s} (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x}))\,(\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x}))\; ds(\boldsymbol{x})$ where $\boldsymbol{u}^s$ and $\boldsymbol{\delta u}^s$ belong to the admissible space of the displacement field of the structure.

2.2 Reduction of the mean computational model

Using n_s structural elastic modes *in vacuo*, the reduced mean computational model of the dynamical system is written as

$$\underline{\boldsymbol{u}}^s(\omega) = [\underline{\Phi}^s]\boldsymbol{q}^s(\omega) \quad , \tag{4}$$

$$\underline{\boldsymbol{f}}^s(\omega) = [\underline{\Phi}^s]^T\underline{\mathbf{f}}^s(\omega) \quad , \tag{5}$$

$$[\,[\underline{A}^s(\omega)] + \underline{a}^s(\omega)[\underline{B}^s]\,] \times \boldsymbol{q}^s(\omega) = \underline{\boldsymbol{f}}^s(\omega) \tag{6}$$

in which $[\underline{\Phi}^s]$ is the $(m_s \times n_s)$ real matrix of the elastic modes of the structure. The complex $(n_s \times n_s)$ matrix $[\underline{A}^s(\omega)]$ is such that $[\underline{A}^s(\omega)] = -\omega^2[\underline{M}^s] + i\omega[\underline{D}^s(\omega)] + [\underline{K}^s(\omega)]$ in which $[\underline{M}^s]$, $[\underline{D}^s(\omega)]$ and $[\underline{K}^s(\omega)]$ are the generalized mass, damping and stiffness matrices of the structure.

3. STOCHASTIC COMPUTATIONAL MODEL WITH A NON-PARAMETRIC MODEL OF UNCERTAINTIES

As explained above, the quality of the predictions of the mean computational model can be improved in implementing a non-parametric probabilistic model of uncertainties encompassing both data and model uncertainties. Therefore, the mean generalized matrices $[\underline{M}^s]$, $[\underline{D}^s(\omega)]$ and $[\underline{K}^s(\omega)]$ are replaced by the random matrices $[\mathbf{M}^s]$, $[\mathbf{D}^s(\omega)]$, $[\mathbf{K}^s(\omega)]$ respectively. It should be noted that the random matrix associated with $\underline{a}^s(\omega)[\underline{B}^s]$ is $\underline{a}^s(\omega)[\mathbf{B}^s]$ in which $[\mathbf{B}^s]$ is the random matrix associated with $[\underline{B}^s]$. The level of uncertainties of these random matrices is controlled by the dispersion parameters δ_{M^s}, δ_{D^s}, δ_{K^s} and δ_{B^s} which are independent of the matrix dimension and of the frequency. The development of the construction of the probability model of all these random matrices will not be detailed here. Such an approach is presented in [15, 16] and its application to a car booming noise analysis without insulation layer can be found in [19]. In this section, we simply remind some properties of these matrices. They are independent second-order random variables. For all ω in $\mathbb{B}$, the random matrices $[\mathbf{M}^s]$, $[\mathbf{D}^s(\omega)]$ and $[\mathbf{K}^s(\omega)]$ are with values in $\mathbb{M}^+_{n_s}(\mathbb{R})$; the random matrix $[\mathbf{B}^s]$ is with values in $\mathbb{M}^{+0}_{n_s}(\mathbb{R})$. The mean values are such that

$$\begin{aligned}
&\mathcal{E}\{[\mathbf{M}^s]\} = [\underline{M}^s] \quad , \\
&\mathcal{E}\{[\mathbf{K}^s(\omega)]\} = [\underline{K}^s(\omega)] \quad , \\
&\mathcal{E}\{[\mathbf{D}^s(\omega)]\} = [\underline{D}^s(\omega)] \quad , \\
&\mathcal{E}\{\underline{a}^s(\omega)[\mathbf{B}^s]\} = \underline{a}^s(\omega)[\underline{B}^s] \quad ,
\end{aligned} \tag{7}$$

where $\mathcal{E}$ is the mathematical expectancy. Moreover those matrices have the required mathematical properties (see. [17]). For example, we give below the detail of the construction for the random matrix $[\mathbf{K}^s(\omega)]$. The matrix $[\underline{K}^s(\omega)]$ can be written as $[\underline{K}^s(\omega)] = [L_{K^s}(\omega)]^T[L_{K^s}(\omega)]$ corresponding to the Choleski decomposition of the positive-definite matrix $[\underline{K}^s(\omega)]$. We then introduce the random matrix $[\mathbf{K}^s(\omega)] = [L_{K^s}(\omega)]^T[\mathbf{G}_{K^s}][L_{K^s}(\omega)]$ where the random matrix $[\mathbf{G}_{K^s}]$ belongs to the $\mathbf{SG}^+$ ensemble defined in [16] and is independent of the frequency. The dispersion parameter δ_{K^s} of this random matrix $[\mathbf{K}^s(\omega)]$ is independent of the dimension and of the frequency and is defined by $\delta_{K^s} = (\mathcal{E}\{||[\mathbf{G}_{K^s}] - [\underline{G}_{K^s}]||_F^2\}/||[\underline{G}_{K^s}]||_F^2)^{1/2}$ in which $||K||_F$ is the Frobenius norm defined by $||K||_F^2 = \mathrm{tr}(K^T K)$. For all $\omega \in \mathbb{B}$, let $\mathbf{Q}^s(\omega)$ be the random vector in $\mathbb{C}^{n_s}$ of the generalized DOF of the structure. In the same way, one introduces the generalized stiffness random matrices written,

$$[\mathbf{A}^s(\omega)] = -\omega^2\ [\mathbf{M}^s] + i\omega\ [\mathbf{D}^s(\omega)] + [\mathbf{K}^s(\omega)] \quad , \tag{8}$$

where $[\mathbf{A}^s(\omega)]$ is a random matrix in $M_{n_s}^S(\mathbb{C})$ for all $\omega \in \mathbb{B}$. Equations. (6) and (8) allow the stochastic reduced computational model to be written as,

$$[\,[\mathbf{A}^s(\omega)] + \underline{a}^s(\omega)[\mathbf{B}^s]\,] \times \mathbf{Q}^s(\omega) = \underline{\boldsymbol{f}}^s(\omega) \quad , \tag{9}$$

where the vector of the random structural displacement $\mathbf{U}^s(\omega)$ is written as, for all $\omega \in \mathbb{B}$,

$$\mathbf{U}^s(\omega) = [\underline{\Phi}^s]\mathbf{Q}^s(\omega) \quad . \tag{10}$$

4. STOCHASTIC SOLVER AND IDENTIFICATION OF THE DISPERSION PARAMETERS - EXPERIMENTAL VALIDATION

4.1 Experiments

Experiments have been made in PSA Peugeot Citroën noise and vibration laboratory. The experimental configuration is made up of homogeneous, isotropic and slightly damped thin plate (steel plate with a constant thickness) connected to an elastic framework on its edges. The connection between the plate and the framework is uncertain. This dynamical system is hung by four springs in order to avoid the rigid body modes. The highest eigenfrequency of suspension is 9 Hz while the lowest eigenfrequency of the elastic modes is 43 Hz. The excitation is a point force applied to the framework and excites the dynamical system mainly in bending mode in the frequency band of analysis $\mathbb{B} =]0, 300]$ Hz. The number of sampling frequencies is $N_f = 300$. The frequency resolution is $\Delta f = 1$ Hz. Only one experiment is performed for this structure. The frequency response functions $\omega \mapsto \gamma_i^{\mathrm{exp}}(\omega)$ are identified on frequency band $\mathbb{B}$ for $n_{\mathrm{obs}} = 60$ normal accelerations in the plate. The following experimental frequency response function $\omega \mapsto r^{\mathrm{exp}}(\omega) = 10\log_{10}(\sum_{i=1}^{n_{\mathrm{obs}}} |\gamma_i^{\mathrm{exp}}(\omega)|^2)$ is then constructed. The mean computational model is a finite element model having $N_{dofs} = 57,768$ structural dofs. The reduced mean computational model is constructed with $n_s = 240$ structural modes. The mean computational model has been updated with respect to the Young modulus, the mass density and the damping coefficient of the plate and of the framework using the experimental values of the two first elastic modes and the ninth elastic mode (first elastic torsion mode of the structure). The updated mean computational model will simply be called below the mean computational model. The following experimental frequency response function $\omega \mapsto r^s(\omega) = 10\log_{10}(\sum_{i=1}^{n_{\mathrm{obs}}} |\gamma_i(\omega)|^2)$is then constructed. The stochastic solver used is based on a Monte Carlo simulation. The methodology used is the following:

(1) The realizations of random variables $\mathbf{U}^s(\omega;\theta_\ell)$ are constructed for all $\ell \in [1, n_r]$. For each realization $\mathbf{U}^s(\omega;\theta_\ell)$, the observation $\mathbf{R}^s(\omega;\theta_l) = 10\log_{10}\left(\sum_{k=1}^{n_{\rm obs}} |\omega^2 \mathbf{U}_k^s(\omega;\theta_l)|^2\right)$ is calculated.
(2) The mathematical statistics are used to construct the estimations and a convergence analysis is performed with respect to the number of realizations and to the number of modes.
(3) An innovative method [20] in order to identify the parameters of dispersion of the non-parametric model of uncertainties is used. This method is based on the maximum likelihood coupled with a statistical reduction of information.

4.2 Convergence analysis

The convergence analysis with respect to n_r and n_s is carried out in studying the convergence of the estimated second-order moment of $\mathbf{Q}^s(\omega)$ for a fixed δ and is defined by $|||\mathbf{Q}^s(\omega)|||_{\mathbb{B}}^2 = \int_{\mathbb{B}} \mathcal{E}\{||\mathbf{Q}^s(\omega)||^2\}$. An estimation is provided by the following function,

$$(n_r, n_s) \mapsto \mathrm{Conv}^s(n_r, n_s) = \frac{1}{n_r}\sum_{j=1}^{n_r}\int_{\mathbb{B}} ||\mathbf{Q}^s(\omega,\theta_j)||^2 d\omega. \tag{11}$$

Graph of function $(n_r, n_s) \mapsto \mathrm{Conv}^s(n_r, n_s)$ is shown at FIG. 1 where one can see the convergence with respect to the number of structural modes taken and to the number of realizations. One can see that the solver is converged for $n_s = 103$ modes and $n_r = 800$ realizations. A high value of the parameter of dispersion $\delta = 0.8$ is used (generally, such a high level of uncertainties is not reached for real applications). The convergence for lower values of delta is then ensured. The convergence with respect to the number of modes is valid in the case of the model of the plate without insulation layer as well as in the case of the model of the plate with the insulation layer.

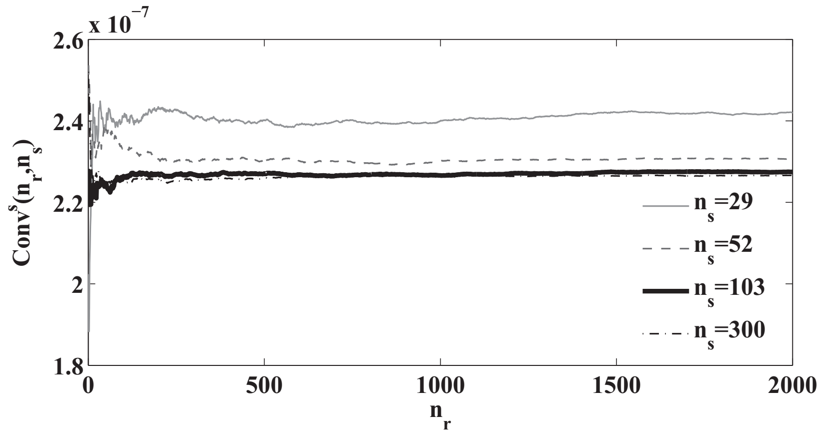

Figure 1. Graph of $(n_r, n_s) \mapsto \mathrm{Conv}^s(n_r, n_s)$

4.3 Estimation of the mean value and of the confidence region

For all ω in $\mathbb{B}$, let $w \mapsto F_{\mathbf{R}^s(\omega)}(w)$ be the distribution function (continuous from the right) of random variable $\mathbf{R}^s(\omega)$ defined in Section 4.1, such that $F_{\mathbf{R}^s(\omega)}(w) = P(\mathbf{R}^s(\omega) \le w)$. For

$0 < p < 1$, the p^{th} quantile or fractile of $F_{\mathbf{R}^s(\omega)}$ is defined as (see [21]),

$$\zeta(p) = \inf_{F_{\mathbf{R}^s(\omega)} \geq p} \{w\} \quad . \tag{12}$$

The upper envelope $w^+(\omega)$ and the lower envelope $w^-(\omega)$ of the confidence region are defined by

$$w^+(\omega) = \zeta\left(\frac{1+P_c}{2}\right) \quad , \quad w^-(\omega) = \zeta\left(\frac{1-P_c}{2}\right) \quad , \tag{13}$$

and the probability P_c can be calculated using the following equation,

$$\mathcal{P}(w^-(\omega) < \mathbf{R}^s(\omega) \leq w^+(\omega)) = P_c \quad . \tag{14}$$

The estimation of $w^+(\omega)$ and $w^-(\omega)$ is performed by using the sample quantiles (see [21]). Let $w_1 = \mathbf{R}^s(\omega;\theta_1), \ldots, w_{n_r} = \mathbf{R}^s(\omega;\theta_{n_r})$ be the n_r independent realizations of the random variable $\mathbf{R}^s(\omega)$. Let $\widetilde{w}_1(\omega) < \ldots < \widetilde{w}_{n_r}(\omega)$ be the order statistics associated with $w_1(\omega), \ldots, w_{n_r}(\omega)$. Therefore, one has the following estimation

$$w^+(\omega) \simeq \widetilde{w}_{j^+}(\omega) \quad , \quad j^+ = \text{fix}\,(n_r(1+P_c)/2) \quad , \tag{15}$$

$$w^-(\omega) \simeq \widetilde{w}_{j^-}(\omega) \quad , \quad j^- = \text{fix}\,(n_r(1-P_c)/2) \quad , \tag{16}$$

where $\text{fix}(z)$ is the integer part of the real number z.

4.4 Identification of the dispersion parameters δ_{M_s}, δ_{D_s} and δ_{K_s}

This section deals with the identification of the dispersion parameters of the master structure δ_{M_s}, δ_{D_s} and δ_{K_s}. The value of the damping dispersion parameter δ_{D_s} is fixed *a priori* to $\delta_{D_s} = 0.3$ according to the conclusion of [19]. In order to verify that the random response is not really sensitive (see [19]) to the value of the parameter of dispersion δ_{D_s}, we have performed a sensitivity analysis with respect to δ_{D_s} varying in the interval $[0.2, 0.4]$ where δ_{M_s} and δ_{K_s} are fixed to the value 0.1 (small value of the dispersion parameter for the structural mass and stiffness matrices). With this sensitivity analysis, we have effectively verified that the influence of this dispersion parameter is negligible. Moreover, it is assumed that $\delta_{M_s} = \delta_{K_s}$ (see [19]) in the identification procedure using the maximum likelihood method coupled with a statistical reduction of information (see [20]). For this step, Eq. (9) is then replaced by the random equation $[\mathbf{A}^s(\omega)]\,\mathbf{Q}^s(\omega) = \underline{\boldsymbol{f}}^s(\omega)$ relative to the uncertain structure without insulation layer. Figure 2 displays the graph of $\omega \mapsto \mathbf{R}^s(\omega)$ for the identified parameters $\delta_{M_s} = \delta_{K_s} = 0.3$ and $\delta_{D_s} = 0.3$.

4.5 Comments on the results of the model without the insulation

Figure 2 shows the stochastic response which encompasses 95% of the measurement. The prediction of the mean model is improved by the stochastic model. Indeed, in the graph displayed in Fig. 2, the wider the gray regions are, the less robust to the uncertainties the mean model is.

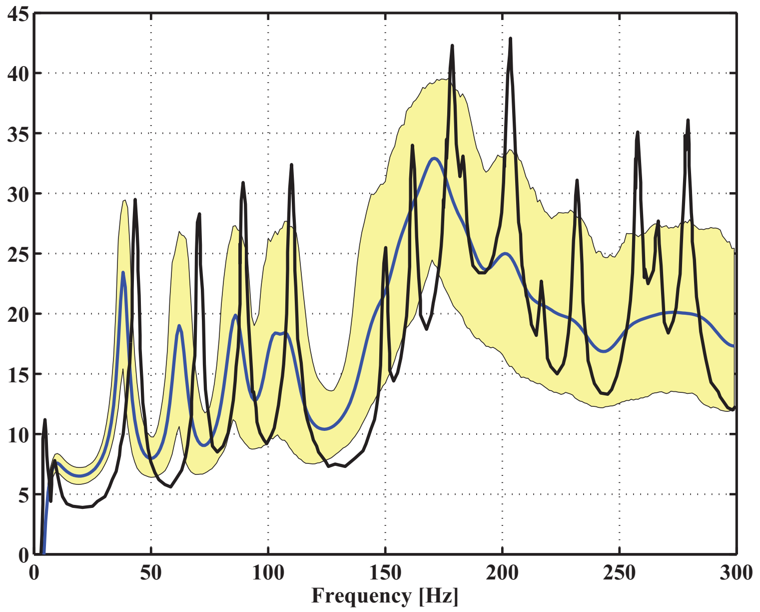

Figure 2: Graph of $\omega \mapsto \mathbf{R}^s(\omega)$ for the structure without insulation layer: measurements (thick black line) ; stochastic confidence zone (gray region) ; mean stochastic response (thick dark gray line)

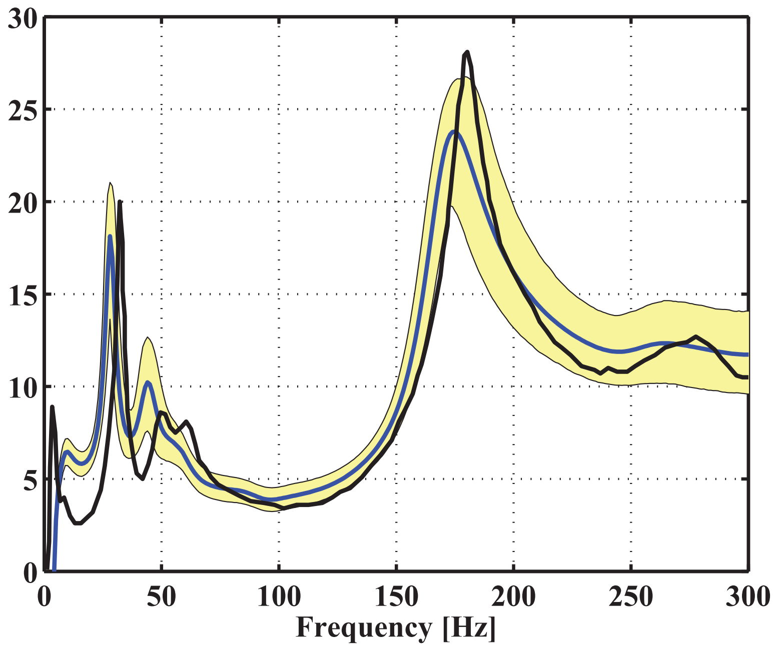

Figure 3: Graph of $\omega \mapsto \mathbf{R}^s(\omega)$ for the structure with insulation layer: measurements (thick black line) ; stochastic confidence zone (gray region) ; mean stochastic response (thick dark gray line)

4.6 Identification of the dispersion parameter δ_{B_s}

This section is devoted to the identification of the dispersion parameter δ_{B_s} relative to the stochastic simplified model of the insulation layer. Numerical simulations have shown that the sensitivity of the response of the structure coupled with the insulation layer is smaller than the sensitivity of the response induced by the dispersion parameters of the structure. Consequently, the identification of δ_{B_s} cannot be carried out with the uncertain structure. Parameter δ_{B_s} must thus be identified with a "reference structure" for which there is no uncertainty (note that the insulation layer cannot be analyzed alone and has to be coupled with a structure). The methodology proposed consists
(1) in defining a "reference structure" and analyzing the response of this "reference structure" coupled with the insulation layer. This reference coupling system is analyzed by the finite element method using a fine mesh for the insulation layer and the "reference structure". This deterministic computational model allows the responses to be computed. These responses are defined below as the "numerical experiments". Note that this computational model does not represent the experimental configuration, but this choice is completely coherent because the stochastic simplified model of the insulation layer is independent of the choice of the structure. This model is constituted of a thin plate similar to the plate of the experimental configuration presented in Section 4.1;
(2) in constructing a stochastic computational model constituted of the computational model above for the "reference structure" and of the stochastic simplified model for the insulation layer which depends on δ_{B_s}. The insulation layer model is similar to the one presented in Section 4.1 For this second step, Eq. (9) is then replaced by the random equation $[\underline{A}^s_{\text{ref}}(\omega) + \underline{a}^s(\omega)[\mathbf{B}^s]]$ $\mathbf{Q}^s(\omega) = \underline{\boldsymbol{f}}^s(\omega)$. The identification of parameter δ_{B_s} is performed following the above methodology. For this identified value of δ_{B_s}, Fig. 3 displays the graph of $\omega \mapsto \mathbf{R}^s(\omega)$ for the uncertain master structure coupled with the uncertain insulation layer with the following identified dispersion parameters $\delta_{M_s} = \delta_{K_s} = 0.3$, $\delta_{D_s} = 0.3$ and $\delta_{B_s} = 0.3$. Figure 3 shows that the comparison of the stochastic model predictions with the measurements is good in the frequency band of analysis.

5. CONCLUSION

In this paper, a method has been presented to model insulation layers with a fuzzy structure approach combined with a non-parametric probabilistic model of uncertainties. This extension of the fuzzy structure theory (1) allows the dynamics of the insulation layer to be taken into account without increasing the number of degrees of freedom, (2) allows a representation of the insulation layer in terms of its physical parameters such as the modal density, the participating mass and the internal damping rate and (3) gives a robust prediction regarding both parameters and model uncertainties. An experimental comparison validated the theory proposed.

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A TABLE OF NOTATIONS

$\underline{a}$: mean value of scalar a

$\underline{\boldsymbol{u}}$	:	mean value of vector $\boldsymbol{u}$
$[\underline{\mathbb{M}}]$	:	mean finite element matrix in physical coordinates
$[\underline{M}]$	:	mean finite element matrix in generalized coordinates
$[\mathbf{M}]$	:	random matrix
$\mathbf{U}$	:	random vector
$\mathcal{E}$	:	mathematical expectation

B FUNCTIONS OF THE FUZZY COEFFICIENTS

For all $\omega \in \mathbb{B}$,

$$\Theta_{R}(\omega) = \frac{1}{4\sqrt{1-\underline{\xi}(\omega)^2}} \ln\left\{ \frac{N^{+}(\widetilde{b}(\omega), \underline{\xi}(\omega))\, N^{-}(\widetilde{a}(\omega), \underline{\xi}(\omega))}{N^{-}(\widetilde{b}(\omega), \underline{\xi}(\omega))\, N^{+}(\widetilde{a}(\omega), \underline{\xi}(\omega))} \right\}, \tag{17}$$

$$\Theta_{I}(\omega) = \frac{1}{2\sqrt{1-\underline{\xi}(\omega)^2}} \left[\Lambda(\widetilde{b}(\omega), \underline{\xi}(\omega)) - \Lambda(\widetilde{a}(\omega), \underline{\xi}(\omega)) \right], \tag{18}$$

$$N^{\pm}(u,\xi) = u^2 \pm 2\,u\,\sqrt{1-\xi^2} + 1 \quad , \quad \Lambda(u,\xi) = \arctan\left\{ \frac{u^2 + 2\xi^2 - 1}{2\xi\sqrt{1-\xi^2}} \right\} \quad , \tag{19}$$

$$\widetilde{a}(\omega) = \sup\left\{ 0, 1 - \frac{1}{2\omega\underline{n}(\omega)} \right\} \quad , \quad \widetilde{b}(\omega) = 1 + \frac{1}{2\omega\underline{n}(\omega)} \quad , \tag{20}$$

$$\widetilde{\ell}(\omega) = \frac{1}{\widetilde{b}(\omega) - \widetilde{a}(\omega)} \quad , \quad \underline{\lambda}(\omega) = \frac{\widetilde{\ell}(\omega)}{\omega\underline{n}(\omega)} \quad . \tag{21}$$

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