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Stochastic inverse problem for the experimental identification in the ultrasonic range of a mechanical model for cortical bones

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Abstract. This paper deals with the construction of a simplified elastoacoustic model which allows the ultrasonic wave propagation to be simulated in a complex biomechanical system. This simplified model consists in a fluid-solid multilayer system. In this simplified model, the main source of uncertainties is due to the constitutive equation for the solid layer which is chosen as a homogeneous transverse isotropic elastic medium. In order to improve this simplified model, a probabilistic model of the effective elasticity tensor of the solid medium is developed. A method is presented for the experimental identification in a statistical sense of the model parameters using the ultrasonic transmission technique. A complete application is presented for the human cortical bone for which an experimental database is available.

1. Introduction

A biomechanical system such as the human cortical bone is often very complex to model in regard to the microstructure of the materials made up of live tissues. Simplifications concerning the geometry and the constitutive equations are used to construct a simplified mechanical model (see for instance [1, 2, 3]) which allows the response of the biomechanical system to be predicted. Nevertheless, this simplified model is a rough approximation of the real biomechanical system and the main source of uncertainties is related to the constitutive equations. In order to improve the simplified model, these uncertainties have to be taken into account. In this paper, the biomechanical system under consideration is the human cortical bone, with the skin and the marrow. The simplified mechanical model presented in [2, 3] is used to predict the transient response of the cortical bone in the ultrasonic range. The skin, the muscle, the cortical bone and the marrow are modeled as a fluid-solid semi-infinite multilayers system. In addition, the mean model of the constitutive equation of the solid is represented by an effective elasticity tensor that corresponds to a homogeneous transverse isotropic solid medium. In order to validate this simplified mechanical model, an experimental database has been constructed using the axial transmission technique [4-6]. The biomechanical system is submitted to an acoustical impulse in the ultrasonic range and the velocity of the first arriving signal is experimentally

measured. Each element of the experimental database is a realization of a random tensor whose probabilistic model is presented in this paper. Then a stochastic simplified model is developed by modeling the effective elasticity tensor with a random tensor whose probabilistic model is presented in [7, 8]. The stochastic simplified model parameters that have to be identified are the mechanical properties of the solid layer i.e, the mass density, the coefficients of the transverse isotropic tensor (the lateral and transversal Young moduli, the lateral and transversal Poisson coefficients and the lateral and transversal shear moduli) and a dispersion coefficient describing the uncertainties level of the tensor. This identification is carried out by solving a stochastic inverse problem. A computational optimization problem is then introduced, consisting in minimizing a cost function with respect to the stochastic simplified model parameters. This cost function is defined by taking into account the type of experimental observations obtained by using the axial transmission technique. The simplex algorithm is used to solve the optimization problem. At each iteration of the simplex algorithm, the value of this cost function is calculated using a stochastic solver based on the Monte Carlo method. Thus, for each realization of the random effective elastic tensor, it is necessary to predict a transient elastic wave response of a fluid-solid semi-infinite multilayer system submitted to an acoustical impulse. Nevertheless, the numerical cost for constructing such a transient elastic wave can be prohibitive for the stochastic inverse problem if usual computational methods are used. Consequently, in order to decrease the computational cost of the optimization problem, a new fast, hybrid numerical method developed in [2] is used. This mechanical solver is based on a time-domain formulation associated with a space Fourier transform for the infinite dimensions and a finite element approximation for the finite dimension. A complete numerical application concerning the human cortical bone is presented.

2. Experimental database

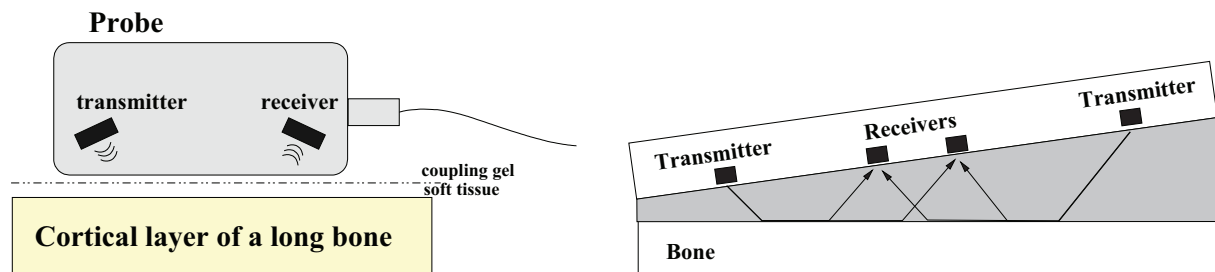


Figure 1. Experimental configuration

The ultrasonic axial transmission technique is used to construct an experimental database. For this, a device has been designed and is made up of several receivers and transmitters. A coupling gel is applied at the interface between the device and the skin of a patient. Each transmitter generates an acoustical impulse in the ultrasonic range that propagates in the coupling gel, the skin, the muscle, the cortical bone and the marrow. Each receivers records the signal at different positions allowing the velocity of the first arriving signal to be experimentally obtained (see Fig. 2). Such experiments are applied to a given set of patient yielding an experimental database that consists of 2747 measurements of the velocity of the first arriving signal on 168 human *radii*. For each specimen, the velocity of the first arriving signal is a realization of a random variable V^{exp} . Thus, the database is made up of $N = 2747$ statistical independent realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ of random variable V^{exp} . Let $\underline{V}^{\text{exp}}$ be the

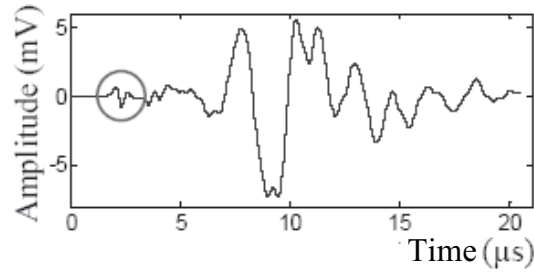


Figure 2. First arriving signal

experimental mean value defined as

$$\underline{V}^{\text{exp}} = \frac{1}{N} \sum_{j=1}^N V^{\text{exp}}(\hat{\theta}_k) \quad .$$

Let $\underline{\delta}^{\text{exp}}$ be the experimental coefficient of variation such that

$$\underline{\delta}^{\text{exp}} = \frac{\sqrt{\sum_{k=1}^N (V^{\text{exp}}(\hat{\theta}_k) - \underline{V}^{\text{exp}})^2}}{\sqrt{N} \underline{V}^{\text{exp}}} \quad .$$

We then construct the random variable V^{exp} such that $E\{V^{\text{exp}}\} = \underline{V}^{\text{exp}}$ and such that the coefficient of variation δ^{exp} of V^{exp} is $\delta^{\text{exp}} = \underline{\delta}^{\text{exp}}$ and where $E\{\cdot\}$ is the mathematical expectation.

3. Simplified model

A simplified model of the biomechanical system made up of the coupling gel, the skin, the cortical bone and the marrow has been developed in [2, 3]. This simplified model is composed of an elastic solid semi-infinite layer between two acoustic fluid semi-infinite layers (see Fig. 3). Let $\mathbf{R}(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the reference Cartesian frame where O is the origin of the space and

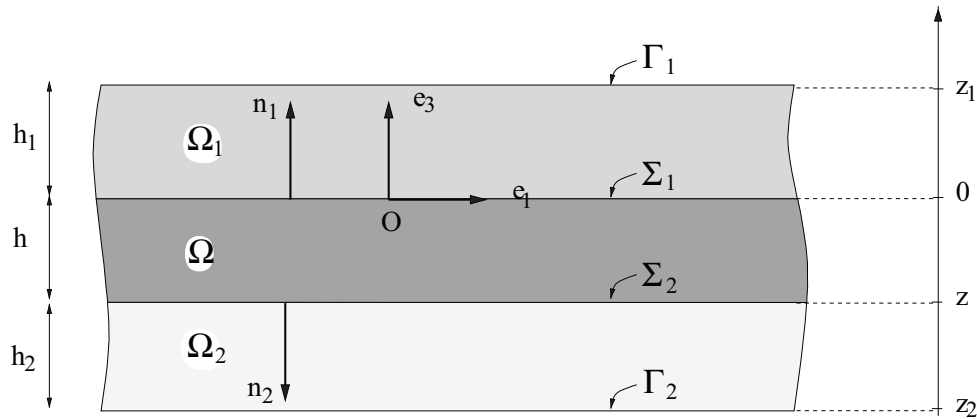


Figure 3. Geometry of the multilayer system

$(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is an orthonormal basis for this space. The coordinate of the generic point \mathbf{x} in \mathbb{R}^3 is (x_1, x_2, x_3) . The thicknesses of the layers are denoted by h_1 , h and h_2 . The first acoustic fluid layer occupies the open unbounded domain Ω_1 , the second acoustic fluid layer occupies the open unbounded domain Ω_2 and the elastic solid layer occupies the open unbounded domain Ω . Let $\partial\Omega_1 = \Gamma_1 \cup \Sigma_1$, $\partial\Omega = \Sigma_1 \cup \Sigma_2$ and $\partial\Omega_2 = \Sigma_2 \cup \Gamma_2$ (see Fig. 3) be respectively the boundaries of Ω_1 , Ω and Ω_2 in which $\Gamma_1, \Sigma_1, \Sigma_2$ and Γ_2 are the planes defined by

$$\Gamma_1 = \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z_1\}$$

$$\Sigma_1 = \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = 0\}$$

$$\Sigma_2 = \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z\}$$

$$\Gamma_2 = \{x_1 \in \mathbb{R}, \quad x_2 \in \mathbb{R}, x_3 = z_2\}$$

in which $z_1 = h_1$, $z = -h$ and $z_2 = -(h + h_2)$. Therefore, the domains Ω_1 , Ω and Ω_2 are unbounded along the transversal directions \mathbf{e}_1 and \mathbf{e}_2 whereas they are bounded along the vertical direction \mathbf{e}_3 . Let \mathbf{u} be the displacement field of the solid elastic and, for $k = 1, 2$, let p_k be the pressure field in the fluid occupying the domain Ω_k . We then have (see for instance [9, 10])

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \text{div} \boldsymbol{\sigma} = \mathbf{0} \quad , \quad \mathbf{x} \in \Omega \quad , \quad t > 0 \quad , \quad (1)$$

$$\frac{1}{c_k^2} \frac{\partial^2 p_k}{\partial t^2} - \Delta p_k = \frac{\partial Q_k}{\partial t} \quad , \quad \mathbf{x} \in \Omega_k \quad , \quad t > 0 \quad , \quad (2)$$

where ρ and $\boldsymbol{\sigma}$ are the mass density and the Cauchy stress tensor field of the solid; div is the divergence operator with respect to \mathbf{x} ; c_k is the wave velocity of the fluid occupying domain Ω_k ; Δ is the Laplacian operator with respect to \mathbf{x} ; Q_k is the source density applied in domain Ω_k such that

$$\frac{\partial Q_1}{\partial t}(\mathbf{x}, t) = \rho_1 F(t) \delta_0(x_1 - x_1^S) \delta_0(x_3 - x_3^S) \quad , \quad (3)$$

$$Q_2(\mathbf{x}, t) = 0 \quad , \quad (4)$$

in which $F(t) = F_1 \sin(2\pi f_c t) e^{-4(t f_c - 1)^2}$ where $f_c = 1$ MHz is the center frequency and $F_1 = 100$ N; ρ_k is the mass density of domain Ω_k ; δ_0 is the Dirac function at the origin and x_1^S and x_3^S are the coordinates of a line source modeling the acoustical impulse. The boundary conditions at time $t > 0$ are written as $\boldsymbol{\sigma} \mathbf{n}_1 = -p_1 \mathbf{n}_1$ in Σ_1 , $\boldsymbol{\sigma} \mathbf{n}_2 = -p_2 \mathbf{n}_2$ in Σ_2 , $p_k = 0$ in Γ_k and $\nabla p_k \cdot \mathbf{n}_k = -\rho_k \ddot{\mathbf{u}} \cdot \mathbf{n}_k$ in Σ_k with $\mathbf{n}_1 = (0, 0, 1)$ and $\mathbf{n}_2 = (0, 0, -1)$. The initial conditions at time $t = 0$ are written as $\mathbf{u} = 0$ and $\dot{\mathbf{u}}$ in $\Omega \cup \Sigma_1 \cup \Sigma_2$ for the displacement field and $p_k = 0$ and \dot{p}_k in $\Omega_k \cup \Gamma_k \cup \Sigma_k$ for the pressure fields. The constitutive equation of the solid elastic medium is written as

$$\boldsymbol{\sigma}(\mathbf{x}, t) = \sum_{i,j,k,h=1}^3 c_{ijkl} \varepsilon_{kh}(\mathbf{x}, t) \mathbf{e}_i \otimes \mathbf{e}_j \quad (5)$$

in which $\sum_{i,j,k,h=1}^3 c_{ijkl} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_h$ is the effective elasticity tensor of the medium and $\varepsilon_{kh} = \frac{1}{2} \left(\frac{\partial u_k}{\partial x_h} + \frac{\partial u_h}{\partial x_k} \right)$ are the components of the linearized strain tensor on basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$. Let $[C]$ be the effective elasticity matrix such that

$$[C] = \begin{pmatrix} c_{1111} & c_{1122} & c_{1133} & \sqrt{2}c_{1123} & \sqrt{2}c_{1131} & \sqrt{2}c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & \sqrt{2}c_{2223} & \sqrt{2}c_{2231} & \sqrt{2}c_{2213} \\ c_{3311} & c_{3322} & c_{3333} & \sqrt{2}c_{3323} & \sqrt{2}c_{3331} & \sqrt{2}c_{3312} \\ \sqrt{2}c_{2311} & \sqrt{2}c_{2322} & \sqrt{2}c_{2333} & 2c_{2323} & 2c_{2331} & 2c_{2312} \\ \sqrt{2}c_{3111} & \sqrt{2}c_{3122} & \sqrt{2}c_{3133} & 2c_{3123} & 2c_{3131} & 2c_{3112} \\ \sqrt{2}c_{1211} & \sqrt{2}c_{1222} & \sqrt{2}c_{1233} & 2c_{1223} & 2c_{1231} & 2c_{1212} \end{pmatrix}. \quad (6)$$

For a transverse isotropic homogeneous medium, all the components $[C]_{ij}$ are zeros except the following

$$[C]_{11} = \frac{e_L^2(1 - \nu_T)}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad [C]_{22} = \frac{e_T(e_L - e_T\nu_L^2)}{(1 + \nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad (7)$$

$$[C]_{12} = \frac{e_T e_L \nu_L}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad [C]_{23} = \frac{e_T(e_L\nu_T + e_T\nu_L^2)}{(1 + \nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} \quad , \quad (8)$$

$$[C]_{44} = g_T \quad , \quad [C]_{55} = g_L \quad , \quad (9)$$

with $[C]_{22} = [C]_{33}$, $[C]_{12} = [C]_{13} = [C]_{21} = [C]_{31}$, $[C]_{23} = [C]_{32}$ and $[C]_{55} = [C]_{66}$ and where (1) e_L and e_T are the longitudinal and transversal Young moduli, (2) g_L and g_T are the longitudinal and transversal shear moduli and (3) ν_L and ν_T are the longitudinal and transversal Poisson coefficients such that $g_T = e_T/2(1 + \nu_T)$. For a given effective elasticity matrix $[C]$, the displacement field \mathbf{u} and the pressure fields p_1 and p_2 are calculated using the solver presented in [2]. Then, the velocity v^{velo} of the first arriving signal is deduced. Consequently, there exists a mapping g_{velo} such that

$$v^{\text{mod}} = g_{\text{velo}}([C]) \quad . \quad (10)$$

4. Stochastic simplified model

It is assumed that uncertainties are only related to the components c_{ijkl} of the effective elasticity tensor. The stochastic simplified model is constructed by substituting matrix $[C]$ in Eq. (10) by a random matrix $[\mathbf{C}]$ for which a probabilistic model is constructed using the information theory. The available information on $[\mathbf{C}]$ is defined as follows: (1) the mean value of random matrix $[\mathbf{C}]$ is the effective elasticity matrix $[C]$; (2) random matrix $[\mathbf{C}]$ is a second-order random variable with values in the set of all the (6×6) real symmetric positive-definite matrices; (3) the inverse matrix of $[\mathbf{C}]$ exists almost surely and is assumed to be a second-order random variable. Thus, random matrix $[\mathbf{C}]$ belongs to the set SE^+ (see [7, 8]) and is written as

$$[\mathbf{C}] = [L]^T [\mathbf{G}] [L] \quad , \quad (11)$$

in which the (6×6) upper triangular matrix $[L]$ corresponds to the Cholesky factorization of the elasticity matrix $[C]$ and where random matrix $[\mathbf{G}]$ belongs to the set SG^+ introduced in [7, 8]. Moreover, it is shown in [7, 8] that probabilistic model of random matrix $[\mathbf{G}]$ only depends on a dispersion coefficient denoted by δ . The probabilistic model of random matrix $[\mathbf{C}]$ is completely defined with matrix $[C]$ (that depends on e_L , ν_L , g_L , e_T and ν_T) and δ . The velocity of the first arriving signal constructed using this stochastic simplified model is a random variable denoted by V^{mod} that corresponds to the random experimental velocity V^{exp} introduced in Section 2 and we have (see Eq. (10))

$$V^{\text{mod}} = g_{\text{velo}}([\mathbf{C}]) \quad . \quad (12)$$

5. Optimization problem for the identification

The stochastic simplified model parameters that have to be identified are the coefficients e_L , ν_L , g_L , e_T and ν_T relative to $[C]$, the mass density ρ and the coefficient δ . Let \mathbf{a} be the vector such that $\mathbf{a} = (\rho, e_L\nu_L, g_L, e_T, \nu_T)$. The identification problem consists in finding vector \mathbf{a} and coefficient δ such that the stochastic model can represent the experimental database in a statistical sense (see for instance [11, 12]). The optimal values $(\mathbf{a}^{\text{opt}}, \delta^{\text{opt}})$ for (\mathbf{a}, δ) is given by solving the following optimization problem

$$(\mathbf{a}^{\text{opt}}, \delta^{\text{opt}}) = \arg \min_{(\mathbf{a}, \delta)} F^{\text{cost}}(\mathbf{a}, \delta) \quad , \quad (13)$$

in which $F^{\text{cost}}(\mathbf{a}, \delta)$ is a cost function which has to be defined. The cost function F^{cost} adapted to the optimization problem is written as

$$F^{\text{cost}}(\mathbf{a}, \delta) = \frac{(\underline{V}^{\text{exp}} - \underline{V}^{\text{mod}})^2}{(\underline{V}^{\text{exp}})^2} + \frac{(\underline{\delta}^{\text{exp}} - \delta^{\text{mod}})^2}{(\underline{\delta}^{\text{exp}})^2},$$

in which

$$\delta^{\text{mod}} = \frac{\sqrt{E\{(V^{\text{mod}} - \underline{V}^{\text{mod}})^2\}}}{\underline{V}^{\text{mod}}}.$$

The optimization problem defined by Eq. (13) is solved by the simplex algorithm. For each iteration of the simplex algorithm, the cost function has to be calculated which requires to solve the stochastic equations with an appropriate method such as the Monte Carlo method.

6. Experimental validation of the stochastic simplified model

This section is devoted to the experimental validation of the stochastic simplified model. The stochastic simplified model must be able to simulate the experimental database in a statistical sense. The experimental validation is performed with the *in vivo* experimental database

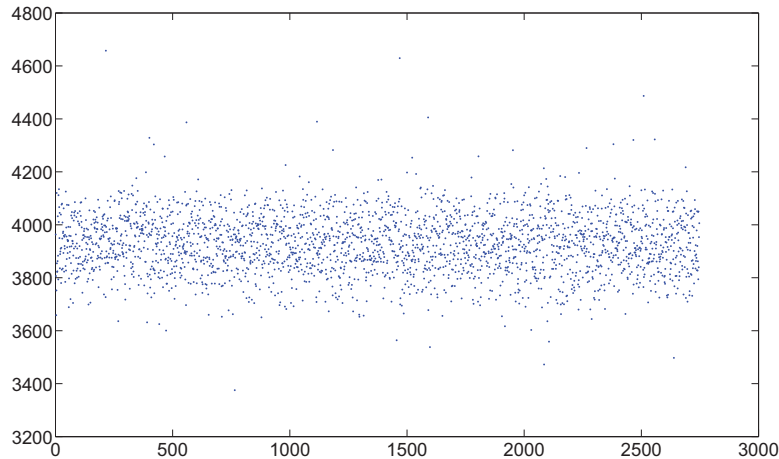


Figure 4. Graph of the realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$

presented in Section 2 and is made up of $N = 2747$ measurements $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ plotted in Fig. 4. The probability density function $v \mapsto p_{V^{\text{exp}}}(v)$ of the random variable V^{exp} estimated with the $N = 2747$ experimental realizations $V^{\text{exp}}(\hat{\theta}_1), \dots, V^{\text{exp}}(\hat{\theta}_N)$ is shown in Fig. 5. The identification of the vector $\mathbf{a} = (\rho, e_L, \nu_L, g_L, e_T, \nu_T)$ and the coefficient δ is carried out using the method presented in Section 5 with $h_1 = 10^{-2}\text{m}$, $h = 4 \times 10^{-3}\text{m}$, $h_2 = 10^{-2}\text{m}$, $\rho_1 = \rho_2 = 1000 \text{ kg.m}^{-3}$ and $c_1 = c_2 = 1500 \text{ m.s}^{-1}$. The solution $\mathbf{a}^{\text{opt}} = (\rho^{\text{opt}}, e_L^{\text{opt}}, \nu_L^{\text{opt}}, g_L^{\text{opt}}, e_T^{\text{opt}}, \nu_T^{\text{opt}})$ and δ^{opt} are such that $\rho^{\text{opt}} = 1598.8 \text{ kg.m}^{-3}$, $e_L^{\text{opt}} = 17.717 \text{ GPa}$, $\nu_L^{\text{opt}} = 0.3816$, $g_L^{\text{opt}} = 4.7950 \text{ GPa}$, $e_T^{\text{opt}} = 9.8254 \text{ GPa}$, $\nu_T^{\text{opt}} = 0.4495$ and $\delta^{\text{opt}} = 0.1029$. For $\mathbf{a} = \mathbf{a}^{\text{opt}}$ and $\delta = \delta^{\text{opt}}$, the realizations $V^{\text{mod}}(\theta_1), \dots, V^{\text{mod}}(\theta_N)$ of random velocity V^{mod} are constructed with the stochastic simplified model and then, the probability density function $v \mapsto p_{V^{\text{mod}}}(v)$ of V^{mod} is estimated. Figure 6 shows the graphs of $v \mapsto p_{V^{\text{mod}}}(v)$. Figure 7 compares the graphs of $v \mapsto p_{V^{\text{mod}}}(v)$ and

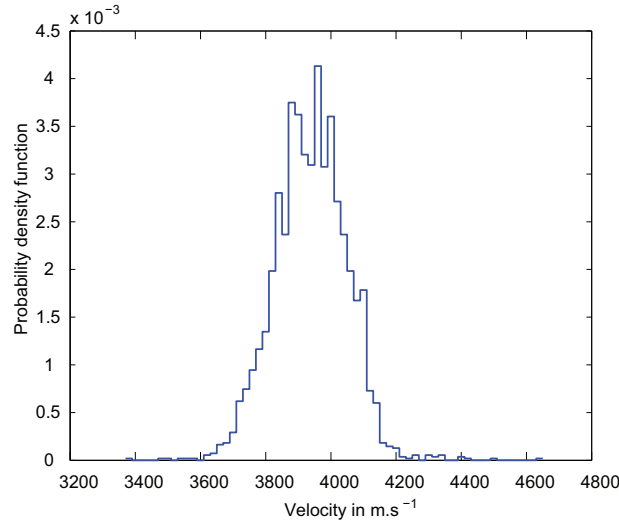


Figure 5. Graph of the probability density function $v \mapsto p_{V^{\text{exp}}}(v)$

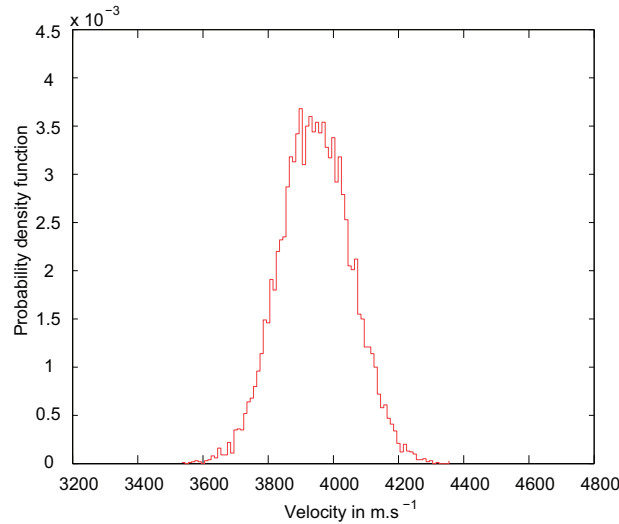


Figure 6. Graph of the density probability density functions $v \mapsto p_{V^{\text{mod}}}(v; \mathbf{a}^{\text{opt}}, \delta^{\text{opt}})$ with $\mathbf{a} = \mathbf{a}^{\text{opt}}$ and $\delta = \delta^{\text{opt}}$

$v \mapsto p_{V^{\text{exp}}}(v)$ in logarithm scale. This figure shows that the stochastic simplified model is able to predict in a statistical sense the velocity of the first arriving signal in a good accordance with the experimental tests.

7. Conclusions

A simplified elastoacoustic model has been developed to simulate the ultrasonic wave propagation in a complex biomechanical system made up of multilayer media. In order to improve the simplified model, the uncertainties related to the solid layer have been taken into account using a probabilistic approach. A method has been presented to identify the parameters of the stochastic simplified model. The capability of the proposed stochastic simplified model to predict the velocity of the first arriving signal in the statistical sense has been demonstrated

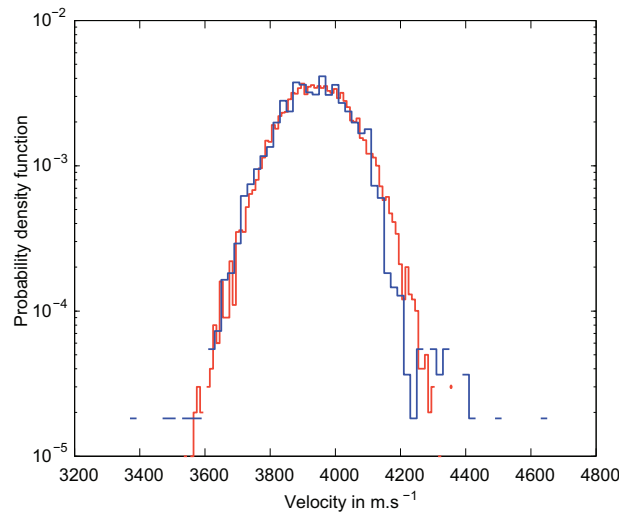


Figure 7. Graphs of $v \mapsto \log(p_{V^{\text{exp}}}(v))$ and $v \mapsto \log(p_{V^{\text{exp}}}(v; \mathbf{a}^{\text{opt}}, \delta^{\text{opt}}))$.

using a large experimental *in vivo* database.

8. Acknowledgments

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9. References

- [1] Macocco, K. , Grimal, Q. , Naili, S., and Soize, C. (2006), "Elastoacoustic model with uncertain mechanical properties for ultrasonic wave velocity prediction: Application to cortical bone evaluation", J. Acoust. Soc. Am., **119**(2), 729–740
- [2] Desceliers, C., Soize, C., Grimal, Q., Haiat, G., Naili, S. (2008), "A time-domain method to solve transient elastic wave propagation in a multilayer medium with a hybrid spectral-finite element space approximation", Wave Motion (in press)
- [3] Desceliers, C., Soize, C., Grimal, Q., Haiat, G., Naili, S. (2008), "Identification of an anisotropic elasticity tensor for an elastic layer using transient wave propagation in a fluid-solid multilayer – Model with uncertainties and Experiments", submitted to J. Acoust. Soc. Am.
- [4] Bossy, E. , Talmant, M. , Laugier, P. (2004),"Effect of bone cortical thickness on velocity measurements using ultrasonic axial transmission: A 2D simulation study", J. Acoust. Soc. Am., **112**(1), 297-307
- [5] Bossy, E. , Talmant, M. , Laugier, P. (2004),"Three-dimensional simulations of ultrasonic axial transmission velocity measurement on cortical bone models", J. Acoust. Soc. Am., **115**(5), 2314-2324
- [6] Camus, E; Talmant, M; Berger, G; *et al.* Analysis of the axial transmission technique for the assessment of skeletal status J. Acoust. Soc. Am., **108**(6), 3058-3065
- [7] Soize, C. (2001), "Maximum entropy approach for modeling random uncertainties in transient elastodynamics", J. Acoust. Soc. Am., **109**(5), 1979–1996
- [8] Soize, C. (2005), "Random matrix theory for modeling uncertainties in computational mechanics", Computer Methods in Applied Mechanics and Engineering, **194**, 1333–1366
- [9] Ohayon, R., Soize, C. (1998) *Structural Acoustics and Vibration*, Academic Press, New York
- [10] Pierce, A.D. (1998) *Acoustics: An Introduction to its Physical Principles and Applications*, Acoust. Soc. Am. Publications on Acoustics, Woodbury, NY, U.S.A (originally published in 1981, McGrawHill, New York)
- [11] Desceliers, C., Ghanem, R. and Soize, C. (2006), "Maximum likelihood estimation of stochastic chaos representations from experimental data", Int. J. for Numerical Methods in Engineering, **66**(6), 978-1001
- [12] Desceliers, C., Soize, C. and Ghanem, R. (2007), "Identification of chaos representations of elastic properties of random media using experimental vibration tests", Computational Mechanics, **39**(6), 831-838