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MODELING SOUND-INSULATION LAYERS IN VIBROACOUSTIC SYSTEMS

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ABSTRACT. This paper presents a new approach in modeling layered sound absorbing materials. This approach builds a probabilistic model based on the fuzzy structure theory and takes into account internal resonances of the structure. An experimental identification of the mean parameters of the model and its experimental validation are presented.

KEYWORDS: sound-insulation layer, fuzzy structure theory, vibroacoustics

1 INTRODUCTION

This paper deals with the probabilistic modeling of sound-insulation layers (denoted by the trim in this paper) in the context of computational vibroacoustics for complex systems. The trim is a complex composite structure which can be either modeled by a full finite element model (e.g. [2]) or by acoustic waves transfer matrices (e.g. [1]). The first method requires a large number of degrees of freedom (for instance, several millions for a whole car vibroacoustic model which have to be added to about one million DOF for the own vehicle). Moreover, as the trim eigenfrequencies are within the frequency range of interest, the number of generalized coordinates may be increased by a thousand. The second method which is proposed does not require additional DOF but in a counter part introduces model uncertainties and data uncertainties as far as only infinite plane layers can be addressed. Usually, wall acoustic impedance models proposed in the literature correspond to asymptotic theory and do not take into account internal resonances such as thickness resonances. The objectives of this paper are (1) to develop a model which takes into account internal resonances by introducing hidden dynamical DOF using the fuzzy structure theory (e.g. [4, 5, 3]) and (2) to present an experimental identification of the mean parameters of the model. The theory and the numerical analysis are presented.

2 VIBROACOUSTIC BOUNDARY VALUE PROBLEM WITH A SOUND-INSULATION LAYER

Definition of the vibroacoustic system. The physical space $\mathbb{R}^3$ is referred to a cartesian system. The generic point of $\mathbb{R}^3$ is denoted by $\boldsymbol{x} = (x_1, x_2, x_3)$. The Fourier transform with respect to time t is denoted by $u(\omega) = \int_{\mathbb{R}} e^{-i\omega t} u(t)\, dt$. The vibroacoustic system is analyzed in the frequency band $\mathbb{B} = [\omega_{min}, \omega_{max}]$ with $0 < \omega_{min} < \omega_{max}$.

The structure occupies a bounded domain $\Omega_s \subset \mathbb{R}^3$ and is modeled by a nonhomogeneous anisotropic viscoelastic material. The boundary of Ω_s is written as $\partial\Omega_s = \Gamma_s \cup \Gamma_0 \cup \Gamma_1 \cup \Gamma_2$ (see figure 1). The outward unit normal to $\partial\Omega_s$ is denoted $\boldsymbol{n}^s(\boldsymbol{x})$. The structure is fixed on Γ_0, a surface force field $\boldsymbol{g}^{surf}(\boldsymbol{x}, \omega)$ is given on Γ_1 and a body force field $\boldsymbol{g}^{vol}(\boldsymbol{x}, \omega)$ is given in Ω_s. The coupling interface between the structure and the acoustic cavity is Γ_2 and the coupling interface between the structure and the trim is Γ_s. It should be noted that the trim only exists on Γ_s and not on Γ_2 (see figure 1). Let $\boldsymbol{x} \mapsto \boldsymbol{u}^s(\boldsymbol{x}, \omega)$ from Ω_s to $\mathbb{C}^3$ be the structure displacement field which is equal to zero on Γ_0. Let $\Omega_h \subset \mathbb{R}^3$ be the bounded domain occupied by the trim with boundary $\partial\Omega_h = \Gamma \cup \Gamma_s$ and $\Gamma \cap \Gamma_s = \emptyset$ (see figure 1). Let $\boldsymbol{x} \mapsto \boldsymbol{u}^h(\boldsymbol{x}, \omega)$ from Ω_h to $\mathbb{C}^3$ be the trim displacement field whose trace on interface Γ is still denoted by $\boldsymbol{x} \mapsto \boldsymbol{u}^h(\boldsymbol{x}, \omega)$. The bounded domain $\Omega_a \subset \mathbb{R}^3$ of the acoustic cavity is filled with a dissipative acoustic fluid (air). The boundary of Ω_a is written $\partial\Omega_a = \Gamma \cup \Gamma_2$ with $\Gamma \cap \Gamma_2 = \emptyset$. Let $\boldsymbol{x} \mapsto p(\boldsymbol{x}, \omega)$ from Ω_a to $\mathbb{C}$ be the acoustic pressure field on the boundary $\Gamma \bigcup \Gamma_2$, the coupling vibroacoustic condition corresponds to the usual continuity condition of the normal velocity fluid. We write a perfect fluid coupling condition between the fluid and the structure and between the fluid and the trim.

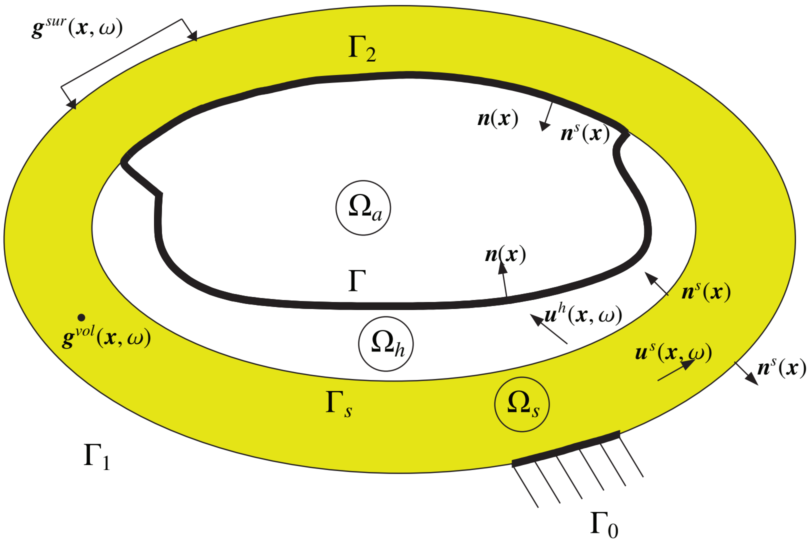

Figure 1: Structural-acoustical problem

Coupling fields on the vibroacoustic interfaces. The coupling force field on boundary Γ_s that the structure exerts on the trim is denoted by $\boldsymbol{x} \mapsto \boldsymbol{f}^s(\boldsymbol{x},\omega) = (f_1^s(\boldsymbol{x},\omega), f_2^s(\boldsymbol{x},\omega), f_3^s(\boldsymbol{x},\omega))$ from Γ_s to $\mathbb{C}^3$ and can be written for all $\boldsymbol{x}$ fixed in Γ_s as $\boldsymbol{f}^s(\boldsymbol{x},\omega) = f^s(\boldsymbol{x},\omega)\boldsymbol{n}^s(\boldsymbol{x})+\boldsymbol{f}^s_{tang}(\boldsymbol{x},\omega)$ with $\boldsymbol{f}^s_{tang}(\boldsymbol{x},\omega).\boldsymbol{n}^s(\boldsymbol{x}) = 0$ and $\boldsymbol{x} \mapsto f^s(\boldsymbol{x},\omega)$ from Γ_s to $\mathbb{C}$ is the normal component such that

$$\boldsymbol{f}^s(\boldsymbol{x},\omega) = f^s(\boldsymbol{x},\omega)\boldsymbol{n}^s(\boldsymbol{x}) \quad . \tag{1}$$

Dimension of $f_i^s(\boldsymbol{x},t)$ is $[M][L]^{-1}[T]^{-2}$. The coupling force field on boundary $\partial\Omega_a = \Gamma \bigcup \Gamma_2$ that the acoustic fluid exerts on the structure (interface Γ_2) and the trim (interface Γ) is denoted by $\boldsymbol{x} \mapsto \boldsymbol{f}^p(\boldsymbol{x},\omega)$ from $\partial\Omega_a$ to $\mathbb{C}^3$ and is written for all $\boldsymbol{x}$ in $\partial\Omega_a$,

$$\boldsymbol{f}^p(\boldsymbol{x},\omega)ds(\boldsymbol{x}) = -p(\boldsymbol{x},\omega)\boldsymbol{n}(\boldsymbol{x})ds(\boldsymbol{x}) \quad , \tag{2}$$

with ds the surface measure relative to $\partial\Omega_a$ (see figure 1).

Weak formulation for the mean boundary value problem of the vibroacoustic system. Let C_0^s be the admissible space of the displacement fields of the structure, C^h be the admissible space of the displacement fields of the trim and C^a be the admissible space of the pressure fields in the cavity. We introduce the following bilinear form defined on $C_0^s \times C^a$,

$$c_s^a(\boldsymbol{u}^s, p;\omega) = \int_{\Gamma_2} \boldsymbol{u}^s(\boldsymbol{x}).\boldsymbol{n}^s(\boldsymbol{x})\, p(\boldsymbol{x})\, ds(\boldsymbol{x}) \quad , \tag{3}$$

the bilinear form defined on $C^h \times C^a$,

$$c_h^a(\boldsymbol{u}^h, p;\omega) = \int_{\Gamma} \boldsymbol{u}^h(\boldsymbol{x}).\boldsymbol{n}(\boldsymbol{x})\, p(\boldsymbol{x})\, ds(\boldsymbol{x}) \quad , \tag{4}$$

and the linear form defined on C_0^s or C^h,

$$c_{f^s}(\boldsymbol{u};\omega) = \int_{\Gamma_s} f_i^s(\boldsymbol{x},\omega)\, u_i(\boldsymbol{x})\, ds(\boldsymbol{x}) \quad . \tag{5}$$

The weak formulation of the vibroacoustic boundary value problem can be written (e.g. [3]).

$$-\omega^2 m^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s) + i\omega\, d^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s;\omega) + k^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s;\omega) + c_s^a(\boldsymbol{\delta u}^s, p;\omega) + c_{f^s}(\boldsymbol{\delta u}^s;\omega) = l^s(\boldsymbol{\delta u}^s;\omega) \quad , \tag{6}$$

$$-\omega^2 m^h(\boldsymbol{u}^h, \boldsymbol{\delta u}^h) + i\omega\, d^h(\boldsymbol{u}^h, \boldsymbol{\delta u}^h;\omega) + k^h(\boldsymbol{u}^h, \boldsymbol{\delta u}^h;\omega) - c_{f^s}(\boldsymbol{\delta u}^h;\omega) + c_h^a(\boldsymbol{\delta u}^h, p;\omega) = 0 \quad , \tag{7}$$

$$-\omega^2 m^a(p,\delta p) + i\omega\, d^a(p,\delta p;\omega) + k^a(p,\delta p) + \omega^2\left\{c_s^a(\boldsymbol{u}^s,\delta p;\omega) + c_h^a(\boldsymbol{u}^h,\delta p;\omega)\right\} = l^a(\delta p;\omega) \quad . \tag{8}$$

3 CONSTRUCTION OF A SIMPLIFIED MEAN MODEL OF THE SOUND-INSULATION LAYER USING FUZZY STRUCTURE THEORY

The principle of the construction consists in replacing Eq. (7) by a simplified model and then in eliminating field $\boldsymbol{u}^h$. The construction of the simplified mean model of the trim is based on the use of the fuzzy structure theory [4, 5, 3]. Therefore, we begin introducing the underlying deterministic model of the trim. Then, we introduce the probabilistic model and we perform its statistical averaging.

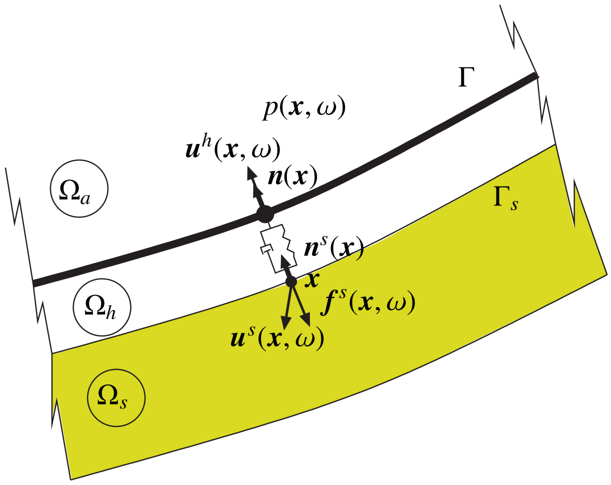

Figure 2: Soundproofing scheme modeling

Underlying deterministic model. We introduce the following hypothesis for the trim (see figure 2): $\Gamma \simeq \Gamma_s$ and consequently, for all $\boldsymbol{x}$ in $\Gamma \simeq \Gamma_s$, $\boldsymbol{n}(\boldsymbol{x}) = \boldsymbol{n}^s(\boldsymbol{x})$. The normal component to Γ_s of the structural displacement field is

$$w^s(\boldsymbol{x},\omega) = \boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x},\omega) \quad , \tag{9}$$

and the normal component to Γ of the trim displacement field is

$$w(\boldsymbol{x},\omega) = \boldsymbol{n}(\boldsymbol{x}).\boldsymbol{u}^h(\boldsymbol{x},\omega) \quad . \tag{10}$$

Using the fuzzy structure theory, the underlying deterministic model is made of a density of damped linear oscillators acting in the normal direction to Γ. At a fixed frequency ω, the mass density distributed on Γ, attached to one oscillator for which the displacement is $w(\boldsymbol{x},\omega)$, is denoted by $\mu(\boldsymbol{x},\omega) > 0$, and where the displacement of its base is $w(\boldsymbol{x},\omega)$. The dimension of $\mu(\boldsymbol{x},\omega)$ is then $[M][L]^{-2}$. The corresponding stiffness density (resp. damping rate density $\xi(\boldsymbol{x},\omega) > 0$) associated with this oscillator is $k(\boldsymbol{x},\omega) = \mu(\boldsymbol{x},\omega)\Omega_p{}^2(\boldsymbol{x},\omega)$ where $\Omega_p(\boldsymbol{x},\omega) > 0$ is the eigenfrequency ($rad.s^{-1}$) of the undamped fixed linear oscillator (*i.e.* for $w^s(\boldsymbol{x},\omega) = 0$). Let $\boldsymbol{f}^p(\boldsymbol{x},\omega)$ be the force applied to the mass of the oscillator and corresponding to the force density induced by the acoustic pressure $p(\boldsymbol{x},\omega)$ on the trim (see Eq. 2). Let $\boldsymbol{f}^s(\boldsymbol{x},\omega)$ be the force applied to the base of the oscillator and corresponding to the force density induced by the structure on the trim (see Eq. 1). Removing $\boldsymbol{x}$ and ω, the equation of the density of oscillators can then be written as

$$\mu\begin{bmatrix} -\omega^2 + 2i\omega\xi\Omega_p + \Omega_p^2 & -2i\omega\xi\Omega_p - \Omega_p^2 \\ -2i\omega\xi\Omega_p - \Omega_p^2 & 2i\omega\xi\Omega_p + \Omega_p^2 \end{bmatrix}\begin{bmatrix} w(\boldsymbol{x},\omega) \\ w^s(\boldsymbol{x},\omega)\end{bmatrix} = \begin{bmatrix} -p(\boldsymbol{x},\omega) \\ f^s(\boldsymbol{x},\omega)\end{bmatrix} . \tag{11}$$

For all $\omega \in \mathbb{B}$, from Eq. (11), it can be deduced that

$$w(\boldsymbol{x}, \omega) = a^c(\boldsymbol{x}, \omega) w^s(\boldsymbol{x}, \omega) + \frac{1}{\omega^2} a^a(\boldsymbol{x}, \omega) p(\boldsymbol{x}, \omega) \quad , \tag{12}$$

$$f^s(\boldsymbol{x}, \omega) = a^s(\boldsymbol{x}, \omega) w^s(\boldsymbol{x}, \omega) + a^c(\boldsymbol{x}, \omega) p(\boldsymbol{x}, \omega) \quad . \tag{13}$$

in which

$$a^s(\boldsymbol{x}, \omega) = \frac{-\omega^2 \, \mu(\boldsymbol{x}, \omega) \left(2i\omega \, \xi(\boldsymbol{x}, \omega) \Omega_p(\boldsymbol{x}, \omega) + \Omega_p(\boldsymbol{x}, \omega)^2\right)}{-\omega^2 + 2i\omega \, \xi(\boldsymbol{x}, \omega) \Omega_p(\boldsymbol{x}, \omega) + \Omega_p(\boldsymbol{x}, \omega)^2} \quad , \tag{14}$$

$$a^a(\boldsymbol{x}, \omega) = \frac{-\omega^2}{\mu(\boldsymbol{x}, \omega) \left(-\omega^2 + 2i\omega \, \xi(\boldsymbol{x}, \omega) \Omega_p(\boldsymbol{x}, \omega) + \Omega_p(\boldsymbol{x}, \omega)^2\right)} \quad , \tag{15}$$

$$a^c(\boldsymbol{x}, \omega) = \frac{2i\omega \, \xi(\boldsymbol{x}, \omega) \Omega_p(\boldsymbol{x}, \omega) + \Omega_p(\boldsymbol{x}, \omega)^2}{-\omega^2 + 2i\omega \, \xi(\boldsymbol{x}, \omega) \Omega_p(\boldsymbol{x}, \omega) + \Omega_p(\boldsymbol{x}, \omega)^2} \quad . \tag{16}$$

Substituting Eq. (12) into Eq. (4), using Eq. (9) and $\Gamma \simeq \Gamma_s$ yield

$$\begin{aligned} \omega^2 c_h^a(\boldsymbol{u}^h, \delta p; \omega) = {} & \omega^2 \int_{\Gamma_s} a^c(\boldsymbol{x}, \omega) \, \boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x}, \omega) \, \delta p(\boldsymbol{x}) \, ds(\boldsymbol{x}) \\ & + \int_{\Gamma_s} a^a(\boldsymbol{x}, \omega) \, p(\boldsymbol{x}, \omega) \, \delta p(\boldsymbol{x}) \, ds(\boldsymbol{x}) \quad . \end{aligned} \tag{17}$$

Substituting Eq. (1) and Eq. (13) into Eq. (5) for $\boldsymbol{u} = \boldsymbol{\delta u}^s$, and using Eq. (9) yield

$$\begin{aligned} c_{f^s}(\boldsymbol{\delta u}^s; \omega) = {} & \int_{\Gamma_s} a^s(\boldsymbol{x}, \omega) \, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x}, \omega)) \, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x})) \, ds(\boldsymbol{x}) \\ + & \int_{\Gamma_s} a^c(\boldsymbol{x}, \omega) p(\boldsymbol{x}, \omega) \, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x})) \, ds(\boldsymbol{x}) \quad . \end{aligned} \tag{18}$$

Probabilistic model. The use of the fuzzy structure theory leads us to model $\Omega_p(\boldsymbol{x}, \omega)$ by a random variable. Then in this section, we introduce the statistical mean part of the previous bilinear form $c_h^a(\boldsymbol{u}^h, \delta p; \omega)$ and the linear form $c_{f^s}(\boldsymbol{\delta u}^s; \omega)$ defined in Eq. (17) and Eq. (18). For all ω in $\mathbb{B}$, we choose to represent $\mu(\boldsymbol{x}, \omega)$ and $\xi(\boldsymbol{x}, \omega)$ by their mean values $\mu(\boldsymbol{x}, \omega) = \underline{\mu}(\omega) > 0$ and $\xi(\boldsymbol{x}, \omega) = \underline{\xi}(\omega) > 0$ where $\omega \mapsto \underline{\mu}(\omega)$ and $\omega \mapsto \underline{\xi}(\omega)$ are two deterministic functions independent of $\boldsymbol{x}$. For all $\boldsymbol{x}$ fixed in Γ_s and ω fixed in $\mathbb{B}$, the eigenfrequency $\Omega_p(\boldsymbol{x}, \omega)$ is modeled by a positive random variable whose probability distribution $P_{\Omega_p(\boldsymbol{x};\omega)}(d\omega_p, \omega)$ is assumed to be independent of $\boldsymbol{x}$ and is defined by the probability density function $p_{\Omega_p(\boldsymbol{x};\omega)}(\omega_p, \omega)$ with respect to $d\omega_p$, such that

$$p_{\Omega_p(\boldsymbol{x};\omega)}(\omega_p, \omega) = \ell(\omega) \mathbb{1}_{[a(\omega), b(\omega)]}(\omega_p) \quad , \tag{19}$$

with $\mathbb{1}_B(\boldsymbol{x}) = 1$ if $\boldsymbol{x} \in B$ and $= 0$ if $\boldsymbol{x} \notin B$ and where

$$a(\omega) = \sup\left\{0, \omega - \frac{1}{2\underline{n}(\omega)}\right\} \quad , \tag{20}$$

$$b(\omega) = \omega + \frac{1}{2\underline{n}(\omega)} \quad , \tag{21}$$

$$\ell(\omega) = \frac{1}{b(\omega) - a(\omega)} \quad , \tag{22}$$

in which $\underline{n}(\omega)$ is the mean modal density of the trim. It should be noted that $\underline{n}(\omega)$ is the main parameter of the trim model. This parameter controls the power flow between the internal dynamical DOF of the trim and the structure and the acoustic cavity. Then for all $\boldsymbol{x}$ fixed in Γ_s and ω fixed in $\mathbb{B}$, $a^s(\boldsymbol{x}, \omega)$, $a^a(\boldsymbol{x}, \omega)$ and $a^c(\boldsymbol{x}, \omega)$ defined by Eq. (14), Eq. (15) and Eq. (16) become random variables denoted by $A^s(\boldsymbol{x}, \omega)$, $A^a(\boldsymbol{x}, \omega)$ et $A^c(\boldsymbol{x}, \omega)$. For all $\boldsymbol{u}^s$ and $\boldsymbol{\delta u}^s$ in C_0^s and for all p and δp in C^a, $c_h^a(\boldsymbol{u}^h, \delta p; \omega)$

and $c_{f^s}(\boldsymbol{\delta u}^s;\omega)$ become random variables denoted by $C_h^a(\boldsymbol{u}^h,\delta p;\omega)$ et $C_{f^s}(\boldsymbol{\delta u}^s;\omega)$ which can then be written as

$$\omega^2 C_h^a(\boldsymbol{u}^h,\delta p;\omega) = \omega^2 \int_{\Gamma_s} A^c(\boldsymbol{x},\omega)\, \boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x},\omega)\, \delta p(\boldsymbol{x})\, ds(\boldsymbol{x}) + \int_{\Gamma_s} A^a(\boldsymbol{x},\omega)\, p(\boldsymbol{x},\omega)\, \delta p(\boldsymbol{x})\, ds(\boldsymbol{x}) \quad , \tag{23}$$

$$C_{f^s}(\boldsymbol{\delta u}^s;\omega) = \int_{\Gamma_s} A^s(\boldsymbol{x},\omega)\; (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x},\omega))\, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x}))\; ds(\boldsymbol{x}) + \int_{\Gamma_s} A^c(\boldsymbol{x},\omega) p(\boldsymbol{x},\omega)\, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x}))\; ds(\boldsymbol{x}) \quad . \tag{24}$$

Statistical averaging of the simplified mean model of the trim. The last step of the fuzzy structure theory consists in defining the mean model taking the statistical averaging of random variables $C_h^a(\boldsymbol{u}^h,\delta p;\omega)$ and $C_{f^s}(\boldsymbol{\delta u}^s;\omega)$, that is to say, in defining

$$\underline{c}_h^a(\boldsymbol{u}^h,\delta p;\omega) = \mathcal{E}\Big\{C_h^a(\boldsymbol{u}^h,\delta p;\omega)\Big\} \quad , \tag{25}$$

$$\underline{c}_{f^s}(\boldsymbol{\delta u}^s;\omega) = \mathcal{E}\Big\{C_{f^s}(\boldsymbol{\delta u}^s;\omega)\Big\} \quad . \tag{26}$$

Analyzing Eq. (23) and Eq. (24) leads us to introduce the following deterministic bilinear forms $b^s(\boldsymbol{u}^s,\boldsymbol{\delta u}^s)$ on $C_0^s \times C_0^s$, $c^s(p,\boldsymbol{\delta u}^s)$ on $C^a \times C_0^s$ and $b^a(p,\delta p)$ on $C^a \times C^a$,

$$b^s(\boldsymbol{u}^s,\boldsymbol{\delta u}^s) = \int_{\Gamma_s} (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{u}^s(\boldsymbol{x}))\, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x}))\; ds(\boldsymbol{x}) \quad , \tag{27}$$

$$c^s(p,\boldsymbol{\delta u}^s) = \int_{\Gamma_s} p(\boldsymbol{x})\, (\boldsymbol{n}^s(\boldsymbol{x}).\boldsymbol{\delta u}^s(\boldsymbol{x}))\; ds(\boldsymbol{x}) \quad , \tag{28}$$

$$b^a(p,\delta p) = \int_{\Gamma_s} p(\boldsymbol{x})\, \delta p(\boldsymbol{x})\, ds(\boldsymbol{x}) \quad . \tag{29}$$

From Eq. (23) and Eq. (24) and using Eq. (19) to Eq. (22), it can be deduced that

$$\omega^2 \underline{c}_h^a(\boldsymbol{u}^h,\delta p;\omega) = \omega^2 \underline{a}^c(\omega)\, c^s(\delta p,\boldsymbol{u}^s) + \underline{a}^a(\omega)\, b^a(p,\delta p) \quad , \tag{30}$$

$$\underline{c}_{f^s}(\boldsymbol{\delta u}^s;\omega) = \underline{a}^s(\omega)\, b^s(\boldsymbol{u}^s,\boldsymbol{\delta u}^s) + \underline{a}^c(\omega)\, c^s(p,\boldsymbol{\delta u}^s) \quad , \tag{31}$$

in which

$$\underline{a}^s(\omega) = -\omega^2 \underline{a}_R^s(\omega) + i\omega \underline{a}_I^s(\omega) \quad , \tag{32}$$

$$\underline{a}^a(\omega) = \underline{a}_R^a(\omega) + i\omega \underline{a}_I^a(\omega) \quad , \tag{33}$$

$$\underline{a}^c(\omega) = \underline{a}_R^c(\omega) + i\underline{a}_I^c(\omega) \quad , \tag{34}$$

with

$$\underline{a}_R^s(\omega) = \underline{\mu}(\omega)\underline{n}(\omega) \left[\frac{1}{\underline{n}(\omega)} - \omega\, \underline{\lambda}(\omega)\, \Theta_R\Big(\widetilde{a}(\omega),\widetilde{b}(\omega),\underline{\xi}(\omega)\Big)\right] \quad , \tag{35}$$

$$\underline{a}_I^s(\omega) = \underline{\mu}(\omega)\underline{n}(\omega)\; \omega^2 \underline{\lambda}(\omega)\; \Theta_I\Big(\widetilde{a}(\omega),\widetilde{b}(\omega),\underline{\xi}(\omega)\Big) \quad , \tag{36}$$

$$\underline{a}_R^a(\omega) = \omega\, \underline{n}(\omega)\, \frac{\underline{\lambda}(\omega)}{\underline{\mu}(\omega)}\; \Theta_R\Big(\widetilde{a}(\omega),\widetilde{b}(\omega),\underline{\xi}(\omega)\Big) \quad , \tag{37}$$

$$\underline{a}_I^a(\omega) = \underline{n}(\omega)\, \frac{\underline{\lambda}(\omega)}{\underline{\mu}(\omega)}\; \Theta_I\Big(\widetilde{a}(\omega),\widetilde{b}(\omega),\underline{\xi}(\omega)\Big) \quad , \tag{38}$$

$$\underline{a}_R^c(\omega) = 1 - \omega\, \underline{n}(\omega)\underline{\lambda}(\omega)\; \Theta_R\Big(\widetilde{a}(\omega),\widetilde{b}(\omega),\underline{\xi}(\omega)\Big) \quad , \tag{39}$$

$$\underline{a}_I^c(\omega) = -\omega\, \underline{n}(\omega)\underline{\lambda}(\omega)\; \Theta_I\Big(\widetilde{a}(\omega),\widetilde{b}(\omega),\underline{\xi}(\omega)\Big) \quad , \tag{40}$$

and where functions $\underline{\lambda}, \widetilde{a}, \widetilde{b}, \Theta_R$ and Θ_I are defined in Appendix. Replacing $c_h^a(\boldsymbol{u}^h, \delta p; \omega)$ by $\underline{c}_h^a(\boldsymbol{u}^h, \delta p; \omega)$ in Eq. (8) yields an equation denoted by (8'). Replacing $c_{f^s}(\boldsymbol{\delta u}^s; \omega)$ by $\underline{c}_{f^s}(\boldsymbol{\delta u}^s; \omega)$ in Eq. (6) yields an equation denoted by (6'). Substituting Eq. (30) into Eq. (8') and substituting Eq. (31) into Eq. (6') yield the weak formulation of the vibroacoustic boundary value problem with a simplified mean model of the trim: find $\boldsymbol{u}^s$ in C_0^s and p in C^a such that, for all $\boldsymbol{\delta u}^s$ in C_0^s and δp in C^a, we have

$$\begin{aligned} -\omega^2 m^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s) + i\omega\, d^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s; \omega) + k^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s; \omega) + c_s^a(\boldsymbol{\delta u}^s, p; \omega) \\ + \underline{a}^s(\omega) b^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s) + \underline{a}^c(\omega) c^s(p, \boldsymbol{\delta u}^s) \;=\; l^s(\boldsymbol{\delta u}^s; \omega) \quad , \end{aligned} \tag{41}$$

and

$$\begin{aligned} -\omega^2 m^a(p, \delta p) + i\omega\, d^a(p, \delta p; \omega) + k^a(p, \delta p) + \omega^2 c_s^a(\boldsymbol{u}^s, \delta p; \omega) \\ + \omega^2 \underline{a}^c(\omega) c^s(\delta p, \boldsymbol{u}^s) + \underline{a}^a(\omega) b^a(p, \delta p) \;=\; l^a(\delta p; \omega) \quad , \end{aligned} \tag{42}$$

in which the bilinear forms $b^s(\boldsymbol{u}^s, \boldsymbol{\delta u}^s)$, $b^a(p, \delta p)$ and $c^s(\delta p, \boldsymbol{u}^s)$ are defined by Eq. (27), Eq. (28) and Eq. (29).

4 COMPUTATIONAL VIBROACOUSTIC MEAN MODEL

The finite element discretization [6] of Eq. (41) and Eq. (42) yields the following matrix equation on $\mathbb{C}^{m_s} \times \mathbb{C}^{m_a}$,

$$\begin{bmatrix} [\underline{\mathbb{A}}^s(\omega)] + \underline{a}^s(\omega)[\underline{\mathbb{B}}^s] & [\underline{\mathbb{C}}] + \underline{a}^c(\omega)[\underline{\mathbb{C}}^s] \\ \omega^2\left\{[\underline{\mathbb{C}}]^T + \underline{a}^c(\omega)[\underline{\mathbb{C}}^s]^T\right\} & [\underline{\mathbb{A}}^a(\omega)] + \underline{a}^a(\omega)[\underline{\mathbb{B}}^a] \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{u}}^s(\omega) \\ \underline{\boldsymbol{p}}(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbb{f}}^s(\omega) \\ \underline{\mathbb{f}}^a(\omega) \end{bmatrix} \quad , \tag{43}$$

where $[\underline{\mathbb{A}}^s(\omega)]$ is the dynamical stiffness $(m_s \times m_s)$ complex matrix of the structure such that

$$[\underline{\mathbb{A}}^s(\omega)] = -\omega^2\, [\underline{\mathbb{M}}^s] + i\omega\, [\underline{\mathbb{D}}^s(\omega)] + [\underline{\mathbb{K}}^s(\omega)] \quad , \tag{44}$$

in which $[\underline{\mathbb{M}}^s]$, $[\underline{\mathbb{D}}^s(\omega)]$ and $[\underline{\mathbb{K}}^s(\omega)]$ are the mass, damping and stiffness matrices of the structure *in vacuo*. In Eq. (43), $[\underline{\mathbb{A}}^a(\omega)]$ is the dynamical stiffness $(m_a \times m_a)$ complex matrix of the acoustic fluid such that

$$[\underline{\mathbb{A}}^a(\omega)] = -\omega^2\, [\underline{\mathbb{M}}^a] + i\omega\, [\underline{\mathbb{D}}^a(\omega)] + [\underline{\mathbb{K}}^a] \quad , \tag{45}$$

in which $[\underline{\mathbb{M}}^a]$, $[\underline{\mathbb{D}}^a(\omega)]$ and $[\underline{\mathbb{K}}^a]$ are the mass, damping and stiffness matrices of the acoustic cavity with fixed coupling interface. Matrix $[\underline{\mathbb{C}}]$ is the usual vibroacoustic coupling $(m_s \times m_a)$ real matrix relative to boundary Γ_2 (which is without trim). Matrices $[\underline{\mathbb{B}}^s]$, $[\underline{\mathbb{C}}^s]$ and $[\underline{\mathbb{B}}^a]$ correspond to the finite element approximation of the bilinear forms defined by Eq. (27), Eq. (28) and Eq. (29) respectively. Using n_s structural modes *in vacuo* and n_a acoustic modes of the cavity with fixed coupling interface, the mean reduced matrix model of the vibroacoustic system can then be written as

$$\underline{\boldsymbol{u}}^s(\omega) = [\underline{\Phi}^s]\underline{\boldsymbol{q}}^s(\omega) \quad , \tag{46}$$

$$\underline{\boldsymbol{p}}(\omega) = [\underline{\Phi}^a]\underline{\boldsymbol{q}}^a(\omega) \quad , \tag{47}$$

$$\begin{bmatrix} [\underline{A}^s(\omega)] + \underline{a}^s(\omega)[\underline{B}^s] & [\underline{C}] + \underline{a}^c(\omega)[\underline{C}^s] \\ \omega^2\left\{[\underline{C}]^T + \underline{a}^c(\omega)[\underline{C}^s]^T\right\} & [\underline{A}^a(\omega)] + \underline{a}^a(\omega)[\underline{B}^a] \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{q}}^s(\omega) \\ \underline{\boldsymbol{q}}^a(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{f}}^s(\omega) \\ \underline{\boldsymbol{f}}^a(\omega) \end{bmatrix} \quad . \tag{48}$$

in which $[\underline{\Phi}^s]$ is the $(m_s \times n_s)$ real matrix of the structural modes and $[\underline{\Phi}^s]$ is the $m_a \times n_a$ real matrix of the acoustic modes.

5 EXPERIMENTAL IDENTIFICATION OF THE MEAN MODAL DENSITY OF THE SOUND-INSULATION LAYER MODEL

The problem is to develop an experimental identification of the mean modal density $\omega \mapsto \underline{n}(\omega)$ introduced in the simplified mean model of the sound-insulation layer (see Section 3).

Experimental configuration and measurements. Experiments were made by PSA Peugeot Citroën engineers. The experimental configuration is made up of a steel thin plate connected to a framework (see figures 3 and 4), the trim is attached to the plate. The structure is hung with four springs (see figure 3).

Figure 3: Experiment

The structure is excited by a point force delivered by an electrodynamic shaker (see figure 3). The out-plane acceleration of the plate have been measured at the sixty points displayed in figure 4.

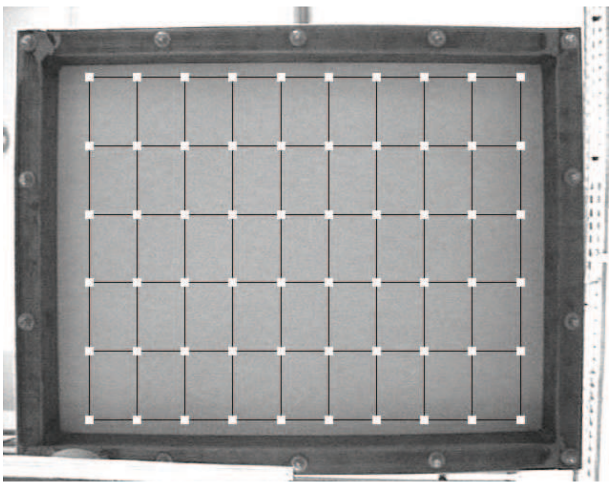

Figure 4: Sixty measurements points

The measured complex vector-valued frequency response function from the input force to the sixty acceleration components is denoted by $\omega \mapsto \boldsymbol{\gamma}^{\exp}(\omega) = (\gamma_1^{\exp}(\omega), \ldots, \gamma_{n_{\text{obs}}}^{\exp}(\omega))$ in which $n_{\text{obs}} = 60$.

Identification method The computed complex vector-valued frequency response function corresponding to the experimental measurements is denoted by $\omega \mapsto \boldsymbol{\gamma}(\omega; \underline{n}(\omega)) = (\gamma_1(\omega; \underline{n}(\omega)), \ldots, \gamma_{n_{\text{obs}}}(\omega; \underline{n}(\omega)))$ in which we have indicated the dependance in $\underline{n}(\omega)$ of the response of the computational vibroacoustic mean model. We introduce the function $\omega \mapsto r^{\text{exp}}(\omega)$ and $\omega \mapsto r(\omega)$ defined by

$$r^{\text{exp}}(\omega) = 10 \log_{10} \left(\sum_{i=1}^{n_{\text{obs}}} |\gamma_i^{\text{exp}}(\omega)|^2 \right) \quad , \quad r(\omega; \underline{n}(\omega)) = 10 \log_{10} \left(\sum_{i=1}^{n_{\text{obs}}} |\gamma_i(\omega)|^2 \right) \quad . \tag{49}$$

We then introduce the cost function J defined by

$$J(\underline{n}) = \int_{\mathbb{B}} \left(r(\omega; \underline{n}(\omega)) - r^{\text{exp}}(\omega) \right)^2 d\omega \quad . \tag{50}$$

The identified mean modal density function $\omega \mapsto \widehat{\underline{n}}(\omega)$ can be estimated in solving the following optimization problem

$$\widehat{\underline{n}} = \arg \min_{\underline{n}} J(\underline{n}) \quad , \tag{51}$$

in which $\underline{n}$ belongs to a class of smooth functions from $\mathbb{B}$ into $\mathbb{R}^+$ such as the space of continuously differentiable functions on $\mathbb{B}$. The following procedure has been used to identify the mean modal density.

(1) First, the updating of the computational mean model of the structure without the trim has been performed using experiments. Figure 5 displays the graphs of functions $\omega \mapsto r^{\text{exp}}(\omega)$ and $\omega \mapsto r(\omega)$ for the updated computational model (note that r does not depend on $\underline{n}$ because the considered system is made up of the structure without the trim). It should be noted that the computational model of the structure without the trim will be used in a second step as a "measurement device" to identify the trim model (the mean modal density). Figure 5 shows that the comparison between experiments and computational model is good in frequency band $[25, 180]$ Hz. Consequently, this "measurement device" will not introduce a bias in the identification of the trim model. It can be seen significant errors in the frequency band $[180, 300]$ Hz and consequently a bias will be introduced in the identification procedure for this part of the frequency band. This is the reason why identification will only be performed for frequency band $[30, 200]$ Hz.

(2) Secondly, the mean modal density $\underline{n}$ of the trim model has to be identified. Such an identification could be performed by using Eq. (51) with the solution of Eq. (46) and Eq. (48) without the acoustic cavity (structure with trim *in vacuo*). Presently, we do not have solved this optimization problem and we have estimated $\widehat{\underline{n}}$ by successive approximations in a class of smooth functions. The thin dashed line in figure 6 shows the graph of the modal density which has been estimated. Figure 6 also displays the comparison of the experimental measurement $\omega \mapsto r^{\text{exp}}(\omega)$ and the response $\omega \mapsto \widehat{\underline{n}}(\omega)$, computed with the model for the estimated value of the mean modal density. It can be seen a good correlation between experiments and computation. Clearly, there is a residual error due to the introduction of a simplified mean model of the trim. This model error can be taken into account introducing a probabilistic model of uncertainties. Such a development is in progress.

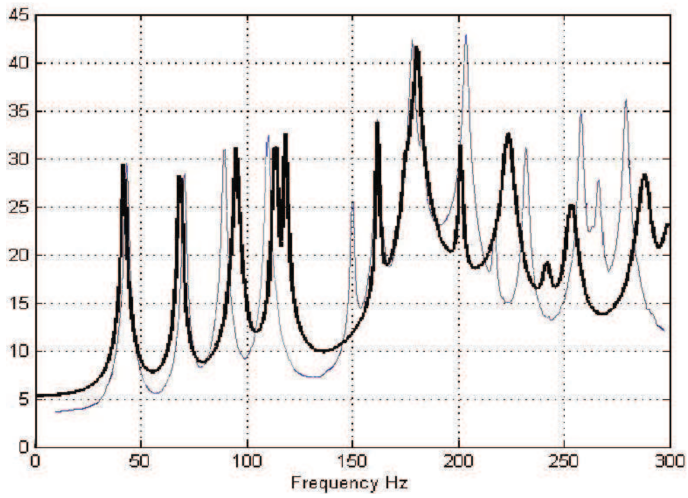

Figure 5: Graph of $\omega \mapsto r(\omega)$ for the structure without trim: measurements (thin line) ; model (thick line)

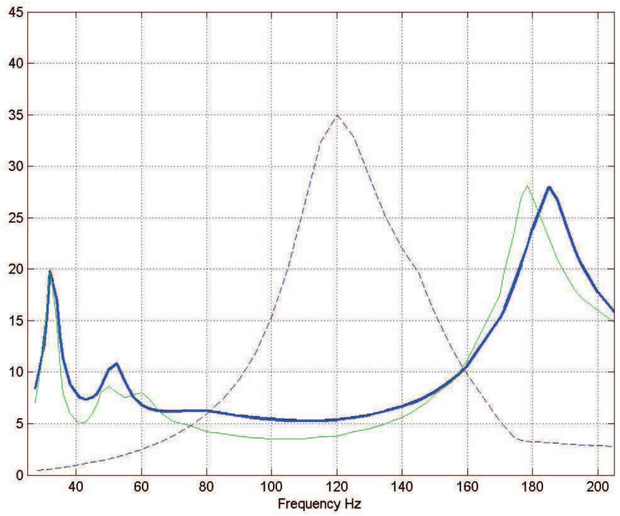

Figure 6: Graph of $\omega \mapsto r(\omega)$ for the structure with trim: measurements (thin solid line) ; calculation (thick solid line) ; modal density $\times 5.10^3$ (thin dashed line)

6 CONCLUSION

We propose a simplified mean model of sound-insulation system constructed using the fuzzy structure theory in the context of vibroacoustics. A first experimental validation of the developed mean model is presented. An identification procedure for the mean modal density is in progress. Clearly, there is a residual error due to the introduction of a simplified mean model of the trim. The introduction of such a model error can be taken into account introducing a probabilistic model of uncertainties. Consequently, an additional probabilistic model allowing model uncertainties to be taken into account has to be implemented. Such an implementation and its experimental validation is in progress.

A APPENDIX

For all $\omega \in \mathbb{B}$,

$$\Theta_R\left(\widetilde{a}(\omega), \widetilde{b}(\omega), \underline{\xi}(\omega)\right) = \frac{1}{4\sqrt{1-\underline{\xi}(\omega)^2}} \ln\left\{\frac{N^+(\widetilde{b}(\omega), \underline{\xi}(\omega))\, N^-(\widetilde{a}(\omega), \underline{\xi}(\omega))}{N^-(\widetilde{b}(\omega), \underline{\xi}(\omega))\, N^+(\widetilde{a}(\omega), \underline{\xi}(\omega))}\right\} \quad , \tag{52}$$

$$\Theta_I\left(\widetilde{a}(\omega), \widetilde{b}(\omega), \underline{\xi}(\omega)\right) = \frac{1}{2\sqrt{1-\underline{\xi}(\omega)^2}} \left[\Lambda(\widetilde{b}(\omega), \underline{\xi}(\omega)) - \Lambda(\widetilde{a}(\omega), \underline{\xi}(\omega))\right] \quad . \tag{53}$$

$$\widetilde{\ell}(\omega) = \omega\, \underline{n}(\omega)\, \underline{\lambda}(\omega) \quad . \tag{54}$$

$$N^{\pm}(u,\xi) = u^2 \pm 2\,u\,\sqrt{1-\xi^2} + 1 \quad , \tag{55}$$

$$\Lambda(u,\xi) = \arctan\left\{\frac{u^2 + 2\xi^2 - 1}{2\xi\sqrt{1-\xi^2}}\right\} \quad , \tag{56}$$

$$\begin{aligned}\widetilde{a}(\omega) &= \frac{1}{\omega} a(\omega) \\ &= \sup\left\{0, 1 - \frac{1}{2\,\omega\,\underline{n}(\omega)}\right\} \quad ,\end{aligned} \tag{57}$$

$$\begin{aligned}\widetilde{b}(\omega) &= \frac{1}{\omega} b(\omega) \\ &= 1 + \frac{1}{2\,\omega\,\underline{n}(\omega)} \quad .\end{aligned} \tag{58}$$

$$\widetilde{\ell}(\omega) = \frac{1}{\widetilde{b}(\omega) - \widetilde{a}(\omega)} \tag{59}$$

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