Uncertain nonlinear dynamical systems subjected to seismic loads
Christophe Desceliers, Christian Soize, S. Cambier

To cite this version:
Christophe Desceliers, Christian Soize, S. Cambier. Uncertain nonlinear dynamical systems subjected to seismic loads. 9th International Conference on Applications of Statistics and Probability in Civil Engineering, Jul 2003, San Francisco, CA, United States. pp.Pages: 251-257. hal-00686219

HAL Id: hal-00686219
https://hal-upec-upem.archives-ouvertes.fr/hal-00686219
Submitted on 8 Apr 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Uncertain nonlinear dynamical systems subjected to seismic loads

C. Desceliers, C. Soize
Laboratoire de Mécanique, Université de Marne-La-Vallée, 5, boulevard Descartes, 77454 Marne-La-Vallée Cedex 2, France

S. Cambier
R&D EDF, Analysis in Mechanics and Acoustics Dpt., 92141, Clamart cedex, France

Keywords: Nonlinear structural dynamics, Earthquake, Random uncertainties

ABSTRACT : This paper deals with the transient response of a nonlinear dynamical system with random uncertainties and subjected to earthquake. The nonparametric probabilistic model of random uncertainties recently published and extended to nonlinear dynamical system analysis is used in order to model random uncertainties related to the linear part of the finite element model. The nonlinearities are due to restoring forces whose parameters are uncertain and are modeled by the parametric approach. Jayne’s maximum entropy principle with the constraints defined by the available information allow the probabilistic model of such random variables to be constructed. Therefore, a nonparametric-parametric formulation is developed in order to model all the sources of uncertainties in such a nonlinear dynamical system. Finally, a numerical application for earthquake engineering analysis is proposed and concerned a reactor coolant system under seismic loads.

1. INTRODUCTION

This paper deals with the transient response of a nonlinear dynamical system with random uncertainties. The source of random uncertainties is induced by the model uncertainties (or the model errors) and the data uncertainties (errors on the parameters of the model). For the problem under consideration, data uncertainties concern the local parameters of the finite element model and the parameters of the nonlinear forces. Usually, parametric approaches are used to model data uncertainties [1, 2] for evaluating and analyzing the response of structures with uncertain parameters under seismic loads, like piping and equipment, power plant installations and industrial structures [3-7]. Nevertheless, such approaches do not allow model uncertainties to be taken into account. It should be noted that model uncertainties can only be modeled by using a nonparametric approach. Recently, a nonparametric model of random uncertainties has been introduced for linear dynamical system analysis [8, 9]. This nonparametric model has also been extended to the transient response of nonlinear dynamical system [10] without having random uncertainties on the nonlinear part. This nonparametric model is constructed by the use of Jayne’s entropy principle which consists in maximizing the probabilistic uncertainties with the constraints defined by the available information (for instance, related to algebraic properties of the finite element matrices). Nevertheless, this nonparametric formulation does not allow the uncertainties related to the nonlinear restoring forces to be modeled. This is the reason why a nonparametric-parametric formulation is proposed for analyzing nonlinear dynamical systems subjected to seismic loads with data uncertainties for the nonlinearities. An application to a multisupported reactor coolant system subjected to seismic loads is presented.

2. MEAN REDUCED MATRIX MODEL OF THE NONLINEAR DYNAMICAL SYSTEM SUBJECTED TO SEISMIC LOADS

We consider a nonlinear dynamical system constituted of a damped structure subjected to $m_s$ deterministic time-dependent dirichlet conditions corresponding to seismic loads applied to the supports...
of the structure. The nonlinearities are due to stops with high stiffness, installed with a given gap at
given points of the structure. Let \( m_f \) be the number of
degrees of freedom of the mean finite element
model of this nonlinear dynamical system. Let \( z \)
be the \( \mathbb{R}^{m_f} \)-vector of the total displacements,
including the displacements of the supports. We are
interested in the transient response \( t \mapsto z(t) \) from
\([0, T]\) into \( \mathbb{R}^{m_f} \). Let \( [\mathcal{M}], [\mathcal{D}] \) and \( [\mathcal{K}] \) be the mass,
damping and stiffness matrices of the linear part of
the mean finite element model. It is assumed that the
structure with fixed supports has no rigid
body modes. Consequently \( [\mathcal{M}], [\mathcal{D}] \) and \( [\mathcal{K}] \) are
positive-definite symmetric \( (m_f \times m_f) \) real matrices.
The real-valued vector \( z(t) \) is rewritten as
\( z(t) = (\tilde{z}(t), z_s(t)) \) in which \( \tilde{z}(t) \in \mathbb{R}^{m_f} \) is the vector of the
constrained DOFs at the supports and where \( z_s(t) \in \mathbb{R}^{m_s} \) is the vector of the \( m \) unconstrained
DOFs with \( m = m_f - m_s \). The block splitting of matrices \( [\mathcal{M}], [\mathcal{D}] \) and \( [\mathcal{K}] \) relative to
\( z(t) = (\tilde{z}(t), z_s(t)) \) are introduced,
\[
[\mathcal{M}] = \begin{bmatrix}
[\mathcal{M}]_T & [\mathcal{M}]_s
\end{bmatrix},
[\mathcal{D}] = \begin{bmatrix}
[\mathcal{D}]_T & [\mathcal{D}]_s
\end{bmatrix},
[\mathcal{K}] = \begin{bmatrix}
[\mathcal{K}]_s
\end{bmatrix}_s.
\]

\[\begin{align}
[\mathcal{M}]_n \dddot{q}^n(t) + [\mathcal{D}]_n \dot{q}^n(t) + [\mathcal{K}]_n q^n(t) + F^n(t, q^n(t), \dot{q}^n(t); w) = \mathbf{F}^n(t), & \quad t \in [0, T], \quad (3) \\
\text{with the initial conditions,} & \quad q^n(0) = \dot{q}^n(0) = 0, \quad (4)
\end{align}\]
in which \( \mathbf{F}^n(t) \in \mathbb{R}^{m_f} \) and \( F^n(t, q, \dot{q}; w) \in \mathbb{R}^{m_f} \) are such that
\[\begin{align}
F^n(t) = -[\Phi_n]^T ([\mathcal{M}] + [\mathcal{D}]_s) \dot{z}_s(t) - [\Phi_n]^T ([\mathcal{D}] + [\mathcal{D}]_s) \dot{z}_s(t), & \quad (5) \\
F^n(t, p, q; w) = [\Phi_n]^T \tilde{f}_c(t, \tilde{z}_c, \tilde{p} + [\mathcal{R}]_s z_s(t), [\Phi_n] q + [\mathcal{R}] \dot{z}_s(t); w), & \quad (6)
\end{align}\]
where \( \tilde{f}_c(t, z(t), \dot{z}(t); w) \) is the nonlinear forces
induced by the stops whose parameters are represented
by vector \( w = (w_1, \ldots, w_n) \in \mathbb{R}^n \). Hereinafter, it
is assumed that the nonlinear dynamical problem
defined by Eqs. (3) and (4) has an unique solution
\( t \mapsto \dot{q}^n(t) \) from \([0, T]\) into \( \mathbb{R}^{m_f} \).

3. STOCHASTIC NONLINEAR DYNAMICAL
SYSTEM SUBJECTED TO SEISMIC LOADS

The nonparametric approach consists in substituting the
mean generalized mass, damping and stiffness matrices \( [\mathcal{M}], [\mathcal{D}], \) and \( [\mathcal{K}] \) of the mean reduced
matrix model by the random matrices \( [\mathcal{M}_n], [\mathcal{D}_n], \) and \( [\mathcal{K}_n] \). For the problem under consideration, the
parametric approach consists in substituting parameter
mean values \( w \) of the nonlinear forces by the \( \mathbb{R}^n \)-valued random variable \( W \). Consequently, the \( m \) unconstrained
DOFs and the \( m \) unconstrained relative DOFs are represented by the \( \mathbb{R}^m \)-valued stochastic processes \( Z^n(t) \) and \( Y^n(t) \) indexed by \([0, T]\), respectively, such that
\[\begin{align}
Z^n(t) = Y^n(t) + [\mathcal{R}] \dot{z}_s, & \quad Y^n(t) = [\Phi_n] Q^n(t), & \quad (7)
\end{align}\]
where the \( \mathbb{R}^n \)-valued stochastic process \( Q(t) \) indexed by \([0, T]\) is the unique second-order solution of the following stochastic nonlinear dynamical system,
\[\begin{align}
[\mathcal{M}_n] \dddot{q}^n(t) + [\mathcal{D}_n] \dot{q}^n(t) + [\mathcal{K}_n] q^n(t) + F^n(t, q^n(t), \dot{q}^n(t); W) = \mathbf{F}^n(t), & \quad t \in [0, T], \quad (8) \\
\text{with the initial conditions,} & \quad Q^n(0) = \dot{Q}^n(0) = 0, \quad a.s. \quad (9)
\end{align}\]
Random matrices \([\mathbf{M}_n], [\mathbf{D}_n]\) and \([\mathbf{K}_n]\) and random vector \(\mathbf{W} = (W_1, \ldots, W_{\nu})\) are second-order random variables subjected to the following constraints defined by the available information,

\[
\begin{align*}
[\mathbf{M}_n], [\mathbf{D}_n], [\mathbf{K}_n] & \in \operatorname{Mat}_R^{+}(n) \quad \text{a.s}, \\
E\{[\mathbf{M}_n]\} & = [\mathbf{M}_n], \\
E\{[\mathbf{D}_n]\} & = [\mathbf{D}_n], \\
E\{[\mathbf{K}_n]\} & = [\mathbf{K}_n], \\
E\{\|\mathbf{M}_n\|^{-1}\} & < +\infty, \\
E\{\|\mathbf{D}_n\|^{-1}\} & < +\infty, \\
E\{\|\mathbf{K}_n\|^{-1}\} & < +\infty, \\
W_1 & \in \mathcal{D}_1, \ldots, W_{\nu} \in \mathcal{D}_{\nu} \quad \text{a.s}, \\
E\{\mathbf{W}\} & = \mathbf{w}, 
\end{align*}
\]

in which \(E\) denotes the mathematical expectation and where \(\|\cdot\|_F = \operatorname{tr}\{(\|\cdot\|_F^2)\}^{1/2}\). In Eq. (10), \(\operatorname{Mat}_R^{+}(n)\) is the set of all the positive-definite symmetric \((n \times n)\) real matrices and in Eq. (15), for all \(1 \leq \ell \leq \nu, \mathcal{D}_\ell\) is a subset of \(\mathbb{R}\).

Let \(Z_1(t), \ldots, Z_n(t)\) be the coordinates of \(\mathbf{Z}^n(t)\). Let \(S_j(\xi, \omega)\) be the random normalized response spectrum of stochastic transient response \(Z^n_j(t)\) with \((\xi, \omega)\) belonging to \(J_\xi \times J_\omega \subseteq \mathbb{R}^2\) where \(J_\xi = [0, 1]\) and \(J_\omega = [\omega_{\min}, \omega_{\max}]\). Then we have

\[
S_j(\xi, \omega) = \frac{\omega^2}{g} \max_{t \in [0, T]} |X_j(t)|, 
\]

in which \(g\) is a normalization constant (for instance, the gravity acceleration) and where \(X_j(t)\) is the solution of the linear dynamical system,

\[
\ddot{X}_j + 2\xi_0 \dot{X}_j + \omega^2 X_j = -\mathbf{Z}^n_j, \quad t \in [0, T] 
\]

\[
X_j(0) = X_j(\theta_k) = 0. 
\]

Normal response spectrum \(S_j(\xi, \omega)\) is a random variable whose mean values \(m_{1j}(\xi, \omega)\), second-order moment \(m_{2j}(\xi, \omega)\), variance \(V_j(\xi, \omega)\) and standard deviation \(\sigma_j(\xi, \omega)\) are estimated by the Monte-Carlo numerical simulation. The stochastic convergence of the probabilistic model is studied with respect to \(n\) and with respect to the number \(n_s\) of samples for the Monte-Carlo numerical simulation, by introducing the norm \(\|\| \mathbf{Z}^n_j\|\|\) defined as

\[
\|\| \mathbf{Z}^n_j\|\|^2 = E\{\int_0^T \mathbf{Z}^n_j(t)^2 dt\},
\]

This norm is estimated by \(\|\| \mathbf{Z}^n_j\|\| \approx \text{Conv}_j(n_s, n)\) with

\[
\text{Conv}_j(n_s, n) = \frac{1}{n_s} \sum_{k=1}^{n_s} \int_0^T \mathbf{Z}^n_j(t, \theta_k)^2 dt.
\]

where \(\theta_1, \ldots, \theta_{n_s}\) denotes the \(n_s\) samples for the Monte-Carlo numerical simulation.

Let \(dB_j(\xi, \omega)\) be the random variable such that \(dB_j(\xi, \omega) = \log_{10}(S_j(\xi, \omega))\). The confidence region associated with the probability level \(P_c\) is delimited by the upper envelope \(dB^+_j(\xi, \omega)\) and the lower envelope \(dB^-_j(\xi, \omega)\) such that \(\Pr\{dB^-_j(\xi, \omega) < dB_j(\xi, \omega) \leq dB^+_j(\xi, \omega)\} \geq P_c\) in which \(dB^+_j\) and \(dB^-_j\) are constructed by using the Tchebychev inequality and which are such that \(dB^-_j(\xi, \omega) = 2dB^0_j(\xi, \omega) - dB^+_j(\xi, \omega)\) and \(dB^+_j(\xi, \omega) = \log_{10}(m_{1j}(\xi, \omega) + a_j(\xi, \omega))\), in which \(dB^0_j(\xi, \omega) = \log_{10}(m_{1j}(\xi, \omega))\) and \(a_j(\xi, \omega) = \sigma_j(\xi, \omega)/(\sqrt{1 - P_c})\).

The probability model of random matrices \([\mathbf{M}_n], [\mathbf{D}_n]\) and \([\mathbf{K}_n]\) are defined in references [8-10]. The probability model of random variable \(\mathbf{W}\) is constructed by using Jayne’s maximum entropy principle with the constraints defined by Eqs. (15) and (16). If \(\mathcal{D}_\ell\) is a bounded interval of \(\mathbb{R}\) such that \(\mathcal{D}_\ell = [a_\ell, b_\ell]\) then it can be shown that, for all \(1 \leq \ell \leq \nu\), the probability density function \(p_{W_\ell}(w)\) of random variable \(W_\ell\) is written as

\[
p_{W_\ell}(w) = \mathbb{1}_{[a_\ell, b_\ell]}(w) \frac{k_\ell}{\alpha_\ell} e^{-k_\ell w},
\]

in which \(\mathbb{1}_{B}(w)\) is is the indicatrix function of the set B and where the positive real \(k_\ell\) is such that \((w_\ell k_\ell - 1) \alpha_\ell k_\ell - k_\ell \beta_\ell k_\ell = 0\) in which \(\alpha_\ell(k) = e^{-a_\ell k} - e^{-b_\ell k}\) and \(\beta_\ell(k) = a_\ell e^{-a_\ell k} - b_\ell e^{-b_\ell k}\). If there exists a real \(a_\ell\) such that \(\mathcal{D}_\ell = [a_\ell, +\infty[\), then it can be shown that

\[
p_{W_\ell}(w) = \mathbb{1}_{[a_\ell, +\infty[}(w) \frac{e^{-(w-a_\ell)/(\delta_\ell^2)}}{w_\ell - a_\ell}.
\]

If the additional constraint \(E\{W_\ell - a_\ell\} < +\infty\) is introduced, then the probability density function \(p_{W_\ell}(w)\) is such that

\[
p_{W_\ell}(w) = \mathbb{1}_{[a_\ell, +\infty[}(w) \times C_{W_\ell}(w - a_\ell)/(\delta_\ell^2)\}^{1/(\delta_\ell^2)}
\times e^{-(w-a_\ell)/(\delta_\ell^2)}
\]

in which

\[C_{W_\ell} = ( \omega_\ell \delta_\ell^2 - a_\ell \delta_\ell^2 ) - \frac{\delta_\ell^2}{\Gamma\left(\frac{1}{\delta_\ell^2}\right)},\]

where the real parameter \(\delta_\ell > 0\) allows the dispersion of random variable \(W_\ell\) to be fixed. Let \(\sigma_\ell\) be the standard deviation of random variable \(W_\ell\). It can be shown that \(\delta_\ell = \sigma_\ell/|\omega_\ell - a_\ell|\).
4. MULTISUPPORTED REACTOR COOLANT SYSTEM SUBJECTED TO SEISMIC LOADS

The structure under consideration is a typical four loops reactor coolant system [11] (see Fig. 1). Each loop is constituted of a reactor, a reactor coolant pump and a steam generator (see Fig. 2). These three elements are connected to each other by three primary coolant pipes: a hot leg which links the reactor with the steam generator, a cold leg which links the reactor with a reactor coolant pump and an intermediate leg which links the reactor coolant with and the steam generator. The structure is multisupported with 36 supports. Its supports are constituted of anchors located under the reactor coolant pumps, the steam generators and the cold legs. Due to seismic loads, the displacement field of all the 36 supports are constrained by time-dependent Dirichlet conditions (mesh nodes 1 of Fig. 2).

The vibrations of each steam generator (see Fig. 3) are limited by three elastic stops located at their connection point with the intermediate leg and the hot leg (mesh nodes 3 of Fig. 3) and by four elastic stops located at the middle of each steam generator (mesh nodes 2 of Fig. 3). These elastic stops induce nonlinear restoring forces. Furthermore, each elastic stops is subjected to seismic load and consequently, at each stop, the displacement is constrained by a time-dependent Dirichlet condition. The mean finite element model of the reactor coolant system is composed of beam finite elements. Let $j_{stop}$ be the DOF number corresponding to the horizontal translation of the steam generator mesh node $B$ which is close to four stops (see Figs.1 to 3). Let $j_{obs}$ be the DOF number corresponding to the horizontal translation of the node $A$ (see Figs. 1 and 2).

![Figure 1. Four loops reactor coolant system.](image1)

![Figure 2. One loop : one reactor, one reactor coolant system, one steam generator and three coolant pipes.](image2)

![Figure 3. Steam generator.](image3)

![Figure 4. Normalized response spectrum of the mean reduced matrix model of the nonlinear dynamical system for DOF number $j_{stop}$](image4)
Figure 4 to 5 show the normalized response spectra of the mean reduced matrix model of the nonlinear dynamical system for DOF number $j_{stop}$ (Fig 4) and for DOF number $j_{obs}$ (Fig 5).

The dispersions of random matrices $[M_n]$, $[D_n]$ or $[K_n]$ are controlled by parameters $\delta_M$, $\delta_D$ and $\delta_K$. The stochastic convergence analysis is performed for $n \geq 5$ which yields $n_0 = 5$. Consequently, parameters $\delta_M$, $\delta_D$ and $\delta_K$ are such that

$$0 < \delta_M, \delta_D, \delta_K < 0.9258 = \frac{\sqrt{n_0 + 1}}{n_0 + 5} \quad (26)$$

Data uncertainties are relative to the stiffnesses of the stops whose probabilistic models are defined by Eqs.(24) to (25). Parameters $\{W_\ell\}_{1 \leq \ell \leq \nu}$ correspond to the 28 stiffnesses of the 28 elastic stops. Therefore, each parameter belongs to $D_\ell = [0, +\infty[$. Since the structure is multisupported and the number of nonlinear elastic stops is large, then the solution is very sensitive to the value of the time-step size $\Delta_t$ of the time integration scheme. Time-step size $\Delta_t$ has the same value for all $n \in \{5, 50, 100, 200\}$. For each sample $\theta_k$, Eq.(8) is solved by using the Euler explicit step-by-step integration scheme with $\Delta t = 1/25000s$ and for a time duration $T = 15s$. The Monte-Carlo numerical simulation is performed with $n_s = 700$ samples with a finite element code [12]. Figure 6 shows the graphs of the functions $n_s \mapsto \log_{10}\{\text{Conv}_{j_{obs}}(n_s, n)\}$ for $n = \{5, 50, 100, 200\}$ and for $\delta_M = \delta_D = \delta_K = 0.2$ and $\delta_1 = \ldots = \delta_28 = 0.2$. It can be seen that convergence with respect to $n$ and $n_s$ is obtained for $n = 100$ and $n_s = 500$. Figures 7 to 10 show the confidence region associated with the probability level $P_c = 0.95$ for $n_s = 700$, $n = 200$, $\xi = 0.001$.

Data uncertainties are relative to the stiffnesses of the stops whose probabilistic models are defined by Eqs.(24) to (25). Parameters $\{W_\ell\}_{1 \leq \ell \leq \nu}$ correspond to the 28 stiffnesses of the 28 elastic stops. Therefore, each parameter belongs to $D_\ell = [0, +\infty[$. Since the structure is multisupported and the number of nonlinear elastic stops is large, then the solution is very sensitive to the value of the time-step size $\Delta_t$ of the time integration scheme. Time-step size $\Delta_t$ has the same value for all $n \in \{5, 50, 100, 200\}$. For each sample $\theta_k$, Eq.(8) is solved by using the Euler explicit step-by-step integration scheme with $\Delta t = 1/25000s$ and for a time duration $T = 15s$. The Monte-Carlo numerical simulation is performed with $n_s = 700$ samples with a finite element code [12]. Figure 6 shows the graphs of the functions $n_s \mapsto \log_{10}\{\text{Conv}_{j_{obs}}(n_s, n)\}$ for $n = \{5, 50, 100, 200\}$ and for $\delta_M = \delta_D = \delta_K = 0.2$ and $\delta_1 = \ldots = \delta_28 = 0.2$. It can be seen that convergence with respect to $n$ and $n_s$ is obtained for $n = 100$ and $n_s = 500$. Figures 7 to 10 show the confidence region associated with the probability level $P_c = 0.95$ for $n_s = 700$, $n = 200$, $\xi = 0.001$.
For such a dynamical system, the nonparametric-parametric approach allows the level of uncertainties to be extended and is well adapted to this kind of problem.

REFERENCES


