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Uncertain nonlinear dynamical systems subjected to seismic loads

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ABSTRACT : This paper deals with the transient response of a nonlinear dynamical system with random uncertainties and subjected to earthquake. The nonparametric probabilistic model of random uncertainties recently published and extended to nonlinear dynamical system analysis is used in order to model random uncertainties related to the linear part of the finite element model. The nonlinearities are due to restoring forces whose parameters are uncertain and are modeled by the parametric approach. Jayne's maximum entropy principle with the constraints defined by the available information allow the probabilistic model of such random variables to be constructed. Therefore, a nonparametric-parametric formulation is developed in order to model all the sources of uncertainties in such a nonlinear dynamical system. Finally, a numerical application for earthquake engineering analysis is proposed and concerned a reactor coolant system under seismic loads.

1. INTRODUCTION

This paper deals with the transient response of a nonlinear dynamical system with random uncertainties. The source of random uncertainties is induced by the model uncertainties (or the model errors) and the data uncertainties (errors on the parameters of the model). For the problem under consideration, data uncertainties concern the local parameters of the finite element model and the parameters of the nonlinear forces. Usually, parametric approaches are used to model data uncertainties [1,2] for evaluating and analyzing the response of structures with uncertain parameters under seismic loads, like piping and equipement, power plant installations and industrial structures [3-7]. Nevertheless, such approaches do not allow model uncertainties to be taken into account. It should be noted that model uncertainties can only be modeled by using a nonparametric approach. Recently, a nonparametric model of random uncertainties has been introduced for linear dynamical system analysis [8,9]. This nonparametric model has also been extended to the transient response of nonlinear dynamical system [10] without having random uncertainties on the nonlinear part. This nonparametric model is constructed by the use of Jayne's entropy principle which consists in maximizing the probabilistic uncertainties with the constraints defined by the available information (for instance, related to algebraic properties of the finite element matrices). Nevertheless, this nonparametric formulation does not allow the uncertainties related to the nonlinear restoring forces to be modeled. This is the reason why a nonparametric-parametric formulation is proposed for analyzing nonlinear dynamical systems subjected to seismic loads with data uncertainties for the nonlinearities. An application to a multisupported reactor coolant system subjected to seismic loads is presented.

2. MEAN REDUCED MATRIX MODEL OF THE NONLINEAR DYNAMICAL SYSTEM SUBJECTED TO SEISMIC LOADS

We consider a nonlinear dynamical system constituted of a damped structure subjected to m_s deterministic time-dependent dirichlet conditions corresponding to seismic loads applied to the supports of the structure. The nonlinearities are due to stops with high stiffness, installed with a given gap at given points of the structure. Let m_f be the number of degrees of freedom of the mean finite element model of this nonlinear dynamical system. Let $\boldsymbol{z}$ be the $\mathbb{R}^{m_f}$-vector of the total displacements, including the displacements of the supports. We are interested in the transient response $t \mapsto \boldsymbol{z}\,(t)$ from $[0,T]$ into $\mathbb{R}^{m_f}$. Let $[\underline{\mathcal{M}}]$, $[\underline{\mathcal{D}}]$ and $[\underline{\mathcal{K}}]$ be the mass, damping and stiffness matrices of the linear part of the mean finite element model. It is assumed that the structure with fixed supports has no rigid body modes. Consequently $[\underline{\mathcal{M}}]$, $[\underline{\mathcal{D}}]$ and $[\underline{\mathcal{K}}]$ are positive-definite symmetric $(m_f \times m_f)$ real matrices. The real-valued vector $\boldsymbol{z}\,(t)$ is rewritten as $\boldsymbol{z}\,(t) = (\underline{\mathbf{z}}(t), \underline{\mathbf{z}}_s(t))$ in which $\underline{\mathbf{z}}_s(t) \in \mathbb{R}^{m_s}$ is the vector of the constrained DOFs at the supports and where $\underline{\mathbf{z}}(t) \in \mathbb{R}^m$ is the vector of the m unconstrained DOFs with $m = m_f - m_s$. The block splitting of matrices $[\underline{\mathcal{M}}]$, $[\underline{\mathcal{D}}]$ and $[\underline{\mathcal{K}}]$ relative to $\boldsymbol{z}\,(t) = (\underline{\mathbf{z}}(t), \underline{\mathbf{z}}_s(t))$ are introduced,

$$[\underline{\mathcal{M}}] = \begin{bmatrix} [\underline{\mathbb{M}}] & [\underline{\mathbb{M}}_{ls}] \\ [\underline{\mathbb{M}}_{ls}]^T & [\underline{\mathbb{M}}_s] \end{bmatrix}, [\underline{\mathcal{D}}] = \begin{bmatrix} [\underline{\mathbb{D}}] & [\underline{\mathbb{D}}_{ls}] \\ [\underline{\mathbb{D}}_{ls}]^T & [\underline{\mathbb{D}}_s] \end{bmatrix},$$

$$[\underline{\mathcal{K}}] = \begin{bmatrix} [\underline{\mathbb{K}}] & [\underline{\mathbb{K}}_{ls}] \\ [\underline{\mathbb{K}}_{ls}]^T & [\underline{\mathbb{K}}_s] \end{bmatrix} \quad . \tag{1}$$

Since $[\underline{\mathcal{M}}]$, $[\underline{\mathcal{D}}]$ and $[\underline{\mathcal{K}}]$ are positive-definite matrices, then $[\underline{\mathbb{M}}]$, $[\underline{\mathbb{D}}]$ and $[\underline{\mathbb{K}}]$ are also positive-definite symmetric $(m \times m)$ real matrices. Consequently, the inverse matrix $[\underline{\mathbb{K}}]^{-1}$ exists and the $(m \times m_s)$ real matrix $[\underline{R}] = -[\underline{\mathbb{K}}]^{-1}[\underline{\mathbb{K}}_{ls}]$ and the relative unconstrained DOFs vector $\underline{\mathbf{y}}(t) = \underline{\mathbf{z}}(t) - [\underline{R}]\ \underline{\mathbf{z}}_s(t)$ belonging to $\mathbb{R}^m$ can be defined. Let $\{\omega_\alpha\}_{0<\alpha\leq n}$ be the eigenfrequencies of the corresponding structural modes $\{\boldsymbol{\varphi}_\alpha\}_{0<\alpha\leq n}$ such that $[\underline{\mathbb{K}}]\boldsymbol{\varphi}_\alpha = \omega_\alpha^2[\underline{\mathbb{M}}]\boldsymbol{\varphi}_\alpha$. Let $\underline{\mathbf{y}}^n(t)$ be the projection of $\underline{\mathbf{y}}(t)$ on the subspace of $\mathbb{R}^m$ spanned by $\{\boldsymbol{\varphi}_\alpha\}_{0<\alpha\leq n}$ with $n \ll m$ such that $\underline{\mathbf{y}}^n(t) = [\underline{\Phi}_n]\ \underline{\mathbf{q}}^n(t)$, in which $[\underline{\Phi}_n]$ is the $(m \times n)$ real matrix whose columns are vectors $\{\boldsymbol{\varphi}_\alpha\}_{0<\alpha\leq n}$ and where $\underline{\mathbf{q}}^n(t)$ is the vector of the generalized coordinates belonging to $\mathbb{R}^n$. Let the positive-definite symmetric $(n \times n)$ real matrices $[\underline{M}_n]$, $[\underline{D}_n]$ and $[\underline{K}_n]$ be the generalized mass, damping and stiffness matrices of the nonlinear dynamical system such that

$$[\underline{M}_n] = [\underline{\Phi}_n]^T\ [\underline{\mathbb{M}}]\ [\underline{\Phi}_n]\ ,\ [\underline{D}_n] = [\underline{\Phi}_n]^T\ [\underline{\mathbb{D}}]\ [\underline{\Phi}_n]\ ,$$
$$[\underline{K}_n] = [\underline{\Phi}_n]^T\ [\underline{\mathbb{K}}]\ [\underline{\Phi}_n] \quad . \tag{2}$$

It can be shown that generalized coordinates $\underline{\mathbf{q}}^n(t)$ is a solution of the reduced matrix model of the nonlinear dynamical system,

$$\begin{aligned}[\underline{M}_n]\ \ddot{\underline{\mathbf{q}}}^n(t) + [\underline{D}_n]\ \dot{\underline{\mathbf{q}}}^n(t) + [\underline{K}_n]\ \underline{\mathbf{q}}^n(t) \\ + \mathbf{F}_c^n(t, \underline{\mathbf{q}}^n(t), \dot{\underline{\mathbf{q}}}^n(t); \underline{\mathbf{w}}) \\ = \underline{\mathbf{F}}^n(t) \quad , \quad t \in [0,T] \quad ,\end{aligned} \tag{3}$$

with the initial conditions,

$$\underline{\mathbf{q}}^n(0) = \dot{\underline{\mathbf{q}}}^n(0) = 0 \quad , \tag{4}$$

in which $\underline{\mathbf{F}}^n(t) \in \mathbb{R}^m$ and $\mathbf{F}_c^n(t, \mathbf{q}\ , \mathbf{p}\ ; \underline{\mathbf{w}}) \in \mathbb{R}^m$ are such that

$$\begin{aligned}\underline{\mathbf{F}}^n(t) = -[\underline{\Phi}_n]^T\ ([\underline{\mathbb{M}}]\ [\underline{R}] + [\underline{\mathbb{M}}_{ls}])\ \ddot{\underline{\mathbf{z}}}_s(t) \\ - [\underline{\Phi}_n]^T\ ([\underline{\mathbb{D}}]\ [\underline{R}] + [\underline{\mathbb{D}}_{ls}])\ \dot{\underline{\mathbf{z}}}_s(t) \quad ,\end{aligned} \tag{5}$$

$$\begin{aligned}\mathbf{F}_c^n(t, \mathbf{p}\ , \mathbf{q}\ ; \underline{\mathbf{w}}) = [\underline{\Phi}_n]^T\ \mathbb{f}_c(t, [\underline{\Phi}_n]\ \mathbf{p}\ + [\underline{R}]\ \underline{\mathbf{z}}_s(t), \\ [\underline{\Phi}_n]\ \mathbf{q}\ + [\underline{R}]\ \dot{\underline{\mathbf{z}}}_s(t); \underline{\mathbf{w}}) \quad ,\end{aligned} \tag{6}$$

where $\mathbb{f}_c(t, \underline{\mathbf{z}}(t), \dot{\underline{\mathbf{z}}}(t); \underline{\mathbf{w}})$ is the nonlinear forces induced by the stops whose parameters are represented by vector $\underline{\mathbf{w}} = (\underline{w}_1, \ldots, \underline{w}_\nu) \in \mathbb{R}^\nu$. Hereinafter, it is assumed that the nonlinear dynamical problem defined by Eqs. (3) and (4) has an unique solution $t \mapsto \underline{\mathbf{q}}^n(t)$ from $[0,T]$ into $\mathbb{R}^n$.

3. STOCHASTIC NONLINEAR DYNAMICAL SYSTEM SUBJECTED TO SEISMIC LOADS

The nonparametric approach consists in substituting the mean generalized mass, damping and stiffness matrices $[\underline{M}_n]$, $[\underline{D}_n]$ and $[\underline{K}_n]$ of the mean reduced matrix model by the random matrices $[\mathbf{M}_n]$, $[\mathbf{D}_n]$ and $[\mathbf{K}_n]$. For the problem under consideration, the parametric approach consists in substituting parameter mean values $\underline{\mathbf{w}}$ of the nonlinear forces by the $\mathbb{R}^\nu$-valued random variable $\mathbf{W}$. Consequently, the m unconstrained DOFs and the m unconstrained relative DOFs are represented by the $\mathbb{R}^m$-valued stochastic processes $\mathbf{Z}^n(t)$ and $\mathbf{Y}^n(t)$ indexed by $[0,T]$, respectively, such that

$$\mathbf{Z}^n(t) = \mathbf{Y}^n(t) + [\underline{R}]\ \underline{\mathbf{z}}_s\ ,\ \mathbf{Y}^n(t) = [\underline{\Phi}_n]\ \mathbf{Q}^n(t)\ , \tag{7}$$

where the $\mathbb{R}^n$-valued stochastic process $\mathbf{Q}(t)$ indexed by $[0,T]$ is the unique second-order solution of the following stochastic nonlinear dynamical system,

$$\begin{aligned}[\mathbf{M}_n]\ \ddot{\mathbf{Q}}^n(t) + [\mathbf{D}_n]\ \dot{\mathbf{Q}}^n(t) + [\mathbf{K}_n]\ \mathbf{Q}^n(t) \\ + \mathbf{F}_c^n(t, \mathbf{Q}^n(t), \dot{\mathbf{Q}}^n(t); \mathbf{W}) \\ = \underline{\mathbf{F}}^n(t) \quad , \quad t \in [0,T] \quad ,\end{aligned} \tag{8}$$

with the initial conditions,

$$\mathbf{Q}^n(0) = \dot{\mathbf{Q}}^n(0) = 0 \quad , \quad \text{a.s} \quad . \tag{9}$$

Random matrices $[\mathbf{M}_n]$, $[\mathbf{D}_n]$ and $[\mathbf{K}_n]$ and random vector $\mathbf{W} = (W_1, \ldots, W_\nu)$ are second-order random variables subjected to the following constraints defined by the available information,

$$[\mathbf{M}_n]\,,\,[\mathbf{D}_n]\,,\,[\mathbf{K}_n] \in \mathrm{Mat}_{\mathbb{R}}^{+}(n) \quad \text{a.s} \quad , \tag{10}$$

$$E\{[\mathbf{M}_n]\} = [\underline{M}_n] \quad , \quad E\{[\mathbf{D}_n]\} = [\underline{D}_n] \quad ,$$
$$E\{[\mathbf{K}_n]\} = [\underline{K}_n] \tag{11}$$

$$E\{\|[\mathbf{M}_n]^{-1}\|_F^2\} < \; +\infty \,, \tag{12}$$

$$E\{\|[\mathbf{D}_n]^{-1}\|_F^2\} < \; +\infty \,, \tag{13}$$

$$E\{\|[\mathbf{K}_n]^{-1}\|_F^2\} < \; +\infty \,, \tag{14}$$

$$W_1 \in \mathcal{D}_1, \ldots, W_\nu \in \mathcal{D}_\nu \quad , \quad \text{a.s} \quad , \tag{15}$$

$$E\{\mathbf{W}\} = \underline{\mathbf{w}} \quad , \tag{16}$$

in which E denotes the mathematial expectation and where $\|[A]\|_F = (\mathrm{tr}\{[A]\;[A]^T\})^{1/2}$. In Eq. (10), $\mathrm{Mat}_{\mathbb{R}}^{+}(n)$ is the set of all the positive-definite symmetric $(n \times n)$ real matrices and in Eq. (15), for all $1 \leq \ell \leq \nu$, $\mathcal{D}_\ell$ is a subset of $\mathbb{R}$.

Let $Z_1(t), \ldots, Z_n(t)$ be the coordinates of $\mathbf{Z}^n(t)$. Let $S_j(\xi, \omega)$ be the random normalized response spectrum of stochastic transient response $Z_j^n(t)$ with (ξ, ω) belonging to $J_\xi \times J_\omega \subset \mathbb{R}^2$ where $J_\xi = [0,1]$ and $J_\omega = [\omega_{min}, \omega_{max}]$. We then have

$$S_j(\xi, \omega) = \frac{\omega^2}{g} \max_{t \in [0,T]} |X_j(t)| \quad , \tag{17}$$

in which g is a normalization constant (for instance, the gravity acceleration) and where $X_j(t)$ is the solution of the linear dynamical system,

$$\ddot{X}_j + 2\xi\omega\dot{X}_j + \omega^2 X_j = -Z_j^n \quad , \quad t \in [0,T] \tag{18}$$

$$X_j(0) = \dot{X}_j(0) = 0 \quad . \tag{19}$$

Normalized response spectrum $S_j(\xi, \omega)$ is a random variable whose mean values $m_{1j}(\xi, \omega)$, second-order moment $m_{2j}(\xi, \omega)$, variance $V_j(\xi, \omega)$ and standard deviation $\sigma_j(\xi, \omega)$ are estimated by the Monte-Carlo numerical simulation. The stochastic convergence of the probabilistic model is studied with respect to n and with respect to the number n_s of samples for the Monte-Carlo numerical simulation, by introducing the norm $|||\ddot{Z}_j^n|||$ defined as

$$|||\ddot{Z}_j^n|||^2 = E\{\int_0^T \ddot{Z}_j^n(t)^2 dt\} \quad . \tag{20}$$

This norm is estimated by $|||\ddot{Z}_j^n||| \simeq \mathrm{Conv}_j(n_s, n)$ with

$$\mathrm{Conv}_j(n_s, n)^2 = \frac{1}{n_s} \sum_{k=1}^{n_s} \int_0^T \ddot{Z}_j^n(t, \theta_k)^2 dt \quad , \tag{21}$$

where $\theta_1, \ldots, \theta_{n_s}$ denotes the n_s samples for the Monte-Carlo numerical simulation.

Let $dB(\xi, \omega)$ be the random variable such that $dB_j(\xi, \omega) = \log_{10}(S_j(\xi, \omega))$. The confidence region associated with the probability level P_c is delimited by the upper envelope $dB_j^+(\xi, \omega)$ and the lower envelope $dB_j^-(\xi, \omega)$ such that $Proba\{dB_j^-(\xi, \omega) < dB_j(\xi, \omega) \leq dB_j^+(\xi, \omega)\} \geq P_c$ in which dB_j^+ and dB_j^- are constructed by using the Tchebychev inequality and which are such that $dB_j^-(\xi, \omega) = 2dB_j^0(\xi, \omega) - dB_j^+(\xi, \omega)$ and $dB_j^+(\xi, \omega) = \log_{10}(m_{1j}(\xi, \omega) + a_j(\xi, \omega))$ in which $dB_j^0(\xi, \omega) = \log_{10}(m_{1j}(\xi, \omega))$ and $a_j(\xi, \omega) = \sigma_j(\xi, \omega)/(\sqrt{1 - P_c})$.

The probability model of random matrices $[\mathbf{M}_n]$, $[\mathbf{D}_n]$ and $[\mathbf{K}_n]$ are defined in references [8-10]. The probability model of random variable $\mathbf{W}$ is constructed by using Jayne's maximum entropy principle with the constraints defined by Eqs. (15) and (16). If $\mathcal{D}_\ell$ is a bounded interval of $\mathbb{R}$ such that $\mathcal{D}_\ell = [a_\ell, b_\ell]$ then it can be shown that, for all $1 \leq \ell \leq \nu$, the probability density function $p_{W_\ell}(w)$ of random variable W_ℓ is written as

$$p_{W_\ell}(w) = \mathbb{1}_{[a_\ell, b_\ell]}(w)\, \frac{k_\ell}{\alpha_\ell(k_\ell)}\, e^{-k_\ell w} \quad , \tag{22}$$

in which $\mathbb{1}_B(\omega)$ is is the indicatrix function of the set B and where the positive real k_ℓ is such that $(\underline{w}_\ell k_\ell - 1)\, \alpha_\ell(k_\ell) - k_\ell \beta_\ell(k_\ell) = 0$ in which $\alpha_\ell(k) = e^{-a_\ell k} - e^{-b_\ell k}$ and $\beta_\ell(k) = a_\ell\, e^{-a_\ell k} - b_\ell\, e^{-b_\ell k}$. If there exists a real a_ℓ such that $\mathcal{D}_\ell = [a_\ell, +\infty[$, then it can be shown that

$$p_{W_\ell}(w) = \mathbb{1}_{[a_\ell, +\infty[}(w)\, \frac{e^{-(w - a_\ell)/(\underline{w}_\ell - a_\ell)}}{\underline{w}_\ell - a_\ell} \quad . \tag{23}$$

If the additional constraint $E\{(W_\ell - a_\ell)^{-2}\} < +\infty$ is introduced, then the probability density function $p_{W_\ell}(w)$ is such that

$$p_{W_\ell}(w) = \mathbb{1}_{[a_\ell, +\infty[}(w) \times C_{W_\ell} \times (w - a_\ell)^{(1 - \delta_\ell^2)/(\delta_\ell^2)}$$
$$\times\, e^{-(w - a_\ell)/((\underline{w}_\ell - a_\ell)\, \delta_\ell^2)} \quad , \tag{24}$$

in which

$$C_{W_\ell} = (\underline{w}_\ell\, \delta_\ell^2 - a_\ell\, \delta_\ell^2)^{-\frac{1}{\delta_\ell^2}} / \Gamma(\frac{1}{\delta_\ell^2}) \quad , \tag{25}$$

where the real parameter $\delta_\ell > 0$ allows the dispersion of random variable W_ℓ to be fixed. Let σ_ℓ be the standard deviation of random variable W_ℓ. It can be shown that $\delta_\ell = \sigma_\ell / |\underline{w}_\ell - a_l|$.

4. MULTISUPPORTED REACTOR COOLANT SYSTEM SUBJECTED TO SEISMIC LOADS

The structure under consideration is a typical four loops reactor coolant system [11] (see Fig. 1). Each loop is constituted of a reactor, a reactor coolant pump and a steam generator (see Fig. 2). These three elements are connected to each other by three primary coolant pipes: a hot leg which links the reactor with the steam generator, a cold leg which links the reactor with a reactor coolant pump and an intermediate leg which links the reactor coolant with and the steam generator. The structure is multisupported with 36 supports. Its supports are constituted of anchors located under the reactor coolant pumps, the steam generators and the cold legs. Due to seismic loads, the displacement field of all the 36 supports are constrained by time-dependent Dirichlet conditions (mesh nodes 1 of Fig. 2).

The vibrations of each steam generator (see Fig. 3) are limited by three elastic stops located at their connection point with the intermediate leg and the hot leg (mesh nodes 3 of Fig. 3) and by four elastic stops located at the middle of each steam generator (mesh nodes 2 of Fig. 3). These elastic stops induce nonlinear restoring forces. Futhermore, each elastic stops is subjected to seismic load and consequently, at each stop, the displacement is constrained by a time-dependent Dirichlet condition. The mean finite element model of the reactor coolant system is composed of beam finite elements. Let j_{stop} be the DOF number corresponding to the horizontal translation of the steam generator mesh node B which is close to four stops (see Figs.1 to 3). Let j_{obs} be the DOF number corresponding to the horizontal translation of the node A (see Figs. 1 and 2).

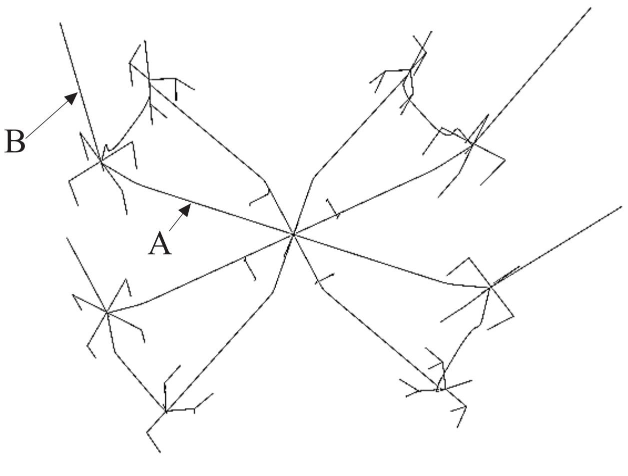

Figure 1. Four loops reactor coolant system.

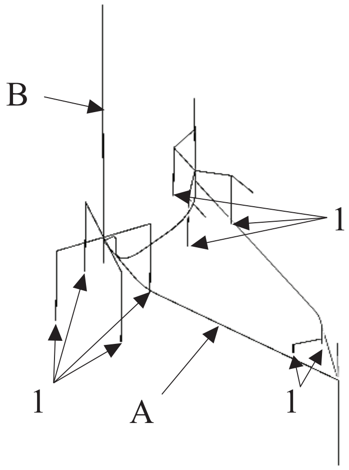

Figure 2. One loop : one reactor, one reactor coolant system, one steam generator and three coolant pipes.

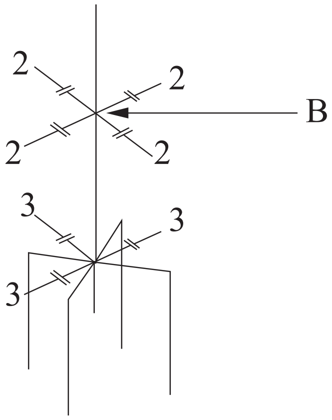

Figure 3. Steam generator.

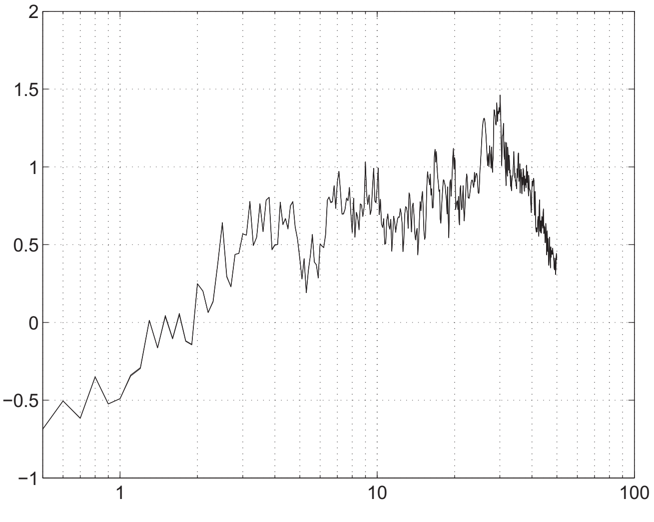

Figure 4. Normalized response spectrum of the mean reduced matrix model of the nonlinear dynamical system for DOF number j_{stop}

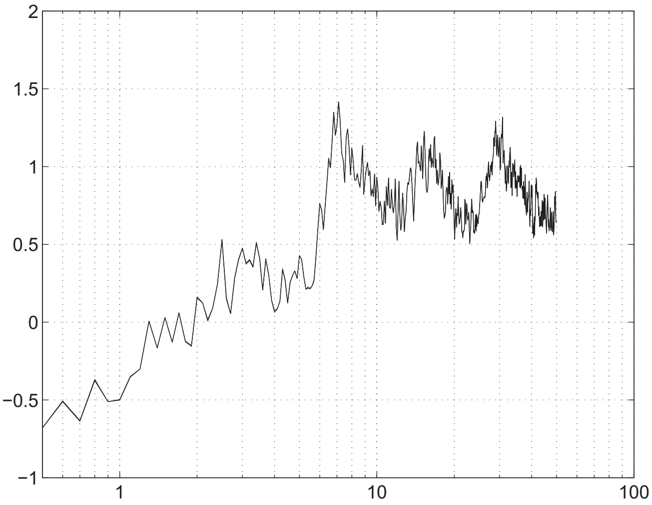

Figure 5. Normalized response spectrum of the mean reduced matrix model of the nonlinear dynamical system for DOF number j_{obs}. The horizontal axis in log scale correspond to the frequency (in Hz).

Figure 4 to 5 show the normalized response spectra of the mean reduced matrix model of the nonlinear dynamical system for DOF number j_{stop} (Fig 4) and for DOF number j_{obs} (Fig. 5).

The dispersions of random matrices $[\mathbf{M}_n]$, $[\mathbf{D}_n]$ or $[\mathbf{K}_n]$ are controlled by parameters δ_M, δ_D and δ_K. The stochastic convergence analysis is performed for $n \geq 5$ which yields $n_0 = 5$. Consequently, parameters δ_M, δ_D and δ_K are such that

$$0 < \delta_M, \delta_D, \delta_K < 0.9258 = \sqrt{\frac{n_0 + 1}{n_0 + 5}} \quad . \tag{26}$$

Data uncertainties are relative to the stiffnesses of the stops whose probabilistic models are defined by Eqs.(24) to (25). Parameters $\{W_\ell\}_{1 \leq \ell \leq \nu}$ correspond to the 28 stiffnesses of the 28 elastic stops. Therefore, each parameter belongs to $\mathcal{D}_\ell = [0, +\infty[$. Since the structure is multisupported and the number of nonlinear elastic stops is large, then the solution is very sensitive to the value of the time-step size Δt of the time integration scheme. Time-step size Δt has the same value for all $n \in \{5, 50, 100, 200\}$. For each sample θ_k, Eq.(8) is solved by using the Euler explicit step-by-step integration scheme with $\Delta t = 1/25000 s$ and for a time duration $T = 15 s$. The Monte-Carlo numerical simulation is performed with $n_s = 700$ samples with a finite element code [12]. Figure 6 shows the graphs of the functions $n_s \mapsto \log_{10}\{\text{Conv}_{j_{obs}}(n_s, n)\}$ for $n = \{5, 50, 100, 200\}$ and for $\delta_M = \delta_D = \delta_K = 0.2$ and $\delta_1 = \ldots = \delta_{28} = 0.2$. It can be seen that convergence with respect to n and n_s is obtained for $n = 100$ and $n_s = 500$. Figures 7 to 10 show the confidence region associated with the probability level $P_c = 0.95$ for $n_s = 700$, $n = 200$, $\xi = 0.001$. Figures 7 and 9 are relative to DOF j_{stop} and Figs. 8 and 10 are relative to DOF j_{obs}.

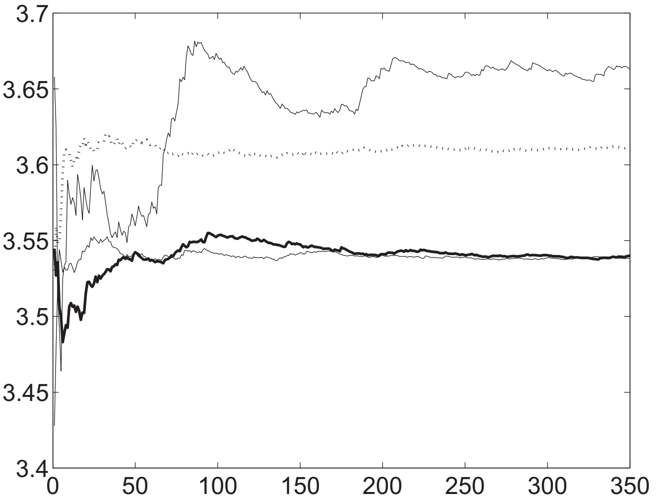

Figure 6. Graph of function $n_s \mapsto \log_{10}\{\text{Conv}_{j_{obs}}(n_s, n)\}$ for $n = 5$ (upper solid lines), $n = 50$ (thick dotted line), $n = 100$ (lower thin solid line) and $n = 200$ (lower thick solid line).

Figures 7 and 8 correspond to a nonparametric and a parametric level of uncertainties equal to 0.2 while Figs. 9 and 10 correspond to a nonparametric level of uncertainties equal to 0.00002 and a parametric level of uncertainties equal to 0.

5. COMMENTS AND CONCLUSIONS

A nonparametric-parametric probabilistic model of random uncertainties has been developed for nonlinear dynamical system in the time domain. A nonparametric probabilistic model of random uncertainties is used for modeling the random uncertainties concerning the linear part of the system. The random uncertainties of the nonlinear part is modeled by the use of a parametric approach.

From the analysis of the normalized response spectra, it can be seen that a DOF close to stops is sensitive to the parametric uncertainties while a DOF not close to a stop is sensitive to the nonparametric uncertainties. In addition, it should be noted that the responses are always very sensitive to uncertainties. For almost zero level of uncertainties, the random responses look like to chaotic responses due to a large number of shocks along the structure. Consequently, it is necessary to model the uncertainties for such a nonlinear dynamical system in order to allow an efficient engineering design process of such a structure to be performed.

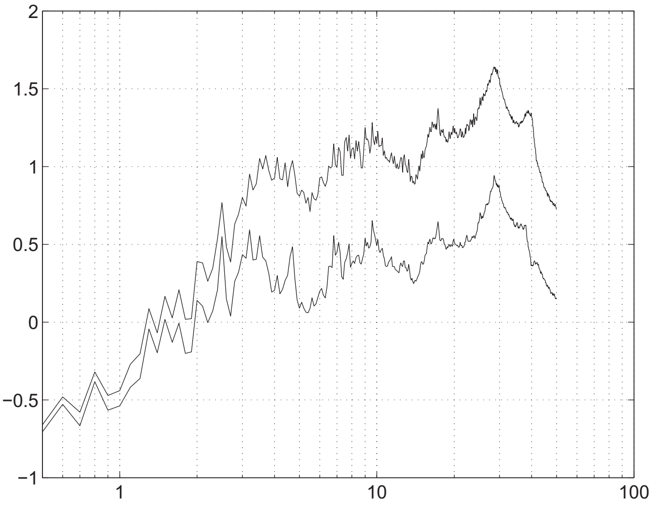

Figure 7. Confidence region of DOF number j_{stop} with a nonparametric and a parametric level of uncertainties equal to 0.2. The upper line corresponds to the upper envelope $dB_j^+(\xi,\nu)$ and the lower line corresponds to the lower envelope $dB_j^-(\xi,\nu)$. The frequency ν is represented on the horizontal axis in log scale (in Hz).

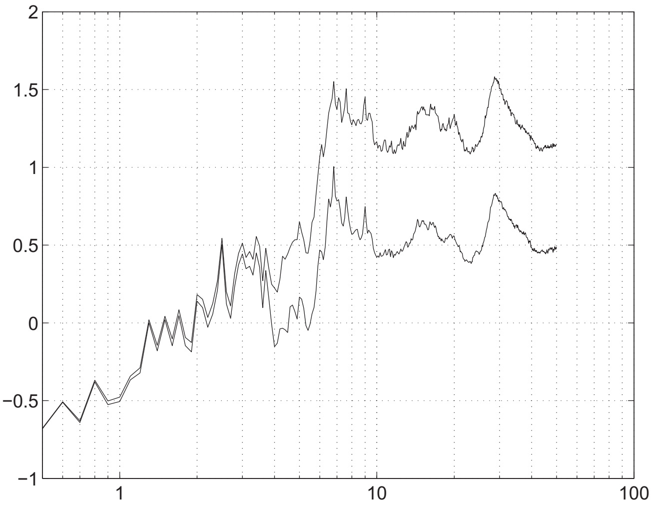

Figure 8. Confidence region of DOF number j_{obs} with a nonparametric and a parametric level of uncertainties equal to 0.2. The upper line corresponds to the upper envelope $dB_j^+(\xi,\nu)$ and the lower line corresponds to the lower envelope $dB_j^-(\xi,\nu)$. The frequency ν is represented on the horizontal axis in log scale (in Hz).

For such a dynamical system, the nonparametric-parametric approach allows the level of uncertainties to be extended and is well adapted to this kind of problem.

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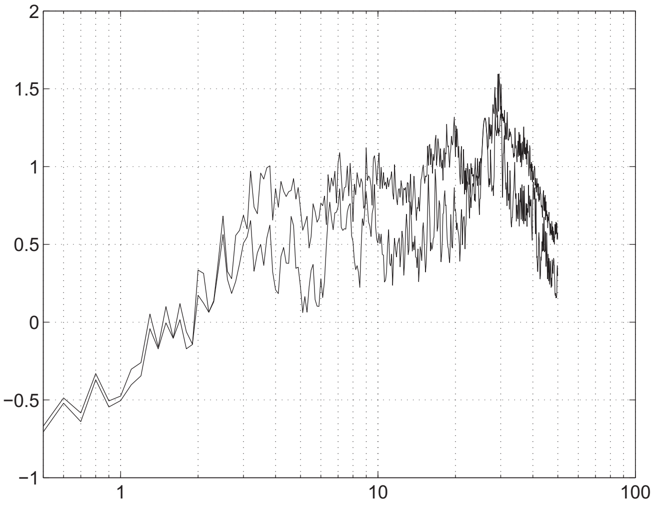

Figure 9. Confidence region of DOF number j_{stop} with a nonparametric equal to 0.00002 and a parametric level of uncertainties equal to 0. The upper line corresponds to the upper envelope $dB_j^+(\xi,\nu)$ and the lower line corresponds to the lower envelope $dB_j^-(\xi,\nu)$. The frequency ν is represented on the horizontal axis in log scale (in Hz).

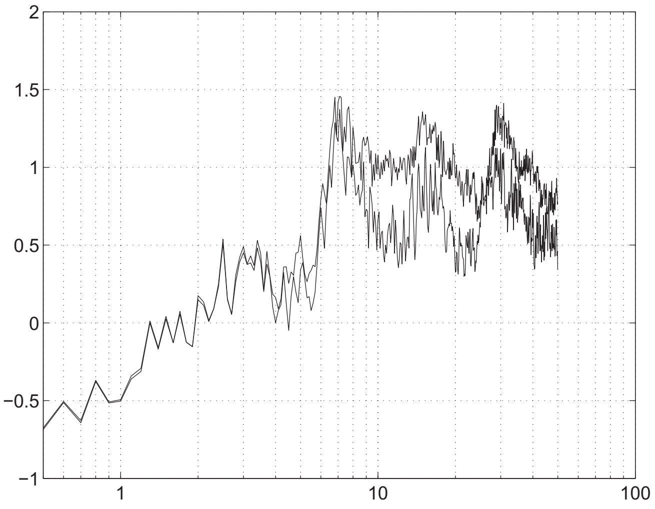

Figure 10. Confidence region of DOF number j_{obs} with a nonparametric equal to 0.00002 and a parametric level of uncertainties equal to 0. The upper line corresponds to the upper envelope $dB_j^+(\xi,\nu)$ and the lower line corresponds to the lower envelope $dB_j^-(\xi,\nu)$. The frequency ν is represented on the horizontal axis in log scale (in Hz).

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