Transient dynamics induced by shocks in stochastic structures

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ABSTRACT: This paper is devoted to numerical models for prediction of transient dynamical response induced by shocks upon structures with areas of non-homogeneous random uncertainties. The usual numerical methods for analyzing such structures in the LF and MF ranges employ reduced matrix models using the elastic modes. The contribution of the higher modes is very sensitive to the modeling errors. Here, a recent nonparametric probabilistic method is applied to construct the random uncertainties matrix model. The paper presents an extension of the nonparametric method to the case of complex structures, modeled by a dynamic substructuring method, in which every substructure gets its own uncertainty level. Examples of numerical prediction of the confidence regions for transient responses are compared with experimental measurements providing a validation of the presented approach.

1 INTRODUCTION

Shock-induced transient dynamics phenomena are widely investigated in the literature. Two kinds of approaches are generally used: either the Finite Elements methods (FEM) (e.g. [1-4]) or the Statistical Energy Analysis method (e.g. [5,6]). The present paper concerns transient phenomena for which elastic eigenmodes can be used for describing the low- and the medium-frequency ranges. In this framework, structures made up with non-homogeneous areas are examined (e.g. two plates attached with a complex junction).

The FE model of such mechanical systems are strongly uncertain due to the model uncertainties (for instance, modeling a complex junction by a simple subsystem) and due to the data uncertainties of the constructed model (geometry, boundary conditions, constitutive equations). In order to improve the reliability of the predictions, the model uncertainties and the data uncertainties have to be considered. In structural dynamics, uncertainties are usually taken into account through parametric models (e.g. [7,8]). Recently, a new nonparametric probabilistic model of random uncertainties has been introduced [9] in structural dynamics. This nonparametric approach allows to handle the modal uncertainties and the data uncertainties.

In the case of nonhomogeneous uncertainties, implementing the nonparametric model need a dynamic substructuring method (for example, the Craig and Bampton method [10]). In this context, works have been carried out in the field of harmonic responses [11]. Here the nonparametric approach is extended to the computation of shock-induced transient responses for structures with spatially nonhomogeneous uncertainties and the developed numerical model is compared with an experimentation [12] for validation.

The steps of this research are : developing a mean model by dynamic substructuring, implementing the probabilistic model from the mean model, analyzing the random responses and comparing them with experimental results.

2 EQUATIONS OF THE MEAN MODEL IN DYNAMIC SUBSTRUCTURING

Let us consider the transient response of a fixed structure with a linear constitutive equation, slightly damped, whose boundary $\partial\mathbb{S}$ is subjected to an impulsive external load $f_{ext}(t)$. There is a Dirichlet condition relative to the displacements $u = 0$ on the part $\partial\mathbb{S}_0$ of boundary $\partial\mathbb{S}$. Here, the dynamic subtructuring method employed is the fixed interface component mode synthesis method (Craig and Bampton [10]). The extension to the case of a free structure is straightforward. Structure $\mathbb{S}$ is divided into N_s substructures $\mathbb{S}^r$, where $r = 1, .., N_s$. One or several substructures are fixed on a part of their boundary $\partial\mathbb{S}^r$. Every substructure is connected through an interface Σ^r to one (or more) substructure. The equation of the mean model of substructure $\mathbb{S}^r$ is

$$[\underline{\mathbb{M}}^r]\ddot{\underline{\mathbf{U}}}^r(t) + [\underline{\mathbb{D}}^r]\dot{\underline{\mathbf{U}}}^r(t) + [\underline{\mathbb{K}}^r]\underline{\mathbf{U}}^r(t) = \underline{\mathbf{F}}^r(t), \quad t \geq 0$$
$$\dot{\underline{\mathbf{U}}}^r(0) = \mathbf{U}_1^r, \quad \underline{\mathbf{U}}^r(0) = \mathbf{U}_0^r, \tag{1}$$

where $\underline{\mathbf{U}}^r(t)$ is the $\mathbb{R}^{n^r}$-valued vector of the n^r degrees of freedom (DOF), $\underline{\mathbf{F}}^r(t)$ is the $\mathbb{R}^{n^r}$-valued vector of the external and coupling forces and $[\underline{\mathbb{M}}^r]$, $[\underline{\mathbb{D}}^r]$ and $[\underline{\mathbb{K}}^r]$ are the mass, damping and stiffness matrices which are symmetric. Matrix $[\underline{\mathbb{M}}^r]$ is positive definite and matrices $[\underline{\mathbb{D}}^r]$ and $[\underline{\mathbb{K}}^r]$ are positive definite (fixed substructure) or positive semidefinite (free substructure) depending on the case.The vectors $\underline{\mathbf{U}}^r(t)$ and $\underline{\mathbf{F}}^r(t)$ are partitioned into n_i^r inner DOFs and $n_\Sigma = n^r - n_i^r$ coupling DOFs,

$$\underline{\mathbf{U}}^r(t) = \begin{bmatrix} \underline{\mathbf{U}}_i^r(t) \\ \underline{\mathbf{U}}_\Sigma^r(t) \end{bmatrix}, \quad \underline{\mathbf{F}}^r(t) = \begin{bmatrix} \underline{\mathbf{F}}_i^r(t) \\ \underline{\mathbf{F}}_j^r(t) + \underline{\mathbf{F}}_\Sigma^r(t) \end{bmatrix}, \tag{2}$$

where $\underline{F}_\Sigma^r(t)$ is relative to the coupling forces upon the coupling interface Σ^r, $\underline{\mathbf{F}}_i^r(t)$ and $\underline{\mathbf{F}}_j^r(t)$ are due to the external forces applied. The equation of motion for the reduced matrix model of substructure $\mathbb{S}^r$ can be written as,

$$[\underline{M}^r]\begin{bmatrix} \ddot{\underline{\mathbf{q}}}^r(t) \\ \ddot{\underline{\mathbf{U}}}_\Sigma^r(t) \end{bmatrix} + [\underline{D}^r]\begin{bmatrix} \dot{\underline{\mathbf{q}}}^r(t) \\ \dot{\underline{\mathbf{U}}}_\Sigma^r(t) \end{bmatrix} + [\underline{K}^r]\begin{bmatrix} \underline{\mathbf{q}}^r(t) \\ \underline{\mathbf{U}}_\Sigma^r(t) \end{bmatrix} = \begin{bmatrix} \underline{\mathcal{F}}_{N^r}(t) \\ \underline{\mathcal{F}}_\Sigma(t) \end{bmatrix}, \tag{3}$$

with the reduced matrices $[\underline{M}^r]$, $[\underline{D}^r]$ and $[\underline{K}^r]$ are such that

$$[\underline{M}^r] = [\underline{H}^r]^T[\underline{\mathbb{M}}^r][\underline{H}^r],$$
$$[\underline{D}^r] = [\underline{H}^r]^T[\underline{\mathbb{D}}^r][\underline{H}^r], \tag{4}$$
$$[\underline{K}^r] = [\underline{H}^r]^T[\underline{\mathbb{K}}^r][\underline{H}^r],$$

where $\underline{\mathbf{q}}^r(t)$ is the vector of the generalized DOFs relative to the N^r first fixed interface eigenmodes, $\underline{\mathbf{U}}_\Sigma^r(t)$ are the coupling DOFs and $[\underline{H}^r]$ is the transformation matrix expressed by means of the $(n_i^r \times N^r)$ modal matrix $[\underline{\Phi}^r]$ and of the $(n_i^r \times n_\Sigma)$ matrix $[\underline{S}_\Sigma^r]$ of the static boundary functions,

$$\begin{bmatrix} \underline{\mathbf{U}}_i^r(t) \\ \underline{\mathbf{U}}_\Sigma^r(t) \end{bmatrix} = [\underline{H}^r]\begin{bmatrix} \underline{\mathbf{q}}^r(t) \\ \underline{\mathbf{U}}_\Sigma^r(t) \end{bmatrix} \text{ and } [\underline{H}^r] = \begin{bmatrix} [\underline{\Phi}^r] & [\underline{S}_\Sigma^r] \\ 0 & [I_{n_\Sigma}] \end{bmatrix}. \tag{5}$$

The reduced matrix models of the substructures are classically assembled using the continuity of displacements and the equilibrium of the interacting forces at the interfaces.

3 NONPARAMETRIC PROBABILISTIC MODEL

The physical structure exhibits uncertainty areas of various levels (nonhomogeneous uncertainties). Consequently, this structure is divided into substructures such that each ones can be considered as homogeneous with respect to its level of uncertainties. Thus the uncertainty level differs from one substructure to another one. The nonparametric probabilistic model of uncertainty is implemented independently for each substructure.

3.1 Construction principle of a nonparametric model of random uncertainties for each substructure

The nonparametric approach [9] proposes to directly construct the probabilistic model of the generalized random matrices for each substructure. Let $\mathbf{U}^r(t)$ be the random vector of the n^r DOFs of substructure $\mathbb{S}^r$. Vector $\mathbf{U}^r(t)$ can be written as,

$$\mathbf{U}^r(t) = \begin{bmatrix} \mathbf{U}_i^r(t) \\ \mathbf{U}_\Sigma^r(t) \end{bmatrix} = [\underline{H}^r]\begin{bmatrix} \mathbf{Q}^r(t) \\ \mathbf{U}_\Sigma^r(t) \end{bmatrix}, \tag{6}$$

where $\mathbf{Q}^r(t)$ is the $\mathbb{R}^{N^r}$-valued random vector of the generalized DOFs. The stochastic process $\mathbf{U}^r(t)$ indexed by $[0, +\infty[$ with values in $\mathbb{R}^{n^r}$ verifies

$$[\mathbf{M}^r]\begin{bmatrix} \ddot{\mathbf{Q}}^r(t) \\ \ddot{\mathbf{U}}_\Sigma^r(t) \end{bmatrix} + [\mathbf{D}^r]\begin{bmatrix} \dot{\mathbf{Q}}^r(t) \\ \dot{\mathbf{U}}_\Sigma^r(t) \end{bmatrix} + [\mathbf{K}^r]\begin{bmatrix} \mathbf{Q}^r(t) \\ \mathbf{U}_\Sigma^r(t) \end{bmatrix} = \begin{bmatrix} \mathcal{F}_N^r(t) \\ \mathcal{F}_\Sigma^r(t) \end{bmatrix}, \tag{7}$$

where $[\mathbf{M}^r]$ is a random reduced matrix with values in the set $\mathbb{M}_{n^r}^+(\mathbb{R})$ of all the symmetric positive-definite $(n^r \times n^r)$ real matrices $(n^r = N^r + n_\Sigma^r)$ and, $[\mathbf{D}^r]$ and $[\mathbf{K}^r]$ are random reduced matrices with values in $\mathbb{M}_{n^r}^+(\mathbb{R})$ (for a fixed substruture) or in the set $\mathbb{M}_{n^r}^{+0}(\mathbb{R})$ of all the symmetric positive-semidefinite $(n^r \times n^r)$ real matrices (for a free substructure). In equation (7), we have $\underline{\mathcal{F}}_{N^r}^r(t) = [\underline{\Phi}^r]^T\underline{\mathbf{F}}_i^r(t)$ and the random vector $\mathcal{F}_\Sigma^r(t) = [\underline{S}_\Sigma^r]^T\underline{\mathbf{F}}_i^r(t) + \underline{\mathbf{F}}_j^r(t) + \mathbf{F}_\Sigma^r(t)$, with $\mathbf{F}_\Sigma^r(t)$ the random vector of the coupling forces at the interface Σ^r.

By construction of the nonparametric model, we have

$$E\{[\mathbf{M}^r]\} = [\underline{M}^r],$$
$$E\{[\mathbf{D}^r]\} = [\underline{D}^r], \tag{8}$$
$$E\{[\mathbf{K}^r]\} = [\underline{K}^r],$$

where E is the mathematical expectation.

In the next section, we give a short description of the probabilistic model for random matrices $[\mathbf{M}^r]$, $[\mathbf{D}^r]$ and $[\mathbf{K}^r]$.

3.2 Probabilistic model for the matrices

Let $[\mathbf{A}^r]$ be $[\mathbf{M}^r]$, $[\mathbf{D}^r]$ or $[\mathbf{K}^r]$. If matrix $[\underline{\mathbf{A}}^r]$ is real positive definite, then an upper triangular $(n^r \times n^r)$ real matrix $[\underline{L}_{A^r}]$ exists such as $[\underline{\mathbf{A}}^r] = [\underline{L}_{A^r}]^T[\underline{L}_{A^r}]$. The random matrix is then written as $[\mathbf{A}^r] = [\underline{L}_{A^r}]^T[\mathbf{G}_{A^r}][\underline{L}_{A^r}]$ with $[\mathbf{G}_{A^r}]$ an $\mathbb{M}_{n^r}^+(\mathbb{R})$-valued random matrix whose probability distribution is defined in reference [9].

If matrix $[\underline{\mathbf{A}}^r]$ is with a rank equal to $\mu^r < n^r$, then a $(\mu^r \times n^r)$ real matrix $[\underline{L}_{A^r}]$ exists such as $[\underline{\mathbf{A}}^r] =$

$[\underline{L}_{A^r}]^T[\underline{L}_{A^r}]$. For instance, this matrix $[\underline{L}_{A^r}]$ can be obtained from the eigenvectors related to the non-zero eigenvalues of matrix $[\underline{L}_{A^r}]$, but some other construction exist. In such a case, random matrix $[\mathbf{A}^r]$ can also be written as $[\mathbf{A}^r] = [\underline{L}_{A^r}]^T[\mathbf{G}_{A^r}][\underline{L}_{A^r}]$ where $[\mathbf{G}_{A^r}]$ is the $\mathbb{M}^+_{\mu^r}(\mathbb{R})$-valued random matrix defined above.

The probability distribution of random matrix $[\mathbf{G}_{A^r}]$ depends only on a real parameter δ_{A^r} which is independent of dimension n^r and of frequency ω, and which allows the dispersion of random matrix $[\mathbf{A}^r]$ to be controlled. This parameter is defined by

$$\delta_{A^r} = \left\{ \frac{E\{\|[\mathbf{G}_{A^r}] - [\underline{\mathbf{G}}_{A^r}]\|_F^2\}}{\|[\underline{\mathbf{G}}_{A^r}]\|_F^2} \right\}^{1/2} \quad \text{such that} \quad 0 < \delta_{A^r} < \sqrt{\frac{n_0+1}{n_0+5}}\,, \tag{9}$$

where n_0 is an integer such as $1 \leq n_0 \leq n^r$ and with $\|[B]\|_F^2 = tr([B][B]^T)$. The dispersion of random matrices $[\mathbf{M}^r]$, $[\mathbf{D}^r]$ and $[\mathbf{K}^r]$ are then controlled by the parameters δ_{M^r}, δ_{D^r} and δ_{K^r} which are independent of n^r and chosen according to inequality (9).

The probabilistic model of the entire structure is obtained by the assemblage of the stochastic substructures and the random unknown vactor is $\mathbf{U}_e = (\mathbf{Q}^1(t), ..., \mathbf{Q}^{N_s}(t), \mathbf{U}_\Sigma(t))$.

4 NUMERICAL ANALYSIS

The analysis and the comparisons of the numerical model developed with the experiments are carried out by means of the Shock Response Spectra [13] calculated for the acceleration reponses and denoted by $S_a(\xi, \omega)$.

In presence of uncertainties, the transient response is a time stochastic process and the SRS's are stochastic process indexed by frequency. The stochastic solution of such a problem can be formally written, but the solution involves high-dimensional multiple integrals which can only be computed by the Monte-Carlo method. Then a direct Monte-Carlo numerical simulation of the stochastic equations is rather performed. Each realization of the random matrices is constructed according to the probabilistic model described in § 3.1 and § 3.2 and using the algorithm specified in [9]. The transient responses for every realization is carried out by employing an unconditionally stable step-by-step numerical integration scheme (Newmark method). The probabilistic magnitudes describing the stochastic SRS are estimated by the usual mathematical statistics.

4.1 *SRS analysis of responses*

Every SRS is computed with a damping ratio $\xi = 0.001$. The SRS are computed by using the Newmark integration scheme of the equation of motion. The SRS's scale is normalized with respect to $g = 9.81 m.s^{-2}$. We then introduce $s(\omega) = S_a(\xi, \omega)/g$. The results of the random model are displayed by an SRS confidence region described with an upper (S^+) and a lower (S^-) envelope of the SRS related to a given probability level $Pc = 0.95$. Let $m_1(\omega) = E\{s(\omega)\}$, $m_2(\omega) = E\{s^2(\omega)\}$ and $\sigma(\omega) = \sqrt{m_2(\omega) - m_1^2(\omega)}$ be the moments estimated by the usual mathematical statistics.

Three curves are defined: (1) $S^0(\omega) = 10log_{10}(m_1(\omega))$, which is related to the mean function of the stochastic response, (2) $S^+ = 10log_{10}\big(m_1(\omega) + \frac{\sigma(\omega)}{\sqrt{1-Pc}}\big)$, which is the the upper envelope of the SRS, and (3) $S^- = 2S^0 - S^+$, which is the lower envelope of the SRS.

5 EXPERIMENTAL AND NUMERICAL MODELS

5.1 *Experimental model description [12]*

It consists of two 3mm-thick Dural plates connected together through a complex junction and excited by an impulsive load over an LF and an MF range. Plate I is 0.4m width and 0.6m length, plate II is 0.5m width and 0.6m length. The complex junction is constituted of 2 smaller plates (2mm-thick, 0.14m width, 0.6m length), tightened by 2 lines of 20 bolts (Figure 1).

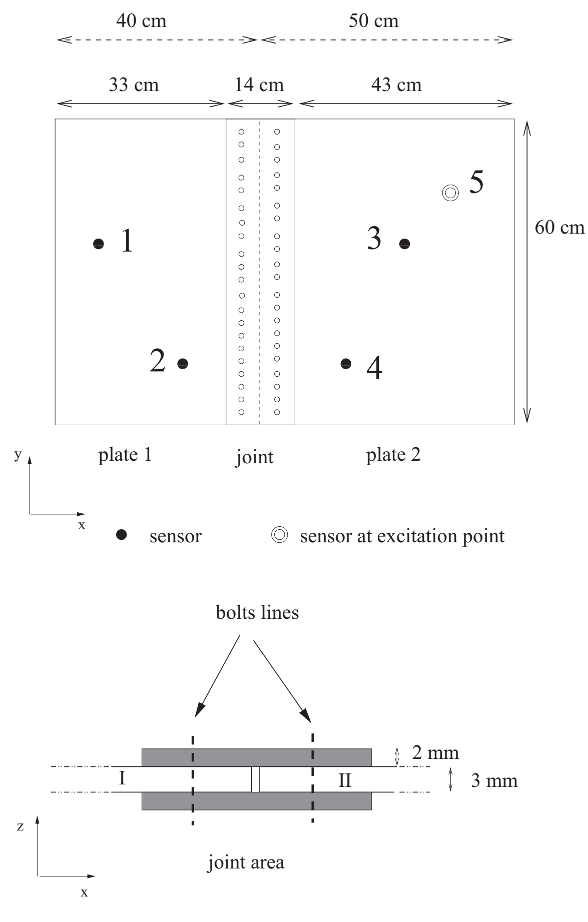

Figure 1. geometry of the experimental configuration

The experimental data base is made with 29 accelerometers for 21 experimental configurations corresponding to 21 random distributions of bolt-prestresses having the same mean values [12]. The experiment conditions correspond to a free-free structure.

Two transient loads are presented in this paper, one corresponding to the low-frequency band [200-400] Hz (see Figure 2) and the other one to the medium-frequency band [1000-1200] Hz (see Figure 3).

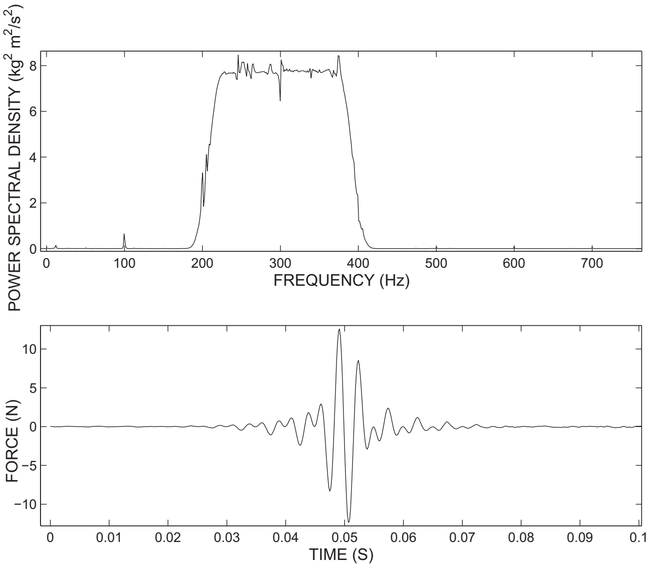

Figure 2. excitation load in [200-400] Hz

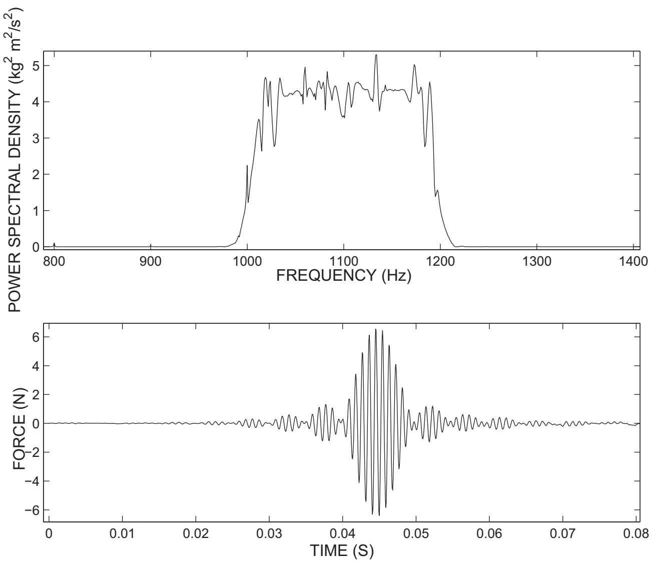

Figure 3. excitation load in [1000-1200] Hz

The experimental results are expressed in terms of SRS and are compared with the confidence regions built up from the numerical stochastic model.

5.2 *Numerical model description*

5.2.1 *Finite Element model of the mean system*

The three substructures of the mean model are represented by a uniform FE model of 4 nodes plates elements following the Kirchhoff thin plate assumptions (e.g. DOFs Tz, Rx and Ry are retained). The main structure consists of the 3 substructures (Figure 1): there are 6222 DOFs in plate 1, 2745 DOFs in the complex junction and 8052 in plate 2, e.g. a total amount of more than 16000 DOFs. Plates 1 and 2 are isotropic whereas the complex junction needs an orthotropic desription. The updating of the first 3 modes and the associated damping ratios of the mean FE model has been performed using experiments.

5.2.2 *Damping model of the mean system*

The damping ratios $\underline{\xi}$ of the mean Fe model has been deduced from the measurements and leads to $\underline{\xi} = 0.0021$. The damping matrix of the mean FE model has then been chosen [12] for each substructure $\mathbb{S}^r$ as,

$$[\underline{\mathbb{D}}^r] = \sum_{\gamma=1}^{N} 2\,\xi_\gamma\,\frac{\omega_\gamma}{\mu_\gamma}\,[\underline{\mathbb{M}}^r]\,\underline{\mathbf{V}}^r_\gamma\,\underline{\mathbf{V}}^{r\,T}_\gamma\,[\underline{\mathbb{M}}^r]$$

where N is such that $1 \le N \le n^r$ and where

$$\underline{\mathbf{V}}^r_\gamma = \begin{bmatrix} \underline{\mathbf{V}}^r_{\gamma i} \\ \underline{\mathbf{V}}^r_{\gamma\Sigma} \end{bmatrix}, \quad \omega_\gamma \quad \text{and} \quad \xi_\gamma$$

are respectively the eigenvector, the natural frequency and the damping ratio relative to the γ^{th} free interface eigenmode. In order to make the computation of the damping matrix efficient, we have chosen $N = N^r + n_\Sigma$, $\xi_\gamma = \underline{\xi}$, $\mu_\gamma = 1$ and we directly construct the reduced damping matrix such that

$$[\underline{D}^r] = \sum_{\gamma=1}^{N} 2\,\underline{\xi}\,\omega_\gamma\,\mathbf{C}_\gamma\,\mathbf{C}_\gamma^{\,T} \text{ with } \mathbf{C}_\gamma = [\underline{H}^r]^T\,[\underline{\mathbb{M}}^r]\,\underline{\mathbf{V}}^r_\gamma.$$

6 NUMERICAL RESULTS

In order to evaluate the role played by the nonhomogeneity of uncertainties which are assumed to be larger in the complex junction, a sensitivity analysis with respect to parameters δ^r has been carried out. Below, we limit the presentation to a sampling of the values used. The results displayed correspond to the following values of the dispersion parameters: $\delta_K^{CJ} = \delta_D^{CJ} = 0.5$, $\delta_M^{CJ} = 0$ and $\delta_K^{plates} = 0.15$, $\delta_D^{plates} = 0.3$, $\delta_M^{plates} = 0$ for [200-400] Hz and $\delta_K^{CJ} = \delta_D^{CJ} = 0.8$, $\delta_M^{CJ} = 0$ and $\delta_K^{plates} = 0.15$, $\delta_D^{plates} = 0.3$, $\delta_M^{plates} = 0$ for [1000-1200] Hz.

The mass of the mean model has been updated with experiments and consequently, there are no errors on the mass matrices. Monte-Carlo simulations have been carried out for a sufficiently high amount of realizations (500 to 1000), depending on the uncertainty level assigned. Convergence anlysis with respect to the number of eigenmodes ν in the reduced matrix models have been performed as well. As a consequence, the results are displayed for the 2 following cases:

$\nu = 48 = 20(N^1) + 2(N^2) + 26(N^3)$ for [200-400] Hz and $\nu = 125 = 50(N^1) + 8(N^2) + 67(N^3)$ for [1000-1200] Hz.

Figures 4 and 5 show the results obtained with these values. The thick dashed lines represent the mean model. The solid lines are related to the 21 experimental configurations. The gray region is the 95% confidence regions of the random responses, computed with the nonparametric model of uncertainties.

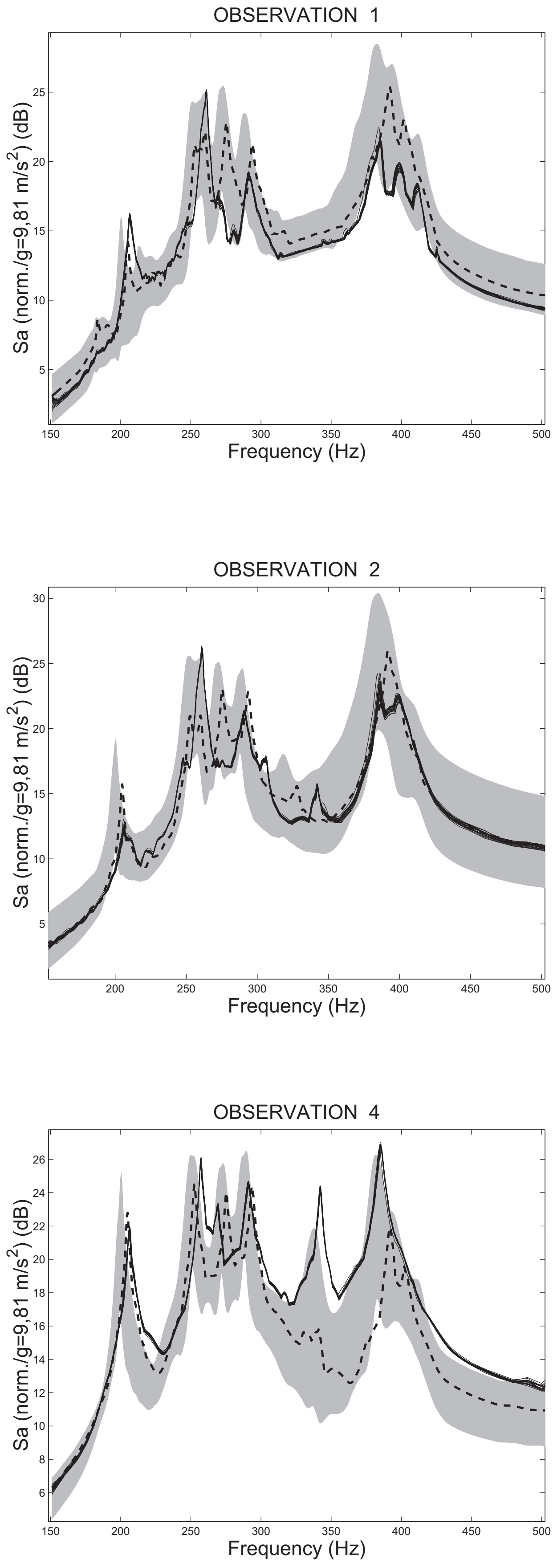

Figure 4. SRS at the observation points 1,2 and 4 for an impulsive load in the band [200-400] Hz with:
$\delta_K^{plates} = 0.15, \delta_D^{plates} = 0.3, \delta_M^{plates} = 0$
$\delta_K^{CJ} = \delta_D^{CJ} = 0.5, \delta_M^{CJ} = 0$

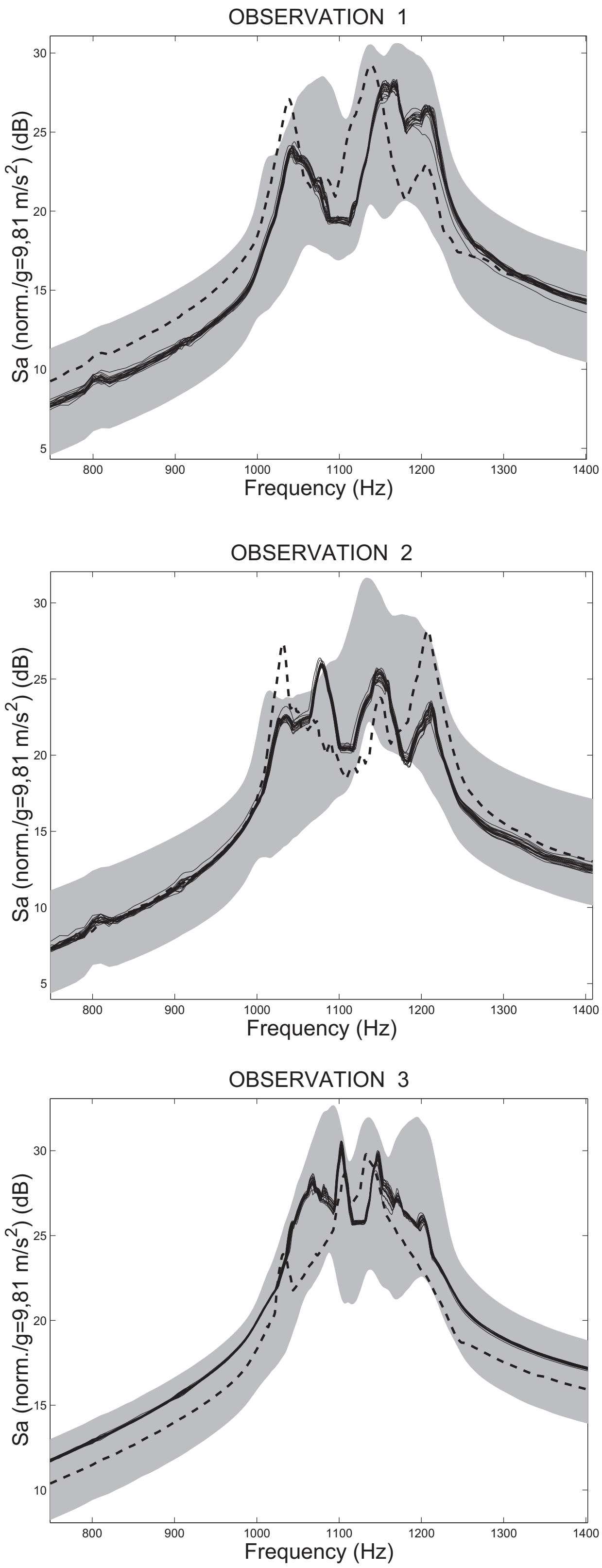

Figure 5. SRS at the observation points 1,2 and 3 for an impulsive load in band [1000-1200] Hz with:
$\delta_K^{plates} = 0.15, \delta_D^{plates} = 0.3, \delta_M^{plates} = 0$
$\delta_K^{CJ} = \delta_D^{CJ} = 0.8, \delta_M^{CJ} = 0$

7 CONCLUSIONS

About uncertainty modeling, the comparisons of the developed model with the experiments show the capability of the nonparametric probabilistic approach to predict the shock-induced transient response in the LF and the MF ranges.About mechanical consideration, in spite of a rather large number of DOFs used in the mean FE model, significant errors appears between experiments and numerical prediction. These errors are mainly due to the simplified model used for modeling the complex junction. It implies that model uncertainties exist in the mean FE model. The nonparametric approach of nonhomogeneous uncertainties proposed in this paper allows the robustness of the numerical prediction to be increased. The comparisons of the experiments with the confidence region predicted by this probabilistic approach is satisfying for an LF and an MF frequency band. A general methodology to estimate the values of the dispersion parameters is in progress.

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