Random uncertainties modeling for the medium-frequency dynamics

Christian Soize

RANDOM UNCERTAINTIES MODELING
FOR THE MEDIUM-FREQUENCY DYNAMICS

Christian SOIZE

Laboratory of Engineering Mechanics, University of Marne-la-Vallée, France,
email: soize@univ-mlv.fr

ABSTRACT

This paper presents a novel probabilistic model of random uncertainties for complex dynamical system in the medium-frequency (MF) range. This approach combines a nonparametric probabilistic model of random uncertainties for the reduced matrix models in structural dynamics with a reduced matrix model method in the MF range. The theory is presented, the random energy matrix relative to a given MF band is studied and a simple numerical example is analyzed.

INTRODUCTION

In structural dynamics, it is known that the higher the eigenfrequency of a structural mode, the lower its accuracy because the uncertainties in the model increase. The effects of uncertainties (geometrical parameters; boundary conditions; mass density; mechanical parameters of constitutive equations; structural complexity; interface and junction modeling; etc.) increase with the frequency and it should be kept in mind that the mechanical model and the finite element model of a complex structure tend to be less reliable in predicting the higher structural modes. Consequently, for the medium-frequency dynamics, random uncertainties in the mechanical model have to be taken into account in order to improve the efficiency and the robustness of the medium-frequency finite element models. An important aspect of the medium-frequency (MF) domain is the construction of an efficient reduced matrix model of the continuous dynamical system. For low-frequency (LF) dynamic analysis in structural dynamics, reduced matrix models are a very efficient tool for constructing the dynamical response (see for instance Clough & Penzien, 1975). These techniques correspond to a Ritz-Galerkin reduction of the structural-dynamics model using the structural modes corresponding to the lowest eigenfrequencies of the associated conservative dynamical system. Unfortunately, this modal method which is very efficient in the LF domain cannot be used in the MF domain for general three-dimensional dynamical systems. In this context, a reduced matrix model in the MF range for general dissipative structural-dynamics systems was proposed by Soize (1998), based on the use of the dominant eigensubspace of the mechanical energy operator related to the MF band as the projection basis.

Recently, a parametric approach of random uncertainties in MF dynamics has been proposed by Sarkar & Ghanem (2001) using the reduced matrix model developed by Soize (1998) combined with the stochastic finite element method (Ghanem & Spanos, 1991), consisting in a stochastic reduction of the random uncertainties utilizing the Karhunen-Loeve expansion and solving the reduced random matrix equation with the polynomial chaos expansion).

In this paper, we propose a novel probabilistic model of random uncertainties for the MF dynamics resulting from the use of the nonparametric probabilistic model of random uncertainties for the reduced matrix models in structural dynamics (Soize, 2000 & 2001) and combined with the reduced matrix model method in the MF range developed in (Soize, 1998). The theory is presented, the random energy matrix relative to a given MF band is studied and a simple numerical example is analyzed and shows the high sensitivity of MF dynamics to uncertainties.

THEORY

Mean Model of the Dynamical System and its Mean Finite Element Model

In the medium-frequency band $B = [\omega_{\text{min}}\,,\,\omega_{\text{max}}]$ with $\omega_{\text{min}} \gg \omega_{\text{max}} - \omega_{\text{min}} > 0$, the mean finite element model of linear vibrations of a viscoelastic bounded structure (fixed or free) around a position of static equilibrium taken as reference configuration without prestresses is written as

$$\left(-\omega^2\,[\,\underline{\mathbb{M}}\,] + i\omega\,[\underline{\mathbb{D}}(\omega)] + [\underline{\mathbb{K}}(\omega)]\,\right)\,\underline{\mathbf{y}}(\omega) = \underline{\mathbf{f}}(\omega) \quad , \quad \omega \in B \quad , \tag{1}$$

in which $\underline{\mathbf{y}}(\omega) = (\underline{y}_1(\omega), \ldots, \underline{y}_m(\omega))$ is the $\mathbb{C}^m$-vector of the m DOFs (displacements and/or rotations) and $\underline{\mathbf{f}}(\omega) = (\underline{f}_1(\omega), \ldots, \underline{f}_m(\omega))$ is the $\mathbb{C}^m$-vector of the m inputs (forces and/or moments). The mean mass matrix $[\,\underline{\mathbb{M}}\,]$ is a positive-definite symmetric $(m \times m)$ real matrix. The mean damping and stiffness matrices $[\underline{\mathbb{D}}(\omega)]$ and $[\underline{\mathbb{K}}(\omega)]$ are symmetric $(m \times m)$ real matrices, depend on ω (viscoelastic structure), are such that $[\underline{\mathbb{D}}(-\omega)] = [\underline{\mathbb{D}}(\omega)]$ and $[\underline{\mathbb{K}}(-\omega)] = [\underline{\mathbb{K}}(\omega)]$, and are either positive definite (fixed structure) or positive semidefinite(free structure). In the case of a free structure, for all ω in B, (1) matrices $[\underline{\mathbb{D}}(\omega)]$ and $[\underline{\mathbb{K}}(\omega)]$ have the same null space having a dimension m_{rig} such that $0 < m_{\text{rig}} \leq 6$ and spanned by the rigid body modes $\{\mathbf{y}_1, \ldots, \mathbf{y}_{m_{\text{rig}}}\}$ and (2) external load vector $\underline{\mathbf{f}}(\omega)$ is in equilibrium, i.e. is such that $<\underline{\mathbf{f}}(\omega)\,,\mathbf{y}_\alpha>= 0$ for all α in $\{1, \ldots, m_{\text{rig}}\}$, in which, for all $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{C}^m$, $<\mathbf{u}\,,\mathbf{v}>= u_1\,v_1 + \ldots + u_m\,v_m$. For all ω in B, Eq. (1) has a unique solution $\underline{\mathbf{y}}(\omega) = [\underline{\mathbb{T}}(\omega)]\,\underline{\mathbf{f}}(\omega)$ in which $[\underline{\mathbb{T}}(\omega)]$ is the matrix-valued FRF (frequency response function) defined by $[\underline{\mathbb{T}}(\omega)] = [\underline{\mathbb{A}}(\omega)]^{-1}$ where $[\underline{\mathbb{A}}(\omega)]$ is the dynamic stiffness matrix such that $[\underline{\mathbb{A}}(\omega)] = -\omega^2\,[\,\underline{\mathbb{M}}\,] + i\omega\,[\underline{\mathbb{D}}(\omega)] + [\underline{\mathbb{K}}(\omega)]$. For a fixed or a free structure, and for every ω fixed in B, the sparse complex matrix $[\underline{\mathbb{A}}(\omega)]$ is invertible.

Mean Reduced Matrix Model Adapted to an MF Band

The energy matrix $[\underline{\mathbb{E}}_B]$ (twice the kinetic energy) of the mean FEM in MF band B is the positive-definite symmetric $(m \times m)$ real matrix which is written (see Soize, 1998) as

$$[\underline{\mathbb{E}}_B] = \frac{1}{\pi}\int_B \omega^2\,\Re e\,\{[\underline{\mathbb{T}}(\omega)]^*[\,\underline{\mathbb{M}}\,][\underline{\mathbb{T}}(\omega)]\}\;d\omega \quad , \tag{2}$$

where $\Re e$ is the real part of complex number and where $[\underline{\mathbb{T}}(\omega)]^* = \overline{[\underline{\mathbb{T}}(\omega)]}^T$ is the adjoint matrix. It should be noted that $[\underline{\mathbb{E}}_B]$ depends on MF band B, but does not depend on the external load vector. The eigenvalue problem for energy matrix $[\underline{\mathbb{E}}_B]$ is written as $[\underline{\mathbb{E}}_B]\,\underline{\mathbf{P}} = \underline{\lambda}\,\underline{\mathbf{P}}$. The normalization condition of the real eigenvectors is chosen as $\|\underline{\mathbf{P}}\|^2 =<\underline{\mathbf{P}}\,,\underline{\mathbf{P}}>= 1$. The dominant eigensubspace of dimension n is spanned by the real eigenvectors $\underline{\mathbf{P}}_1, \underline{\mathbf{P}}_2, \ldots, \underline{\mathbf{P}}_n$ associated with the n highest eigenvalues $\underline{\lambda}_1 \geq \underline{\lambda}_2 \geq \ldots \geq \underline{\lambda}_n$. Introducing the rectangular $(m \times n)$ real matrix $[\underline{P}_n^B] = [\underline{\mathbf{P}}_1 \ldots, \underline{\mathbf{P}}_n]$ whose columns are constituted of eigenvectors $\underline{\mathbf{P}}_1, \ldots, \underline{\mathbf{P}}_n$ and introducing the diagonal square $(n \times n)$ real matrix $[\underline{\Lambda}_n]$ whose diagonal entries are $\underline{\lambda}_1, \ldots, \underline{\lambda}_n$, the eigenvalue problem for constructing the dominant eigenspace of energy matrix $[\underline{\mathbb{E}}_B]$ is written as

$$[\underline{\mathbb{E}}_B]\,[\underline{P}_n^B] = [\underline{P}_n^B]\,[\underline{\Lambda}_n] \quad , \quad [\underline{P}_n^B]^T\,[\underline{P}_n^B] = [I_n] \quad , \tag{3}$$

where $[I_n]$ is the $(n \times n)$ identity matrix. Matrix $[\underline{P}_n^B]$ is calculated by solving Eq. (3). For the computation, matrix $[\underline{\mathbb{E}}_B]$ is not directly calculated by Eq. (2) using a direct calculation of matrix-valued frequency response function $\{[\underline{\mathbb{T}}(\omega)], \omega \in B\}$. An indirect procedure based on the substace iteration method coupled with a solution method in the time domain (see Soize, 1998) is used. Such a procedure does not use the knowledge of complex matrix $[\underline{\mathbb{T}}(\omega)]$ which is generally full.

The mean reduced matrix model is obtained in projecting the mean finite element model defined by Eq. (1) on the dominant eigensubspace of energy matrix $[\mathbb{E}_B]$. The approximation $\underline{\mathbf{y}}^n(\omega)$ of $\underline{\mathbf{y}}(\omega)$ is then written as

$$\underline{\mathbf{y}}^n(\omega) = [\underline{P}_n^B]\,\underline{\mathbf{q}}^n(\omega) \quad , \tag{4}$$

in which, for all ω fixed in B, the $\mathbb{C}^n$-vector $\underline{\mathbf{q}}^n(\omega)$ of the generalized coordinates is the unique solution of the mean reduced matrix equation,

$$\left(-\omega^2\,[\,\underline{M}_n\,] + i\omega\,[\underline{D}_n(\omega)] + [\underline{K}_n(\omega)]\right)\,\underline{\mathbf{q}}^n(\omega) = \underline{\mathbf{F}}^n(\omega) \quad , \quad \omega \in B \quad , \tag{5}$$

with $\underline{\mathbf{F}}^n(\omega) = [\underline{P}_n^B]^T\,\underline{\mathbf{f}}(\omega) \in \mathbb{C}^n$ and where the mean generalized mass, damping and stiffness matrices are the positive-definite symmetric $(n \times n)$ real full matrices such that $[\,\underline{M}_n\,] = [\underline{P}_n^B]^T\,[\,\underline{\mathbb{M}}\,]\,[\underline{P}_n^B]$, $[\underline{D}_n(\omega)] = [\underline{P}_n^B]^T\,[\underline{\mathbb{D}}(\omega)]\,[\underline{P}_n^B]$ and $[\underline{K}_n(\omega)] = [\underline{P}_n^B]^T\,[\underline{\mathbb{K}}(\omega)]\,[\underline{P}_n^B]$.

Nonparametric Model of Random Uncertainties in the MF Band

Using the idea of the nonparametric model of random uncertainties introduced by Soize (2000), the principle of construction of the nonparametric model of random uncertainties in the MF band consists in modeling the generalized mass, damping and stiffness matrices of the mean reduced model defined by Eqs. (4) and (5) by random matrices $[\mathbf{M}_n]$, $[\mathbf{D}_n(\omega)]$ and $[\mathbf{K}_n(\omega)]$. Consequently, the nonparametric model of random uncertainties in the MF band is written as

$$\mathbf{Y}^n(\omega) = [\underline{P}_n^B]\,\mathbf{Q}^n(\omega) \quad , \tag{6}$$

in which, for all ω fixed in B, the $\mathbb{C}^n$-valued random variable $\mathbf{Q}^n(\omega)$ of the random generalized coordinates is the unique solution of the random reduced matrix equation,

$$\left(-\omega^2\,[\mathbf{M}_n] + i\omega\,[\mathbf{D}_n(\omega)] + [\mathbf{K}_n(\omega)]\right)\,\mathbf{Q}^n(\omega) = \underline{\mathbf{F}}^n(\omega) \quad , \quad \omega \in B \quad . \tag{7}$$

From (Soize 2000 & 2001), these random matrices are written as

$$[\mathbf{M}_n] = [\underline{L}_{M_n}]^T\,[\mathbf{G}_{M_n}]\,[\underline{L}_{M_n}] \quad , \tag{8}$$

$$[\mathbf{D}_n(\omega)] = [\underline{L}_{D_n}(\omega)]^T\,[\mathbf{G}_{D_n}]\,[\underline{L}_{D_n}(\omega)] \quad , \tag{9}$$

$$[\mathbf{K}_n] = [\underline{L}_{K_n}(\omega)]^T\,[\mathbf{G}_{K_n}]\,[\underline{L}_{K_n}(\omega)] \quad , \tag{10}$$

in which the upper triangular $(n \times n)$ real matrices $[\underline{L}_{M_n}]$, $[\underline{L}_{D_n}(\omega)]$ and $[\underline{L}_{K_n}(\omega)]$ correspond to the Chowlesky factorization $[\,\underline{M}_n\,] = [\underline{L}_{M_n}]^T\,[\underline{L}_{M_n}]$, $[\underline{D}_n(\omega)] = [\underline{L}_{D_n}(\omega)]^T\,[\underline{L}_{D_n}(\omega)]$ and $[\underline{K}_n(\omega)] = [\underline{L}_{K_n}(\omega)]^T\,[\underline{L}_{K_n}(\omega)]$ of positive-definite symmetric $(n \times n)$ real matrices $[\,\underline{M}_n\,]$, $[\underline{D}_n(\omega)]$ and $[\underline{K}_n(\omega)]$ respectively. Random matrices $[\mathbf{G}_{M_n}]$, $[\mathbf{G}_{D_n}]$ or $[\mathbf{G}_{K_n}]$ are independent and the dispersion of random matrices $[\mathbf{G}_{M_n}]$, $[\mathbf{G}_{D_n}]$ and $[\mathbf{G}_{K_n}]$ are controlled by the positive real parameters δ_M, δ_D and δ_K which are independent of dimension n and do not depend on frequency ω. If A_n is M_n, D_n or K_n, then random matrix $[\mathbf{G}_{A_n}]$, with dispersion parameter δ_A, is such that $[\mathbf{G}_{A_n}] = [\mathbf{L}_{A_n}]^T\,[\mathbf{L}_{A_n}]$, in which $[\mathbf{L}_{A_n}]$ is an upper triangular random $(n \times n)$ real matrix such that the random variables $\{[\mathbf{L}_{A_n}]_{jj'}, j \leq j'\}$ are independent and such that

(1) for $j < j'$, real-valued random variable $[\mathbf{L}_{A_n}]_{jj'}$ is written as $[\mathbf{L}_{A_n}]_{jj'} = \sigma_n U_{jj'}$ in which $\sigma_n = \delta_A(n+1)^{-1/2}$ and where $U_{jj'}$ is a real-valued Gaussian random variable with zero mean and variance equal to 1;

(2) for $j = j'$, positive-valued random variable $[\mathbf{L}_{A_n}]_{jj}$ is written as $[\mathbf{L}_{A_n}]_{jj} = \sigma_n\sqrt{2V_j}$ in which σ_n is defined above and where V_j is a positive-valued gamma random variable whose probability density function $p_{V_j}(v)$ with respect to dv is written as $p_{V_j}(v) = \mathbb{1}_{\mathbb{R}^+}(v)\{\Gamma(\frac{n+1}{2\delta_A^2} + \frac{1-j}{2})\}^{-1}\, v^{\frac{n+1}{2\delta_A^2} - \frac{1+j}{2}}\; e^{-v}$.

Convergence Analysis of the Random Response and Random Energy Matrix

For every $\omega \in B$, let $\mathbf{Y}^n(\omega)$ be the $\mathbb{C}^n$-valued second-order random variable which is the solution of Eqs. (6) and (7), that is to say, the random response of the stochastic system subjected to the prescribed external load. The norm of the $\mathbb{C}^n$-valued second-order stochastic process $\{\mathbf{Y}^n(\omega), \omega \in B\}$ is defined by $|||\mathbf{Y}^n||| = \left(E\{\int_B \|\mathbf{Y}^n(\omega)\|^2 \, d\omega\}\right)^{1/2}$. From Eqs. (3) and (6), it can be deduced that $|||\mathbf{Y}^n||| = |||\mathbf{Q}^n|||$. Consequently, the mean-square convergence with respect to n of the sequence of stochastic processes $\{\mathbf{Y}^n(\omega), \omega \in B\}_n$ can be studied considering the mapping $n \mapsto |||\mathbf{Q}^n|||$. Since $|||\mathbf{Y}^n|||$ depends on the prescribed external load, it is interesting to introduce the approximation $[\mathbb{E}_B^n]$ of order n of the random energy matrix relative to MF band B, which is independent of the prescribed external load. This is a random positive semidefinite symmetric $(m \times m)$ real matrix defined (see Eq. (2)) by

$$[\mathbb{E}_B^n] = [\underline{P}_n^B]\,[\mathbf{E}_n]\,[\underline{P}_n^B]^T \quad , \quad [\mathbf{E}_n] = \frac{1}{\pi}\int_B \omega^2\, \Re e\,\{[\mathbf{T}_n(\omega)]^*[\mathbf{M}_n][\mathbf{T}_n(\omega)]\}\; d\omega \quad , \tag{11}$$

where $[\mathbf{T}_n(\omega)]$ is the random symmetric $(n \times n)$ complex matrix defined by

$$[\mathbf{T}_n(\omega)] = \left(-\omega^2\,[\mathbf{M}_n] + i\omega\,[\mathbf{D}_n(\omega)] + [\mathbf{K}_n(\omega)]\,\right)^{-1} \quad . \tag{12}$$

Let $\mathcal{E}_n$ be the trace of random energy matrix $[\mathbb{E}_B^n]$. Consequently, $\mathcal{E}_n$ is a positive-valued random variable such that $\mathcal{E}_n = \text{tr}\{[\mathbb{E}_B^n]\} = \text{tr}\{[\mathbf{E}_n]\}$ whose probability density function is the mapping $e \mapsto p_{\mathcal{E}_n}(e)$ defined in $\mathbb{R}^+$ with values in $\mathbb{R}^+$. In the next Section, the convergence with respect to n of the sequence of probability density functions $\{e \mapsto p_{\mathcal{E}_n}(e)\}_n$ is studied.

APPLICATION

Mean Model of the Dynamical System and its Mean Finite Element Model

The mean model of the nonhomogeneous dynamical system is composed of a homogeneus thin plane with two attached point masses and two springs. The thin plate is rectangular, homogeneous, isotropic and located in the plane (Ox_1, Ox_2) of a Cartesian coordinate system $(Ox_1x_2x_3)$, in bending mode (the outplane displacement is x_3), with constant thickness $1 \times 10^{-3}\, m$, width along Ox_2 is $0.40\, m$, length along Ox_1 is $0.50\, m$, mass density $7800\, kg/m^3$, Young's modulus $2.1 \times 10^{11}\, N/m^2$ and Poisson ratio 0.29 . This plate is simply supported on 3 edges and free on the fourth edge corresponding to $x_2 = 0$ (see Figure 1). To this plate are attached (1) two point masses having a mass of $10\, kg$ and $6\, kg$ located at points $(0.15, 0.15, 0)$ and $(0.31, 0.20, 0)$ respectively, and (2) two springs having a stiffness coefficient $k = 1.2090 \times 10^9\, N/m$ and $k = 7.0893 \times 10^8\, N/m$ located at the same points that the point masses, that is to say at points $(0.15, 0.15, 0)$ and $(0.31, 0.20, 0)$ respectively.

The mean finite element model of the plate is composed of a regular rectangular mesh with a constant step size of $0.01\, m$ in x_1 and x_2 (41 nodes in the width, 51 nodes in the length). Consequently, all the finite elements have the same size and each one is a 4-node square plate element. There are 2000 finite elements and $m = 6009$ degrees of freedom (x_3-translations and x_1- and x_2-rotations). The two first eigenfrequencies of the mean undamped dynamical system, calculated with the mean finite element model, are $27.73\, Hz$ and $57.35\, Hz$. There are 82 eigenfrequencies in the frequency band $[0\,, 1400]\, Hz$ and respectively, 13, 20 and 11 eigenfrequencies in the frequency bands $[1400\,, 1600]\, Hz$, $[1600\,, 1900]\, Hz$ and $[1900\,, 2100]\, Hz$. The medium-frequency band of analysis is defined as $B = 2\pi \times [1600\,, 1900]\, rad/s$. In the frequency domain, for all $\omega \in B$, the prescribed external load vector $\underline{\mathbf{f}}(\omega) \in \mathbb{C}^m$ is written as $\underline{\mathbf{f}}(\omega) = \mathbf{Z}$ in which the spatial part $\mathbf{Z} = (Z_1, \ldots, Z_m) \in \mathbb{R}^m$ is independent of ω and is such that $Z_j = 0$ for all j in $\{1, \ldots, m\}$ except for the nine DOFs in x_3-translations corresponding to the nodes whose (x_1, x_2) coordinates are $(0.21, 0.23)$, $(0.21, 0.24)$, $(0.21, 0.25)$, $(0.22, 0.23)$, $(0.22, 0.24)$, $(0.22, 0.25)$, $(0.23, 0.23)$, $(0.23, 0.24)$ and $(0.23,\, 0.25)$, for which $Z_j = 1$. The damping matrix $[\,\underline{\mathbb{D}}(\omega)]$ of the mean finite element model depends on the frequency and is written as $[\,\underline{\mathbb{D}}(\omega)] = 2\,\underline{\xi}\,\omega\,[\,\underline{\mathbb{M}}]$ in which $\underline{\xi} = 0.002$. In the system, the observations are the three DOFs number j_{obs1}, j_{obs2} and j_{obs3} corresponding to the x_3-translation of the mesh node located at points of coordinates $(0.22, 0.24, 0)$ (excitation central point), $(0.31, 0, 0)$ (in the free edge) and $(0.37, 0.15, 0)$ (inside the plate) respectively.

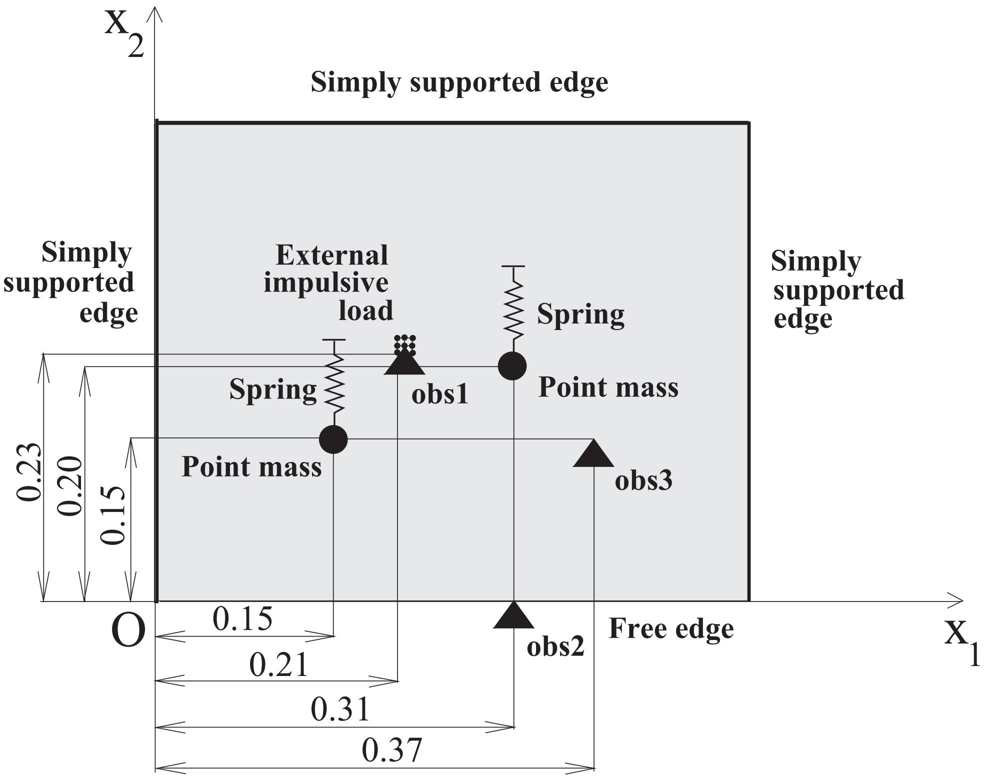


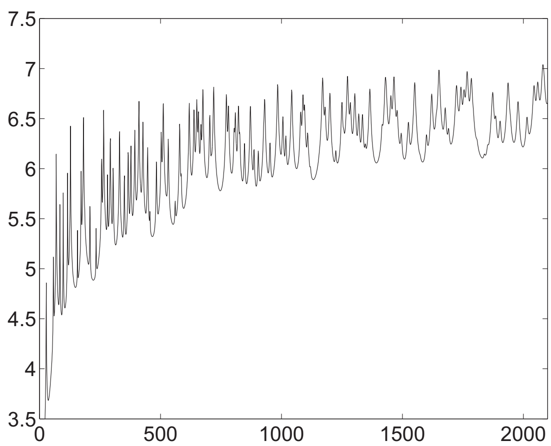
Figure 1 Definition of the mean dynamical system.

Reference Solution for the Mean Model on a Broad Frequency Band

For the mean model, the reference solution is obtained by solving Eq. (1) with the direct frequency-by-frequency method with 2100 sample points in the frequency band $[0\,,2100]$ Hertz. Figure 2 shows the graph of the function $\omega \mapsto \log_{10}(\| - \omega^2\underline{\mathbf{y}}(\omega)\|)$. In this figure, it can be seen that frequency band $B = 2\pi \times [1600\,,1900]\,rad/s$ belongs to the medium-frequency range.

Mean Reduced Matrix Model on MF Band B

The dominant eigensubspace of energy matrix $[\underline{\mathbb{E}}_B]$ relative to MF band $B = [1600\ ,\ 1900]$ Hz is calculated by solving the symmetric eigenvalue problem defined by Eq. (3) as explained above. The 50 highest eigenvalues are calculated. Figure 3 shows the graph $\alpha \mapsto \underline{\lambda}_\alpha$ of the distribution of the eigenvalues of $[\underline{\mathbb{E}}_B]$. Horizontal axis is the rank $\alpha = 1, \ldots, 50$ of the eigenvalues. It can be seen the strong decrease of eigenvalues when the order of the eigenvalues is greater than 20 allowing the construction of a mean reduced matrix model having a small dimension.



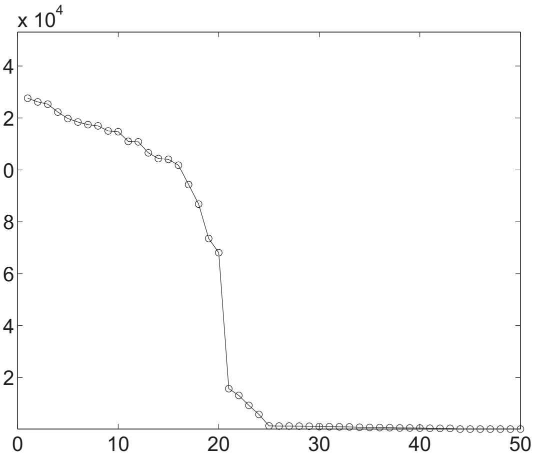


Figure 2 (left) Reference solution of the mean model. Graph of function $\nu \mapsto \log_{10}(\|-(2\pi\nu)^2 \underline{\mathbf{y}}(2\pi\nu)\|)$ on the frequency band [0 , 2100] Hertz (horizontal axis).
Figure 3 (right) Graph $\alpha \mapsto \underline{\lambda}_\alpha$ of the distribution of the eigenvalues of the energy matrix relative to band $B =$ [1600 , 1900] Hz for the mean dynamical system. Horizontal axis is the order $\alpha =$1,..., 50 of the eigenvalues.

Stochastic System with Nonparametric Model of Random Uncertainties

It is assumed that the dispersion parameters which control the mass, damping and stiffness uncertainties are such that $\delta_M = \delta_D = \delta_K = 0.02$. The Monte Carlo numerical simulation is used for solving random Eqs. (6) and (7). Let n_s be the number of simulations. Since $|||\mathbf{Y}^n||| = |||\mathbf{Q}^n|||$, norm $|||\mathbf{Y}^n|||$ can be estimated by Conv$(n_s, n) = \left\{\frac{1}{n_s}\sum_{k=1}^{n_s}\int_B \|\mathbf{Q}^n(\omega,\theta_k)\|^2\, d\omega\right\}^{1/2}$ in which $\theta_1, \ldots, \theta_{n_s}$ correspond to the n_s realizations. Probability density function $e \mapsto p_{\mathcal{E}_n}(e)$ of random variable $\mathcal{E}_n$ is estimated by the usual method. Let be $\mathbf{Y}^n(\omega) = (Y_1^n(\omega), \ldots, Y_m^n(\omega))$ and let $S(\omega) = |-\omega^2 Y_j^n(\omega)|$ be the random response corresponding to the acceleration of DOF j. Let dB(ω) be the random variable defined by dB$(\omega) = \log_{10}(S(\omega))$. For a given probability level P_c (for instance $P_c = 0.95$), the confidence region Proba$\{\text{dB}^-(\omega) < \text{dB}(\omega) \leq \text{dB}^+(\omega)\} = P_c$ of stochastic process $\{\text{dB}(\omega), \omega \in B\}$ is defined by the lower and upper envelopes dB$^-(\omega)$ and dB$^+(\omega)$ such that dB$^+(\omega) = \log_{10}\left(E\{S(\omega)\} + \sigma(\omega)/\sqrt{1-P_c}\right)$ and dB$^-(\omega) = 2\,\text{dB}_0(\omega) - \text{dB}^+(\omega)$ in which dB$_0(\omega) = \log_{10}(E\{S(\omega)\})$ and where $\sigma(\omega)$ is the standard deviation of $S(\omega)$.

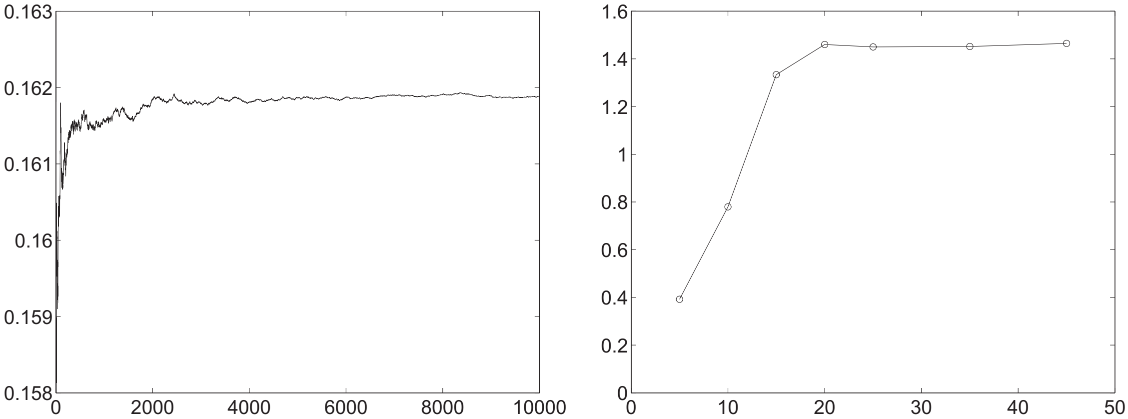


Figure 4 Convergence analysis. **(Left):** for $n = 35$, graph of function $n_s \mapsto \log_{10}\{\text{Conv}(n_s, n)\}$. **(Right):** for $n_s = 10000$, graph of function $n \mapsto \log_{10}\{\text{Conv}(n_s, n)\}$

Figures 4 (left and right) are related to the convergence of $|||\mathbf{Y}^n|||$ with respect to dimension n and to the number n_s of realization. It can be seen that convergence is reached for $n = 35$ and $n_s = 10000$. Figure 5 on the left is related to the convergence of the probability density function $e \mapsto p_{\mathcal{E}_n}(e)$ with respect to dimension n when $n_s = 10000$. It should be noted that convergence is also reached for $n = 35$. Figure 5 on the right shows the graph of $e \mapsto p_{\mathcal{E}_n}(e)$ for $n = 35$ and $n_s = 10000$ (it is not a Gaussian pdf).

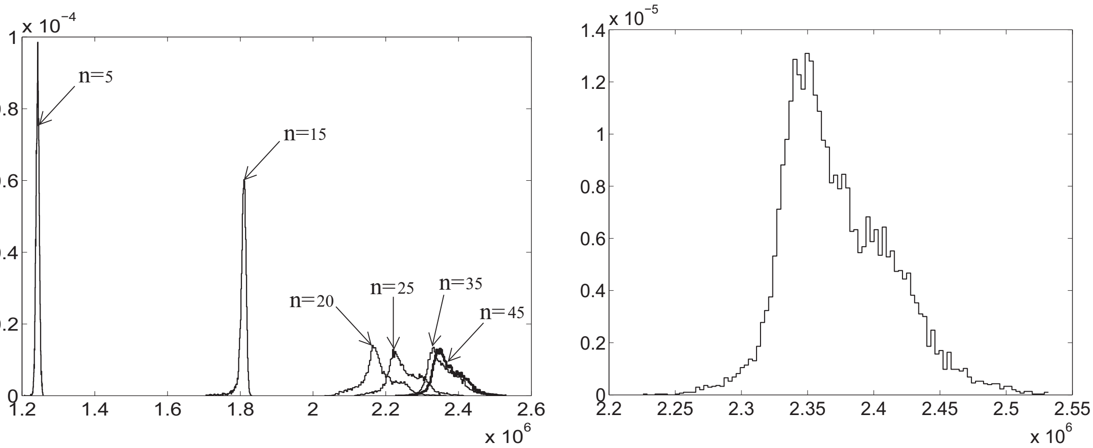


Figure 5 Probability density function of the trace of the random energy operator. **(Left):** Graphs of probability density functions $\{e \mapsto p_{\mathcal{E}_n}(e)\}_n$ for n = 5,15,20,25,35 (thin solid lines) and for n = 45 (thick solid line), corresponding to δ_M=δ_D=δ_K= 0.02 and n_s = 10000. **(Right):** Zoom of the graph of probability density function function $e \mapsto p_{\mathcal{E}_n}(e)$ for n = 35 shown in Figure 5 on the left.

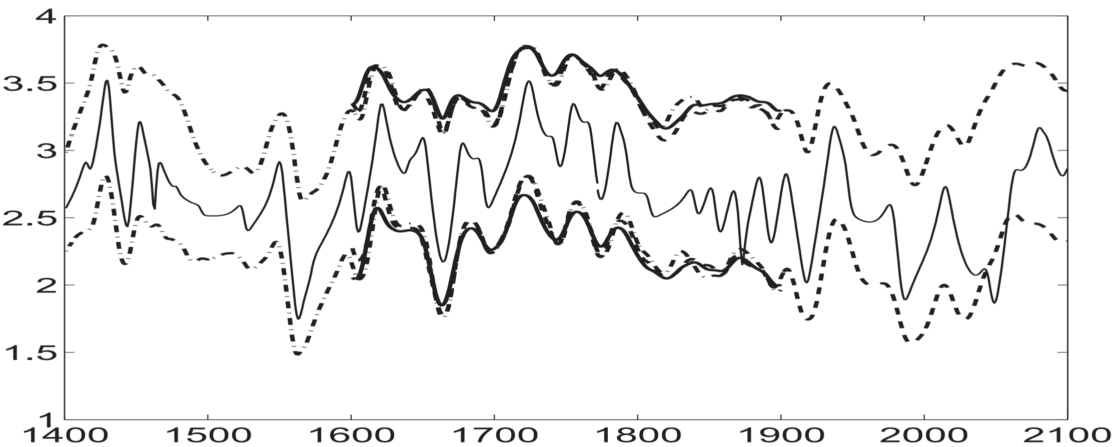


Figure 6 Coherence of the results for three overlapped MF bands B = [1600 , 1900] Hz, B' = [1400 , 1900] Hz and B'' = [1600 , 2100] Hz. Acceleration for DOF number $j_{\rm obs3}$ of the stochastic system subjected to the prescribed external load for bands B, B' and B'', for δ_M=δ_D=δ_K= 0.02 and n_s = 10000. Graphs of the random acceleration dB (vertical axis) as a function of the frequency (horizontal axis in Hz). Deterministic response of the mean model on frequency band [1400 , 2100] Hz (mid irregular thin solid line). Lower and upper envelopes of the confidence region corresponding to the probability level 0.95: band B' (lower and upper thick dashdot lines), band B'' (lower and upper thick dashed lines), band B (lower and upper thick solid lines).

Figure 6 is related to the coherence of the proposed nonparametric model of random uncertainties in the medium-frequency range. Let $B = [1600\,,1900]$ Hz, $B' = [1400\,,1900]$ Hz and $B'' = [1600\,,2100]$

Hz be the three MF bands such that $B \subset B'$, $B \subset B''$ and $B = B' \cap B''$. Let $\{\mathbf{Y}_B^n(\omega), \omega \in B\}$, $\{\mathbf{Y}_B^{n'}(\omega), \omega \in B'\}$ and $\{\mathbf{Y}_B^{n''}(\omega), \omega \in B''\}$ be the solutions of the stochastic problem for a prescribed external load defined on MF bands B, B' and B'' respectively. These three stochastic processes are converged for $n = 35$, $n' = 58$ and $n'' = 58$ respectively. It has to be verified that, over MF band B,the confidence regions of stochastic processes $\{\mathbf{Y}_B^n(\omega), \omega \in B\}$, $\{\mathbf{Y}_B^{n'}(\omega), \omega \in B'\}$ and $\{\mathbf{Y}_B^{n''}(\omega), \omega \in B''\}$ coincide. Figure 6 shows that the coincidence is obtained with a good accuracy.

CONCLUSIONS

We have presented a novel approach for modeling random uncertainties for the medium-frequency dynamics. Convergence properties have been verified and the proposed method is coherent with respect to the bandwith of the MF band of analysis.

REFERENCES

Clough R.W. & Penzien J. 1975. *Dynamics of Structures*. New York: McGraw-Hill.

Ghanem R. & Spanos P. D. 1991. *Stochastic Finite Elements: A spectral Approach*. New York: Springer-Verlag.

Sarkar A. & Ghanem R. 2001. Mid-frequency Structural Dynamics with Parameter Uncertainty. *42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Seatle.

Soize C. 1998. Reduced Models in the Medium-Frequency Range for General Dissipative Structural-Dynamics Systems. *European Journal of Mechanics, A/Solids* **17**(4): 657-685.

Soize C. 2000. A Nonparametric Model of Random Uncertainties for Reduced Matrix Models in Structural Dynamics. *Probabilistic Engineering Mechanics* **15**(3): 277-294.

Soize C. 2001. Maximum Entropy Approach for Modeling Random Uncertainties in Transient Elastodynamics. *J. Acoust. Soc. Am.* **109**(5): 1979-1996.