Uncertain rotating dynamical systems with cyclic geometry

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ABSTRACT: The aim of this paper is to propose probabilistic methodologies for the dynamic analysis of the mistuning of rotating bladed-disks with cyclic symmetry in the low frequency range. A recent nonparametric probabilistic model of random uncertainties is used for modeling the random uncertainties in the context of the blade mistuning. An inverse probabilistic approach, based on the identification of the dispersion parameters controlling the nonparametric probability model with respect to the blade tolerances is constructed in order to define the blade geometric tolerances yielding a given probability level of the dynamic amplification of the forced response. A numerical application is presented in details.

1 INTRODUCTION

It is well known that the description of a bladed disk using its cyclic symmetry properties is not sufficient to predict accurately its dynamic forced response. The blade mistuning terminology is then introduced because the blades of a bladed disk are slightly different from one to another one, due to the manufacturing process of the blades. Such a mistuning is considered in a probabilistic context and is not without consequence when analyzing the forced response of the bladed disk. Indeed, the energy of the mistuned bladed disk can be localized on a few blades, inducing large dynamic amplifications, see for instance (Whitehead 1966)... with respect to the nominal structure. Various researches have been carried out, see for instance (Griffin and Hoosac 1984, Mignolet et al. 2001)... in order to understand and to control this phenomenon.

This paper deals with the inverse problem related to the definition of the blade manufacturing tolerances for a given confidence region of the dynamic amplification factor of the blades. The nonparametric probabilistic model of random uncertainties developed for linear elastodynamics (Soize 2000, Soize 2001) is used for modeling the blade mistuning. The main steps concerning the resolution of such an inverse problem (Capiez-Lernout et al. 2005) are based on (1) the construction of a mean reduced matrix model for each blade because the probability model related to the nonparametric approach is implemented from reduced matrices; (2) the construction of the probability model by using the maximum entropy principle with the available information; three scalar parameters allowing the dispersion of the random mass, damping and stiffness matrices to be controlled constitute the data input of the nonparametric approach; (3) the identification of these dispersion parameters with respect to the geometric parameters controlling the blade geometric mistuning. It should be noted that the geometric mistuning is completely controlled by the probabilistic nonparametric model of the blade uncertainties. This nonparametric model is itself completely controlled by two scalar dispersion parameters δ_M and δ_K related to the mass and the stiffness operators of the blades. The proposed estimator of these dispersion parameters with respect to the blade geometry uncertainties is weakly sensitive to the choice of the probability model used for generating the blade random geometry. It is important to note that the probabilistic model of the blade random geometry is not directly used for predicting the mistuning but only for estimating the dispersion parameters δ_M and δ_K. In Section 2, a mean reduced matrix model both adapted to the nonparametric approach and to large finite element models is briefly recalled. The probabilistic methodology concerning the inverse problem of blade tolerances definition is presented in Section 3. Finally, Section 4 is devoted to the numerical application consisting in a large finite element model of an industrial fan stage.

2 MEAN REDUCED MATRIX MODEL

In this paper, $\mathbb{M}_{m,n}(\mathbb{R})$, $\mathbb{M}_n(\mathbb{R})$ and $\mathbb{M}_n^+(\mathbb{R})$ are the set of all the $(m \times n)$ real matrices, the set of all the $(n \times n)$ real matrices and the set of all the positive-definite symmetric $(n \times n)$ real matrices.

The structure considered is a three dimensional rotating fan stage modeled by a bladed disk with N blades. The bladed disk is rotating around a fixed axis with the constant velocity Ω and is submitted to external forces. The vibration analysis is carried out in the rotating frame related to the reference configuration in the low-frequency band $\mathbb{B}$. The centrifugal terms are taken into account in the stiffness terms. Gyroscopic coupling due to rotating motion is not taken into account. It is assumed that the uncertainty level is the same for each blade and that the random uncertainties are statistically independent from one blade to another one. The construction of a mean reduced matrix model for each blade is thus required in order to model the random uncertainties with the nonparametric probabilistic approach. The mean reduced matrix model of the structure is based on the Craig and Bampton decomposition method (Craig and Bampton 1968) for each blade with an additional reduction for the disk (Benfield and Hruda 1971). The efficiency of this dynamic substructuring method has been proved for mistuned industrial bladed disks (Seinturier et al. 2002).

The matrix equation related to the mean finite element model of the bladed disk is written as

$$\Big(-\omega^2\,[\underline{\mathbb{M}}] + i\,\omega\,[\underline{\mathbb{D}}] + [\underline{\mathbb{K}}]\Big)\,\underline{\mathbf{u}}(\omega) = \underline{\mathbf{f}}(\omega)\,, \tag{1}$$

in which $\underline{\mathbf{u}}(\omega)$ and $\underline{\mathbf{f}}(\omega)$ are the $\mathbb{C}^n$ vectors of the DOF and of the external loads and where $[\underline{\mathbb{M}}]$, $[\underline{\mathbb{D}}]$ and $[\underline{\mathbb{K}}]$ are the mean finite element mass, damping and stiffness matrices of the bladed-disk which are positive-definite matrices. The projection basis resulting from the substructuring method is written with the following block decomposition as

$$\begin{bmatrix} \underline{\mathbf{u}}_i^d \\ \underline{\mathbf{u}}_\Sigma \\ \underline{\mathbf{u}}_i^b \end{bmatrix} = [\underline{H}] \begin{bmatrix} \underline{\mathbf{q}}^d \\ \underline{\mathbf{q}}^b \end{bmatrix},\, [\underline{H}] = \begin{bmatrix} [\underline{\Phi}_i^{d,c}] & [\mathbb{0}] \\ [\underline{\Phi}_\Sigma^{d,c}] & [\mathbb{0}] \\ [\underline{S}^b][\underline{\Phi}_\Sigma^{d,c}] & [\underline{\Phi}^b] \end{bmatrix}. \tag{2}$$

In Eq. (2), $\underline{\mathbf{u}}_i^d$, $\underline{\mathbf{u}}_\Sigma$ and $\underline{\mathbf{u}}_i^b$ are the $\mathbb{C}^{n_i^d}$, $\mathbb{C}^{N\,n_\Sigma}$ and $\mathbb{C}^{N\,n_i}$ vectors of the n_i^d internal DOF of the disk, of the $N\,n_\Sigma$ coupling interface DOF and of the $N\,n_i$ internal DOF of the blades. The vectors $\underline{\mathbf{q}}^d$ and $\underline{\mathbf{q}}^b$ are the $\mathbb{C}^{N_d}$ and the $\mathbb{C}^{N_b}$ vectors of the generalized coordinates of the disk and of the blades. The block decompositions of matrices $[\underline{S}^b]$ and $[\underline{\Phi}^b]$ with respect to the blades are constituted of matrices $[\underline{S}^b_{jk}] = [\underline{S}^j]\,\delta_{jk}$ and $[\underline{\Phi}^b_{jk}] = [\underline{\Phi}^j]\,\delta_{jk}$ where subscript jk is related to blade j and blade k. Matrix $[\underline{\Phi}^j]$ is the matrix in $\mathbb{M}_{n_i,n_b}(\mathbb{R})$ whose columns are the eigenvectors related to the n_b first eigenvalues of blade j with fixed coupling interface. The matrix $[\underline{S}^j]$ is the matrix in $\mathbb{M}_{n_i,n_\Sigma}(\mathbb{R})$ representing the static boundary functions of blade j. The matrices $[\underline{\Phi}_i^{d,c}]$ and $[\underline{\Phi}_\Sigma^{d,c}]$ correspond to the block decomposition of matrix $[\underline{\Phi}^{d,c}]$ with respect to the internal DOF and the coupling interface DOF of the disk. This matrix $[\underline{\Phi}^{d,c}]$ is the matrix in $\mathbb{M}_{N\,n_\Sigma+n_i^d,N_d}(\mathbb{R})$ whose columns are the eigenvectors of the sub-system related to the DOF of the disk and extracted from the assembled system of the mean finite element matrices of the disk with the mean reduced matrices of the blades. The integer N_d is such that $N_d = N\,n_d$, in which n_d is the number of loaded disk modes related to the n_d first eigenvalues of the disk for a given circumferential wave number. It should be noted that such eigenvectors can be calculated by using the cyclic symmetry of the disk (Ohayon and Soize 1998). Using Eqs. (1,2) yields the mean reduced matrix equation

$$[\underline{\mathbb{A}}_{\text{red}}(\omega)] \begin{bmatrix} \underline{\mathbf{q}}^d(\omega) \\ \underline{\mathbf{q}}^b(\omega) \end{bmatrix} = [\underline{H}]^T\,\underline{\mathbf{f}}(\omega)\,, \tag{3}$$

where $[\underline{\mathbb{A}}_{\text{red}}(\omega)] = -\omega^2\,[\underline{\mathbb{M}}_{\text{red}}] + i\,\omega\,[\underline{\mathbb{D}}_{\text{red}}] + [\underline{\mathbb{K}}_{\text{red}}]$ is the mean reduced dynamic stiffness matrix of the bladed disk such that, for $\mathbb{E}$ representing $\mathbb{M}, \mathbb{D}$ or $\mathbb{K}$,

$$[\underline{\mathbb{E}}_{\text{red}}] = [\underline{H}]^T\,[\underline{\mathbb{E}}]\,[\underline{H}]\,,\,[\underline{\mathbb{E}}_{\text{red}}] \in \mathbb{M}^+_{N_b+N_d}(\mathbb{R})\,. \tag{4}$$

Introducing the block decomposition associated with Eq. (3) yields

$$[\underline{\mathbb{A}}_{\text{red}}(\omega)] = \begin{bmatrix} [\underline{\mathcal{A}}^d(\omega)] & [\underline{A}_c(\omega)] \\ [\underline{A}_c(\omega)]^T & [\underline{\mathcal{A}}^b(\omega)] \end{bmatrix}, \tag{5}$$

in which $[\underline{\mathcal{A}}^d(\omega)]$ is the generalized dynamic stiffness matrix of the disk constructed with eigenvectors matrix $[\underline{\Phi}^{d,c}]$ and where the block jk of $[\underline{\mathcal{A}}^b(\omega)]$ is such that $[\underline{\mathcal{A}}^b(\omega)_{jk}] = [\underline{\mathcal{A}}^j(\omega)]\,\delta_{jk}$ in which $[\underline{\mathcal{A}}^j(\omega)] = -\omega^2[\underline{\mathcal{M}}^j] + i\,\omega[\underline{\mathcal{D}}^j] + [\underline{\mathcal{K}}^j]$ is the generalized dynamic stiffness matrix of each blade with fixed coupling interface and constructed with eigenvectors matrix $[\underline{\Phi}^j]$.

3 MISTUNING ANALYSIS OF THE BLADED DISK WITH RANDOM UNCERTAINTIES

The nonparametric probabilistic model of random uncertainties, whose theory has been completely developed in linear elastodynamics for the low-frequency range (Soize 2000, Soize 2001) and has been extended and validated for modeling non homogeneous random uncertainties (Soize and Chebli 2003) and for mistuning problematics (Capiez-Lernout and Soize 2004), is used.

3.1 Nonparametric probabilistic model of random uncertainties

Since the blade is reduced by using the Craig and Bampton method, it can be shown that the reduced matrix equation of tuned blade j with free coupling interface is written as

$$\begin{bmatrix} [\underline{\mathcal{A}}^j(\omega)] & [\underline{A}_s^j(\omega)] \\ [\underline{A}_s^j(\omega)]^T & [\underline{A}_\Sigma^j(\omega)] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{q}}^j(\omega) \\ \underline{\mathbf{u}}_\Sigma^j(\omega) \end{bmatrix} = \underline{\mathcal{F}}^j(\omega)\,, \qquad (6)$$

in which for blade j, $\underline{\mathbf{u}}_\Sigma^j$ and $\underline{\mathbf{q}}^j$ are the vectors of the coupling interface DOF and of the generalized coordinates. Matrix block $[\underline{\mathcal{A}}^j(\omega)]$ corresponds to the dynamic part of the mean reduced matrix model for blade j with fixed coupling interface. For each blade, it is assumed that random uncertainties mainly affect the dynamics of the blade with fixed coupling interface. Consequently, the nonparametric probabilistic approach is implemented with respect to the matrix $[\underline{\mathcal{A}}^j(\omega)]$ and is written as

$$\mathbf{U}(\omega) = [\underline{H}] \begin{bmatrix} \mathbf{Q}^d(\omega) \\ \mathbf{Q}^b(\omega) \end{bmatrix}, \qquad (7)$$

in which $\mathbf{Q}^d$ is the $\mathbb{C}^{N_d}$-valued vector of the random generalized coordinates of the disk and where $\mathbf{Q}^b = (\mathbf{Q}^0, \ldots, \mathbf{Q}^{N-1})$ is the $\mathbb{C}^{N_b}$-valued vector of the random generalized coordinates of the blades. Random vector $(\mathbf{Q}^d, \mathbf{Q}^b)$ is the solution of the random reduced matrix equation

$$\begin{bmatrix} [\underline{\mathcal{A}}^d(\omega)] & [\underline{A}_c(\omega)] \\ [\underline{A}_c(\omega)]^T & [\boldsymbol{\mathcal{A}}^b(\omega)] \end{bmatrix} \begin{bmatrix} \mathbf{Q}^d(\omega) \\ \mathbf{Q}^b(\omega) \end{bmatrix} = [\underline{H}]^T\, \underline{\mathbf{f}}(\omega)\,, \qquad (8)$$

in which the block jk of $[\boldsymbol{\mathcal{A}}^b(\omega)]$ is such that $[\boldsymbol{\mathcal{A}}^b(\omega)_{jk}] = [\boldsymbol{\mathcal{A}}^j(\omega)]\,\delta_{jk}$ and where $[\boldsymbol{\mathcal{A}}^j(\omega)]$ is the random reduced dynamic stiffness matrix of blade j defined by $[\boldsymbol{\mathcal{A}}^j(\omega)] = -\omega^2\,[\boldsymbol{\mathcal{M}}^j] + i\,\omega\,[\boldsymbol{\mathcal{D}}^j] + [\boldsymbol{\mathcal{K}}^j]$. The available information for random matrices $[\boldsymbol{\mathcal{M}}^j]$, $[\boldsymbol{\mathcal{D}}^j]$ and $[\boldsymbol{\mathcal{K}}^j]$ is

$$\mathcal{E}\{[\boldsymbol{\mathcal{M}}^j]\} = [\underline{\mathcal{M}}^j]\,,\ \mathcal{E}\{[\boldsymbol{\mathcal{D}}^j]\} = [\underline{\mathcal{D}}^j]$$

$$\mathcal{E}\{[\boldsymbol{\mathcal{K}}^j]\} = [\underline{\mathcal{K}}^j] \qquad (9)$$

$$[\boldsymbol{\mathcal{M}}^j]\,, [\boldsymbol{\mathcal{D}}^j]\,, [\boldsymbol{\mathcal{K}}^j] \text{ are } \mathbb{M}_{n_a}^+(\mathbb{R})\text{-valued}\,, \qquad (10)$$

$$\mathcal{E}\{||[\boldsymbol{\mathcal{M}}^j]^{-1}||_F^2\} < +\infty\,,\ \mathcal{E}\{||[\boldsymbol{\mathcal{D}}^j]^{-1}||_F^2\} < +\infty$$

$$\mathcal{E}\{||[\boldsymbol{\mathcal{K}}^j]^{-1}||_F^2\} < +\infty \qquad (11)$$

in which $\mathcal{E}$ is the mathematical expectation and where $||[A]||_F$ denotes the Frobenius norm of matrix $[A]$. The use of the maximum entropy principle with the available information allows the probability distribution of each random matrix to be constructed (Soize 2000, Soize 2001) and it can be proved that $[\boldsymbol{\mathcal{M}}^j], [\boldsymbol{\mathcal{D}}^j], [\boldsymbol{\mathcal{K}}^j], j \in \{0, \ldots, N-1\}$ are independent random variables.

3.2 Identification of the dispersion parameters in the context of geometric mistuning

For each blade j, the probability distribution of each random matrix depends only on dimension n_a and on a positive parameter δ^j called the dispersion parameter. Consequently, for a given blade j, there are three positive numbers δ_M^j, δ_D^j and δ_K^j which are the dispersion parameters controlling the dispersion level of the three independent random matrices $[\boldsymbol{\mathcal{M}}^j]$, $[\boldsymbol{\mathcal{D}}^j]$ and $[\boldsymbol{\mathcal{K}}^j]$. Since we are interested in studying the effects of mistuning due to blade manufacturing tolerances, the dispersion parameters have to be quantified with respect to the geometric tolerances. Such an identification is achieved in constructing a blade random geometry model which respects the tolerances specifications. The random geometry model allows the random mass matrix $[\mathbf{M}^{\text{para},j}]$ and the random stiffness matrix $[\mathbf{K}^{\text{para},j}]$ to be constructed for blade j. It can be shown (Capiez-Lernout et al. 2005) that the identification of dispersion parameters δ_M^j and δ_K^j yields

$$\delta_M^j = \sqrt{\frac{W_M^{\text{tol},j}\,(n_b+1)}{tr([\underline{\mathcal{M}}^j]^2) + tr([\underline{\mathcal{M}}^j])^2}}\,, \qquad (12)$$

$$\delta_K^j = \sqrt{\frac{W_K^{\text{tol},j}\,(n_b+1)}{tr([\underline{\mathcal{K}}^j]^2) + tr([\underline{\mathcal{K}}^j])^2}}\,, \qquad (13)$$

in which tr denotes the trace and where $W_M^{\text{tol},j}$ and $W_K^{\text{tol},j}$ are defined by

$$W_M^{\text{tol},j} = \mathcal{E}\{||[\underline{\Phi}^j]^T\,[\mathbf{M}^{\text{para},j}]\,[\underline{\Phi}^j] - [\underline{\mathcal{M}}^j]||_F^2\}\,, \qquad (14)$$

$$W_K^{\text{tol},j} = \mathcal{E}\{||[\underline{\Phi}^j]^T\,[\mathbf{K}^{\text{para},j}]\,[\underline{\Phi}^j] - [\underline{\mathcal{K}}^j]||_F^2\}. \qquad (15)$$

The Monte Carlo numerical simulation is used for computing δ_M^j and δ_K^j. Once the dispersion parameters are identified, the stochastic equation defined by Eq. (8) is solved with the Monte Carlo numerical simulation.

3.3 Random dynamic magnification factor

Let $[\underline{K}^j]$ be the mean finite element stiffness matrix of blade j. The elastic energy of blade j related to the mean dynamical system is written as $\underline{e}^j(\omega) = \frac{1}{2}\underline{\mathbf{u}}^j(\omega)^*\,[\underline{K}^j]\,\underline{\mathbf{u}}^j(\omega)$, in which $\underline{\mathbf{u}}^j$ is the vector of the DOF related to tuned blade j. Due to the cyclic symmetry, we have $\underline{e}^0(\omega) = \ldots = \underline{e}^{N-1}(\omega)$ denoted as $\underline{e}(\omega)$. The similar quantity related to the stochastic dynamic system is defined as $\mathbf{E}^j(\omega) = \frac{1}{2}\mathbf{U}^j(\omega)^*\,[\underline{K}^j]\,\mathbf{U}^j(\omega)$, in which $\mathbf{U}^j$ is the random vector of the DOF related to mistuned blade j. For ω fixed in $\mathbb{B}$, the random dynamic analysis is carried out by introducing the random dynamic magnification factor over frequency band $\mathbb{B}$ B_∞ such that

$$B_\infty = \max_{\omega\in\mathbb{B}} \max_{j=0,\ldots,N-1} \sqrt{\frac{\mathbf{E}^j(\omega)}{\underline{e}_\infty}}\,,\; \underline{e}_\infty = \max_{\omega\in\mathbb{B}} \underline{e}(\omega). \quad (16)$$

We are interested in estimating the probability $\mathcal{P}(B_\infty > b_c)$, in which b_c is a given amplification level. A second-order convergence analysis of random variable B_∞ with respect to parameters n_d and n_b is carried out. We introduce the mapping $(n_d, n_b) \mapsto |||B_\infty|||$ with $|||B_\infty|||^2 = \mathcal{E}\{B_\infty^2\}$. This function $|||B_\infty|||$ is estimated by $\mathrm{C}(n_s, n_d, n_b)$ such that

$$\mathrm{C}^2(n_s, n_d, n_b) = \frac{1}{n_s}\sum_{i=1}^{n_s} B_\infty^2(\theta_i)\,, \quad (17)$$

in which $\theta_1, \ldots, \theta_{n_s}$ are the n_s realizations of the Monte Carlo numerical simulation.

4 APPLICATION TO AN INDUSTRIAL FAN STAGE

4.1 *Description of the structure*

The structure considered is a wide chord supersonic fan geometry called SGC1. The fan has 22 blades. The finite element model of the bladed disk is shown in Fig. 1.

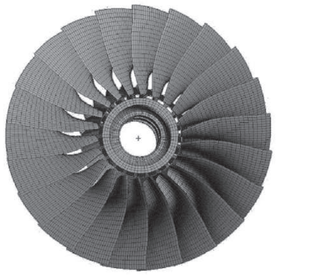


Figure 1. Finite element mesh of the bladed disk.

It is constructed with 31812 solid elements and is constituted of $n = 473\,814$ DOF. Each sector contains 8133 nodes which corresponds to 22947 DOF. The structure is characterized by the following parameters $n_i = 16\,197$, $n_\Sigma = 414$ and $n_i^d = 6036$. The structure is in rotation around its revolution axis with the constant velocity $\Omega = 4500\,rpm$. Since the dynamic analysis is carried out in the rotating frame of the structure, the rigid-body motion due to the rotation of the structure corresponds to a fixed boundary condition at the inner radius of the structure. The bladed disk is made of titanium. Figure 2 displays the eigenfrequencies of the tuned bladed disk with respect to the circumferential wave number.

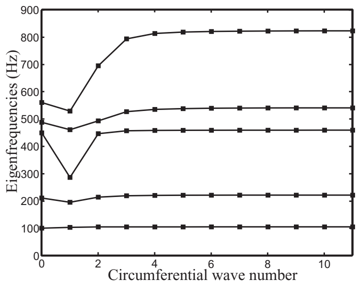


Figure 2. Graph of the tuned eigenfrequencies with respect to the circumferential wave number .

The mistuned forced response of the structure subjected to a third engine order excitation in the low-frequency band $\mathbb{B} = [515\,, 545]\,Hz$ is analyzed. A hysteretical damping model with the mean loss factor $\eta = 0.002$ is added to the bladed disk.

4.2 *Identification of the dispersion parameters*

In the context of the blade manufacturing, the tolerances are specified for profiles at a given height from the base of the blade. For a given profile, the leading edge (LE) of the blade profile is assumed to be fixed in order to define the tolerances parameters (see Fig. 3). Let dL and $d\alpha$ be the parameters controlling the length and the angular position of the chord such that $dL \in [-dL_m; dL_M]$, $d\alpha \in [-d\alpha_m\; d\alpha_M]$, in which dL_m, dL_M, $d\alpha_m$ and $d\alpha_M$ are positive scalars.

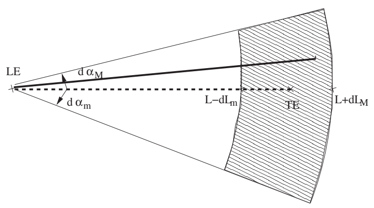
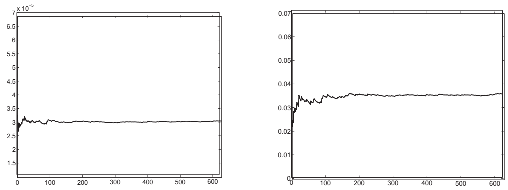


Figure 3. Description of the tolerance parameters. Chord of the nominal profile (dashed line) - Chord of the manufactured profile (solid line) - Localisation of the trailing edge (TE) (gray zone).

A random blade geometry model is constructed in order to identify the dispersion parameters of the non-parametric model used for modeling the blade mistuning. For its construction, it is assumed that the coupling interface between the disk and the blades remains deterministic. Let $\underline{\mathbf{x}}$ be the vector in $\mathbb{R}^{3n_i}$ defining the position of the nodes belonging to the nominal mesh of the blade. Let then $\mathbf{x}$ be the vector in $\mathbb{R}^{3n_i}$ defining the position of the nodes belonging to the mesh of the manufactured blade. In the probabilistic context of tolerancing, vector $\mathbf{x}$ is modeled by the $\mathbb{R}^{3\,n_i}$-valued random vector $\mathbf{X}$ such that

$$\mathbf{X} = \underline{\mathbf{x}} + \sum_{\alpha=1}^{r} \xi_\alpha \, \mathbf{b}_\alpha \, , \tag{18}$$

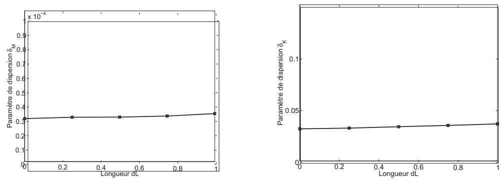
in which $\mathbf{b}_\alpha$, $\alpha \in \{1, \ldots, r\}$ are r deterministic vectors of $\mathbb{R}^{3\,n_i}$ and where ξ_α , $\alpha \in \{1, \ldots, r\}$ are r independent random variables. For convenience the chosen basis vectors $\mathbf{b}_\alpha$ are the blade eigenmodes with fixed coupling interface. The probability distribution of random variable ξ_α is chosen as uniform and its support is calculated in order to respect a given set $\{dL_m, dL_M, d\alpha_m, d\alpha_M\}$ of tolerances. In addition, such a probability model allows the blade random shape to be regular. It should be noted that the construction of such a blade random geometry model remains arbitrary and does not correspond to any given manufacturing process. It is important to note that the random dynamic magnification factor is strongly sensitive to the choice of the probability model which is constructed with the nonparametric approach and which does not directly depend on the probabilistic model of the geometry: this nonparametric model depends only on the dispersion parameters which are estimated as a function of the geometric tolerances by using the blade random geometry model. Let n_s be the number of realizations used in the Monte Carlo numerical simulation and let $\tilde{\delta}_M$ and $\tilde{\delta}_k$ be the statistical estimators of dispersion parameters δ_M and δ_K defined by Eqs. (12,13). The Fig. 4 displays the graphs $n_s \mapsto \tilde{\delta}_M$ and $n_s \mapsto \tilde{\delta}_K$ for $dL_m = 0.55\,mm$, $dL_M = 0.75\,mm$, $d\alpha_m = d\alpha_M = 0.55°$. A reasonable convergence is obtained with $n_s = 300$ realizations and yields $\delta_M = 3\,10^{-5}$ and $\delta_K = 3.5\,10^{-2}$.



(a) $n_s \mapsto \tilde{\delta}_M$ (b) $n_s \mapsto \tilde{\delta}_K$

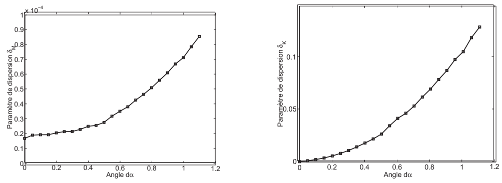
Figure 4. Stochastic convergence of the dispersion parameters with respect to the number of Monte Carlo realizations.

A sensitivity analysis of the dispersion parameters is then performed with respect to the tolerances. For a set of tolerances such that $d\alpha_m = d\alpha_M = 0.55°$ and $dL = dL_m = dL_M$, the Fig. 5 shows the graphs of $dL \mapsto \delta_M$, $dL \mapsto \delta_K$. For a set of tolerances such that $d\alpha = d\alpha_m = d\alpha_M$ and $dL_m = 0.55\,mm$, $dL_M = 0.75\,mm$, the Fig. 6 shows the graphs of $d\alpha \mapsto \delta_M$, $d\alpha \mapsto \delta_K$.



(a) $dL \mapsto \delta_M$ (b) $dL \mapsto \delta_K$

Figure 5. Identification of the dispersion parameters for $d\alpha_m = d\alpha_M = 0.55°$ and $dL \in [0\,,1\,mm]$.



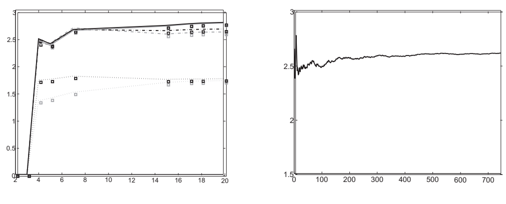
(a) $d\alpha \mapsto \delta_M$ (b) $d\alpha \mapsto \delta_K$

Figure 6. Identification of the dispersion parameters for $dL_m = 0.55\,mm$, $dL_M = 0.75\,mm$ and $d\alpha \in [0\,,1.1°]$.

It is then deduced that the dispersion parameters are more sensitive to the tolerances related to the angular position of the chord than to the tolerances related to the length of the chord.

4.3 *Stochastic convergence analysis for the random reduced model*

For $\delta_M^j = \delta_D^j = 0$ and $\delta_K^j = 0.05$, Fig. 7 presents the convergence analysis of random variable $|||B_\infty|||$ with respect to parameters n_d and n_a allowing the dimension of the random reduced model to be controlled and with respect to the number n_s of realizations of the Monte Carlo numerical simulation.



(a) $n_s \mapsto \mathrm{C}^2(n_s, 20, 10)$ (b) $n_b \mapsto \mathrm{C}^2(500, n_d, n_b)$

Figure 7. Stochastic convergence analysis of the random reduced matrix model.

The Fig. 7(a) displays the graph $n_s \mapsto \mathrm{C}^2(n_s, n_d, n_b)$ for $n_d = 10$ and $n_b = 20$. A reasonable convergence is obtained for $n_s = 350$. For $n_s = 500$, the Fig. 7(b) shows the graph $n_b \mapsto \mathrm{C}^2(500, n_d, n_b)$ for $n_d > 7$ (solid line), for $n_d = 5$ (black dashed-dotted line), for $n_d = 4$ (gray dashed-dotted line), for $n_d = 3$ (black dotted line) and for $n_d = 2$ (gray dotted line). It is deduced that parameters n_d and n_b have to be chosen such that $n_d = 7$ and $n_b = 7$.

4.4 *Probabilistic analysis of the blade mistuning due to geometric uncertainties.*

The random mistuning analysis is carried out for the complete bladed disk. Assuming the uncertainty level to be homogeneous from one blade to another one yields $\delta_M^j = \delta_M$, $\delta_D^j = \delta_D$ and $\delta_K^j = \delta_K$, for all $j \in \{0, \ldots, N-1\}$. The mass and the stiffness dispersion parameters are identified with respect to a given set of tolerances. In the context of blade tolerancing, there is no uncertainty on the damping of the blades and the damping dispersion parameter is chosen as $\delta_D = 0$. The Monte Carlo numerical simulation is performed with $n_s = 1500$ realizations.

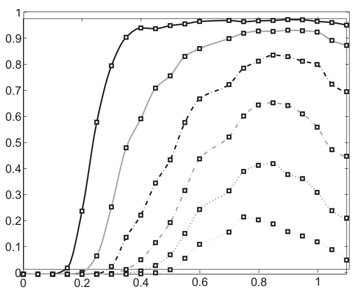


Figure 8. Graph of the function $d\alpha \mapsto \mathcal{P}(B_\infty > b_c)$ for $b_c = 1.2$ (black solid line), $b_c = 1.3$ (gray solid line), $b_c = 1.4$ (black dashed line), $b_c = 1.5$ (gray dashed line), $b_c = 1.6$ (black dotted line), $b_c = 1.7$ (gray dotted line).

Figure 8 displays the graphs $d\alpha \mapsto \mathcal{P}(B_\infty > b_c)$ for several values of $b_c = 1.2$ (black solid line), $b_c = 1.3$ (gray solid line), $b_c = 1.4$ (black dashed-dotted line), $b_c = 1.5$ (gray dashed-dotted line), $b_c = 1.6$ (black dotted line) and $b_c = 1.7$ (gray dotted line). It is seen that this graph has a maximum which allows two types of specifications to be considered. One of these specifications consists in defining tolerances of the blade with high precision, whereas the other one consists in intentionally mistuning the blade. For example, the confidence region defined by $\mathcal{P}(B_\infty > 1.7) \leq 0.1$ is obtained for parameter $d\alpha < 0.48°$ or $d\alpha > 1.04°$.

5 CONCLUSIONS

We have presented a methodology allowing the blade geometric tolerances of a mistuned bladed disk for a given confidence region of the random dynamic magnification factor to be specified. Such a methodology uses a nonparametric probabilistic model of random uncertainties for modeling the mistuning of the blades and requires to relate the dispersion parameters of the nonparametric probabilistic approach to the tolerance parameters of the blades. The efficiency of this methodology is proved through a complex numerical model of a bladed disk.

REFERENCES

Benfield, W.A. & Hruda, R.F. 1971. Vibration analysis of structures by component mode substitution. *AIAA Journal* Vol. 9 (No. 7): 1255–1261.

Capiez-Lernout, E. & Soize, C. 2004. Nonparametric modeling of random uncertainties for dynamic response of mistuned bladed-disks. *ASME Journal of Engineering for Gas Turbines and Power* Vol. 126 (No. 3): 610–618.

Capiez-Lernout, E., Soize, C., Lombard, J.-P., Dupont, C., & Seinturier, E. 2005. Blade manufacturing tolerances definition for a mistuned industrial bladed disk. *ASME Journal of Engineering for Gas Turbines and Power (In press)* Vol. 127 (No. 3):.

Craig, R.R.Jr. & Bampton, M.C.C. 1968. Coupling of substructures for dynamic analyses. *AIAA Journal* Vol. 6 (No. 7): 1313–1319.

Griffin, J.H. & Hoosac, T.M. 1984. Model development and statistical investigation of turbine blade mistuning. *ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design 106* 204–210.

Mignolet, M.P., Lin, C.C., & LaBorde, B.H. 2001. A novel limit distribution for the analysis of randomly mistuned bladed disks. *ASME Journal of Engineering for Gas Turbines and Power 123* 388–394.

Ohayon, R. & Soize, C. 1998. *Structural acoustics and vibration*. Academic press.

Seinturier, E., Dupont, C., Berthillier, M., & Dumas, M. 2002. A new aeroelastic model for mistuned bladed disks. *AIAA paper 2002-1533*.43rd AIAA/-ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference.

Soize, C. 2000. A nonparametric model of random uncertainties for reduced matrix models in structural dynamics. *Probabilistic Engineering Mechanics* Vol. 15 (No. 3): 277–294.

Soize, C. 2001. Maximum entropy approach for modeling random uncertainties in transient elastodynamics. *Journal of the Acoustical Society of America* Vol. 109 (No. 5): 1979–1996.

Soize, C. & Chebli, H. 2003. Random uncertainties model in dynamic substructuring using a nonparametric probabilistic model. *ASCE Journal of Engineering Mechanics* Vol. 129 (No. 4): 449–457.

Whitehead, D.S. 1966. Effects of mistuning on the vibration of turbomachine blades induced by wakes. *Journal of Mechanical Engineering Science* Vol. 8 (No. 1): 15–21.