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ABSTRACT: In order to improve the numerical prediction robustness of the vehicle models, one introduces a model of random uncertainties with the non-parametric approach. Such an approach takes into account data errors as well as model errors. This paper deals with classical low frequency vibroacoustic modelling such as used for booming noise prediction. We are especially interested in vibroacoustic coupling uncertainties which are relevant to model errors. Introducing two different fluid-structure interface meshes, two estimations of acoustic energy are computed. Applying random uncertainties on the coupling interface, the results show that the non-parametric approach of vibroacoustic coupling uncertainties is able to model this class of uncertainties.

1 INTRODUCTION

In automotive industry, numerical simulation is intensively used to predict the dynamical behavior of vehicles. Computational means capacity has increased in the last last years, allowing models more and more detailed to be run, but there is still a need for design tools that are able to provide robustness of the models. Hence, to improve the robustness of the models, one has to introduce uncertainties in numerical computations. Uncertainties can be classified into two classes:

(1) Data uncertainties which concern the parameters of the mechanical model such as the geometrical parameters, the parameters allowing the boundary conditions to be prescribed, the constitutive equations, etc.

(2) Model uncertainties which are introduced during the construction of the model: the constructed model cannot exactly represent a structure as complex as a car body for instance, due to the introduction of approximations and simplifications which make the modeling practicable. This class of uncertainties is not relevant to the parametric approach because, by definition, the model uncertainties cannot be taken into account by the parameters of the model under consideration.

For a classical low-frequency vibroacoustic model of a car, uncertainties may affect the structural part, the acoustic cavity, or the coupling interface. This paper is especially dedicated to the specific case of coupling interface uncertainties. Considering different interface meshes will lead to different computations of a given acoustic quantity. How can we take into account these errors? and in a more general way, how can we account for effects that lie at the coupling interface?

Recently, a new approach called the non-parametric approach has been introduced to model random uncertainties [Soize C. (2000, 2001, 2005)]. This approach, allowing the direct construction of a probabilistic model of a matrix model (modal reduction) of the structure, does not require identifying the numerous uncertain parameters as usually done for parametric approach but is governed by a single uncertainty parameter.

An implementation of the nonparametric approach in the case of acoustic energy prediction for an industrial model of vehicle is presented in this paper. This includes the adaptation of the nonparametric approach to vibroacoustic problems. Since, we are interested in vibroacoustic coupling errors using the non-parametric approach of uncertainties, we introduce coupling uncertainties by using two different interface meshes leading to two computations of acoustic energy. Finally, random acoustic energy is computed, and its confidence region is numerically constructed.

2 STATEMENT OF THE STRUCTURAL-ACOUSTIC PROBLEM

2.1 *Finite element discretization*

The finite element method [Zienkiewicz O.C. and al. (1989)] is used to solve numerically the classical equations of a structural-acoustic boundary value problem. We consider a finite-element mesh of the structure Ω_S and of the internal fluid Ω_F. Let $\underline{\mathbf{U}}^S = (\underline{U}_1^S, \ldots, \underline{U}_{n_S}^S)$ be the complex vector of the n_S degrees of freedom (DOF) of the structure corresponding to the finite element discretization of the displacement field **u**. Let $\underline{\mathbf{P}}^F = (\underline{P}_1^F, \ldots, \underline{P}_{n_F}^F)$ be the complex vector of the n_F DOF of the fluid corresponding to the finite element discretization of the pressure field p. Therefore, the finite element discretization of the boundary value problem in terms of **u** and p [Ohayon R. and al. (1998)] yields the following matrix equation

$$\begin{bmatrix} [\underline{A}^S(\omega)] & [\underline{C}] \\ \omega^2[\underline{C}]^T & [\underline{A}^F(\omega)] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{U}}^S(\omega) \\ \underline{\mathbf{P}}^F(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{F}}^S(\omega) \\ \underline{\mathbf{S}}^F(\omega) \end{bmatrix} \quad (1)$$

where $[\underline{A}^S(\omega)]$ is the dynamical stiffness matrix of the structure which is a symmetric $(n_S \times n_S)$ complex matrix such that

$$[\underline{A}^S(\omega)] = -\omega^2[\underline{M}_{n_S}^S] + i\omega[\underline{D}_{n_S}^S] + [\underline{K}_{n_S}^S] \quad ,$$

in which $[\underline{M}_{n_S}^S]$, $[\underline{D}_{n_S}^S]$, $[\underline{K}_{n_S}^S]$ are the mass, damping and stiffness matrices of the structure *in vacuo*. Matrix $[\underline{M}_{n_S}^S]$ is a positive-definite symmetric $(n_S \times n_S)$ real matrix, $[\underline{D}_{n_S}^S]$ and $[\underline{K}_{n_S}^S]$ are semipositive-definite symmetric $(n_S \times n_S)$ real matrices. In Eq. (1), $[\underline{A}^F(\omega)]$ is the dynamical stiffness matrix of the acoustic fluid which is a symmetric $(n_F \times n_F)$ complex matrix such that

$$[\underline{A}^F(\omega)] = -\omega^2[\underline{M}_{n_F}^F] + i\omega[\underline{D}_{n_F}^F] + [\underline{K}_{n_S}^F] \quad ,$$

in which $[\underline{M}_{n_F}^F]$, $[\underline{D}_{n_F}^F]$, $[\underline{K}_{n_F}^F]$ are the mass, damping and stiffness matrices of the cavity with fixed coupling interface. Matrix $[\underline{M}_{n_F}^F]$ is a positive-definite symmetric $(n_F \times n_F)$ real matrix, $[\underline{D}_{n_F}^F]$ and $[\underline{K}_{n_F}^F]$ are semipositive-definite symmetric $(n_F \times n_F)$ real matrices.

Matrix $[C]$ is the vibroacoustic coupling matrix which is a $(n_S \times n_F)$ real matrix. The vibroacoustic coupling matrix results from the discretization of the sesquilinear form $c(p, \mathbf{u})$, $c(p, \mathbf{u}) = \int_{\Gamma_F} p\overline{u_i} n_i \, d\Gamma_F$ which accounts for the continuity of the pressure and displacement fields at the fluid structure interface (where Γ_F denotes the fluid-structure interface).

Introducing the change of variable $\underline{\mathbf{P}}^F = i\omega\widetilde{\underline{\mathbf{P}}}^F$, Eq. (1) can be rewritten as the following symmetric matrix equation,

$$\begin{bmatrix} [\underline{A}^S(\omega)] & i\omega[\underline{C}] \\ i\omega[\underline{C}]^T & -[\underline{A}^F(\omega)] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{U}}^S(\omega) \\ \widetilde{\underline{\mathbf{P}}}^F(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{F}}^S(\omega) \\ -\frac{1}{i\omega}\underline{\mathbf{S}}^F(\omega) \end{bmatrix} \quad , (2$$

2.2 *Reduced matrix model of the structural-acoustic system*

The structural modes *in vacuo* and the acoustic modes of the cavity with fixed coupling interface are calculated by solving the two generalized eigenvalue problems,

$$[\underline{K}_{n_S}^S]\underline{\psi} = \underline{\lambda}^S[\underline{M}_{n_S}^S]\underline{\psi} \quad , \quad (3)$$

$$[\underline{K}_{n_F}^F]\underline{\phi} = \underline{\lambda}^F[\underline{M}_{n_F}^F]\underline{\phi} \quad . \quad (4)$$

The eigenvectors verify the usual orthogonal properties [Bathe K.J. and al. (1976), Dautray R. and al. (1992)]. The structure *in vacuo* has six rigid body modes corresponding to a zero eigenvalue and $n-6$ structural modes. Since we are only interested in the elastic deformation of the structure, the structural displacement is written as

$$\underline{\mathbf{U}}^S(\omega) = [\underline{\Psi}]\,\underline{\mathbf{q}}^S(\omega) \quad , \quad (5)$$

in which $[\underline{\Psi}]$ is the $(n_S \times n)$ real matrix whose columns are constituted of the n structural modes associated with the n first positive eigenvalues (the n first structural eigenfrequencies). The internal acoustic cavity has one constant pressure mode and $m-1$ acoustic modes. The internal acoustic pressure is written as

$$\underline{\mathbf{P}}^F(\omega) = [\underline{\Phi}]\,\underline{\mathbf{q}}^F(\omega) \quad , \quad (6)$$

in which $[\underline{\Phi}]$ is the $(n_F \times m)$ real matrix whose columns are constituted (1) of the constant pressure mode associated with zero eigenvalue and (2) of the acoustic modes associated with the positive eigenvalues (the $m-1$ first acoustical eigenfrequencies). It should be noted that the constant pressure mode is kept in order to model the quasi-static variation of the internal fluid pressure induced by the deformation of the coupling interface [Ohayon R. and al. (1998)].

Using Eqs. (3) and (4), the projection of Eq. (1) yields the mean reduced matrix model of the structural-acoustic system,

$$\begin{bmatrix} [\underline{\mathcal{A}}^S(\omega)] & i\omega[\underline{\mathcal{C}}] \\ i\omega[\underline{\mathcal{C}}]^T & -[\underline{\mathcal{A}}^F(\omega)] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{q}}^S(\omega) \\ \widetilde{\underline{\mathbf{q}}}^F(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathcal{F}}^S(\omega) \\ -\frac{1}{i\omega}\underline{\mathcal{S}}^F(\omega) \end{bmatrix} \quad (7)$$

3 CONSTRUCTION OF THE STOCHASTIC STRUCTURAL-ACOUSTIC PROBLEM WITH THE NONPARAMETRIC APPROACH OF UNCERTAINTIES

We consider only uncertainties on the vibroacoustic coupling matrix. These uncertainties are induced by model errors associated with incompatible geometries of the structural part and acoustic cavity. Due to the presence of model errors, the usual parametric probabilistic approach which is perfectly adapted for modeling data errors, cannot be used for modelling model errors. Such a non-parametric probabilistic method has been proposed in [Soize C. (2000, 2001, 2005)] and is used below.

From this point, the structural-acoustic system will be named mean structural-acoustic system in opposition to the stochastic structural-acoustic system described below. The first step, in order to use the non-parametric approach, is to factorize the mean reduced coupling matrix.

3.1 Factorization of the reduced vibroacoustic coupling matrix of the mean structural-acoustic system

For most of vibroacoustic problems, coupling matrix is rectangular due to different modal densities for the structural and acoustic parts in a given frequency band. The following factorization method for rectangular matrices has been proposed in [Soize C. (2005)].

Let $\mathbb{M}_{n,m}(\mathbb{R})$, be the set of all the $(n \times m)$ real matrices. Let $[\underline{\mathcal{C}}]$ be a rectangular matrix in $\mathbb{M}_{n,m}(\mathbb{R})$ for which its null space is reduced to 0. Then, $[\underline{\mathcal{C}}]$ can be written as

$$[\underline{\mathcal{C}}] = [\underline{\mathcal{U}}_{n,m}][\underline{\mathcal{T}}_m] \tag{8}$$

in which the rectangular matrix $[\underline{\mathcal{U}}_{n,m}]$ and the symmetric square matrix $[\underline{\mathcal{T}}_m]$ are such that $[\underline{\mathcal{T}}_m]$ belongs to $\mathbb{M}_m^+(\mathbb{R})$ and $[\underline{\mathcal{U}}_{n,m}]$ belongs to $\mathbb{M}_{n,m}(\mathbb{R})$ such that $[\underline{\mathcal{U}}_{n,m}]^T[\underline{\mathcal{U}}_{n,m}] = [I_m]$.

For a given rectangular matrix $[\underline{\mathcal{C}}]$ belonging to $\mathbb{M}_{n,m}(\mathbb{R})$, let $[\underline{\mathcal{B}}_m]$ be the positive-definite square matrix such that

$$[\underline{\mathcal{B}}_m] = [\underline{\mathcal{C}}]^T[\underline{\mathcal{C}}] \quad \in \quad \mathbb{M}_m^+(\mathbb{R}) \tag{9}$$

Since $[\underline{\mathcal{B}}_m]$ belongs to $\mathbb{M}_m^+(\mathbb{R})$, matrix $[\underline{\mathcal{B}}_m]$ is diagonalizable and can be written as

$$[\underline{\mathcal{B}}_m] = [\underline{\Theta}_m][\underline{\Sigma}_m][\underline{\Theta}_m]^T \tag{10}$$

in which $[\underline{\Sigma}_m]$ is the diagonal matrix of the positive eigenvalues of matrix $[\underline{\mathcal{B}}_m]$ and where $[\underline{\Theta}_m]$ is the orthogonal matrix of the corresponding eigenvectors. Consequently, $[\underline{\Sigma}_m]^{1/2}$ is the matrix of singular values of matrix $[\underline{\mathcal{C}}]$ and we define matrix $[\underline{\mathcal{T}}_m]$ by

$$[\underline{\mathcal{T}}_m] = [\underline{\mathcal{B}}_m]^{1/2} = [\underline{\Theta}_m][\underline{\Sigma}_m]^{1/2}[\underline{\Theta}_m]^T \in \mathbb{M}_m^+(\mathbb{R}) \tag{11}$$

$$[\underline{\mathcal{U}}_{n,m}] = [\underline{\mathcal{C}}][\underline{\mathcal{T}}_m]^{-1} \quad \in \mathbb{M}_{n,m}(\mathbb{R}) \tag{12}$$

Since $[\underline{\mathcal{T}}_m]$ belongs to $\mathbb{M}_m^+(\mathbb{R})$ it can be written as

$$[\underline{\mathcal{T}}_m] = [\underline{L}_{\mathcal{T}_m}]^T[\underline{L}_{\mathcal{T}_m}] \tag{13}$$

The non-parametric probabilistic approach of vibroacoustic coupling uncertainties consists in substituting the generalized matrix $[\underline{\mathcal{C}}]$ by the $(n \times m)$ random matrix $[\mathbf{C}]$, which is $\mathbb{M}_{n,m}(\mathbb{R})$-valued random variable written as

$$[\mathbf{C}] = [\underline{\mathcal{U}}_{n,m}][\underline{L}_{\mathcal{T}_m}]^T[\mathbf{G}_{\mathcal{C}}][\underline{L}_{\mathcal{T}_m}] \quad . \tag{14}$$

where $[\mathbf{G}_{\mathcal{C}}]$, is a $\mathbb{M}_m^+(\mathbb{R})$-valued random variable.

3.2 Stochastic process implementation

The basic available information in order to build a stochastic process, is constituted of the mean vibroacoustic coupling matrix.

$$E\{[\mathbf{C}]\} = [\underline{\mathcal{C}}] \quad ,$$

The probability distribution on the set $\mathbb{M}_m^+(\mathbb{R})$ is explicitly constructed in [Soize C. (2000, 2001)]. The following algebraic representation of a random matrix $[\mathbf{G}_{\mathcal{C}}]$, allows an easy construction of its realizations. We then have

$$[\mathbf{G}_{\mathcal{C}}] = [\mathbf{L}_{\mathcal{C}}]^T[\mathbf{L}_{\mathcal{C}}] \quad ,$$

in which $[\mathbf{L}_{\mathcal{C}}]$ is a random upper triangular $(m \times m)$ real matrix whose random elements are independent random variables defined as follows:

(1) for $j < j'$, the real-valued random variable $[\mathbf{L}_{\mathcal{C}}]_{jj'}$ is written as $[\mathbf{L}_{\mathcal{C}}]_{jj'} = \sigma_m U_{jj'}$ in which $\sigma_m = \delta_{\mathcal{C}}\,(m+1)^{-1/2}$ and where $U_{jj'}$ is a real-valued Gaussian random variable with zero mean and variance equal to 1. $\delta_{\mathcal{C}}$ is the uncertainty parameter that controls the actual matrix variance ;

(2) for $j = j'$, the positive-valued random variable $[\mathbf{L}_{\mathcal{C}}]_{jj'}$ is written as $[\mathbf{L}_{\mathcal{C}}]_{jj'} = \sigma_m\sqrt{2V_j}$ in which σ_m is defined above and where V_j is a positive-valued gamma random variable whose probability density function p_{V_j} with respect to dv is written as

$$p_{V_j}(v) = \mathbf{1}_{\mathbb{R}^+}(v)\,\frac{1}{\Gamma(\frac{m+1}{2\delta_{\mathcal{C}}^2} + \frac{1-j}{2})}\, v^{(\frac{m+1}{2\delta_{\mathcal{C}}^2} - \frac{1-j}{2})} e^{-v} \quad .$$

By construction, the matrix $[\mathbf{G}_{\mathcal{C}}]$ has a mean value equal to the identity matrix : $E\{[\mathbf{G}_{\mathcal{C}}]\} = [I_m]$.

and its standard deviation is controlled by the parameter $\delta_{\mathcal{C}}$ defined as

$$\delta_{\mathcal{C}} = \sqrt{\frac{E\{\|[\mathbf{G}_{\mathcal{C}}] - [\underline{G}_{\mathcal{C}}]\|_F^2\}}{\|[\underline{G}_{\mathcal{C}}]\|_F^2}} \quad ,$$

where $[\underline{G}_{\mathcal{C}}] = [I_m]$ and where $\|.\|_F$ is the Frobenius norm.

Consequently, the mean reduced vibroacoustic coupling matrix $[\underline{\mathcal{C}}]$ may be changed into a random $(n \times m)$ real matrix $[\mathbf{C}]$. With such a non-parametric probabilistic model, Eq. (7) is replaced by the following random matrix equation,

$$\begin{bmatrix} [\underline{\mathcal{A}}_n^S(\omega)] & i\omega[\mathbf{C}] \\ i\omega[\mathbf{C}]^T & -[\underline{\mathcal{A}}_m^F(\omega)] \end{bmatrix} \begin{bmatrix} \mathbf{Q}^S(\omega) \\ \widetilde{\mathbf{Q}}^F(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathcal{F}}^S(\omega) \\ -\frac{1}{i\omega}\underline{\mathcal{S}}^F(\omega) \end{bmatrix} \quad (15)$$

where, for fixed ω, $\mathbf{Q}^S(\omega)$ and $\widetilde{\mathbf{Q}}^F(\omega)$ are complex random vectors. For each realization of the stochastic process, Eq.(14) is solved for every ω.

3.3 Confidence region of the random vibroacoustic frequency response

Managing results of a random process is an important issue. We choose to use confidence interval in order to be compatible with any distribution law of the results.

Let $\omega \mapsto V(\omega) : B \to \mathbb{R}^+$ be the modulus of an observed vibroacoustic frequency response (either the modulus of the frequency response in displacement at a given point in the structure or the modulus of the frequency response in pressure at a given point in the internal acoustic cavity). The confidence region associated with the probability level P_c for the random function $\{V(\omega), \omega \in B\}$ is constructed by using the quantiles. For fixed ω in B, let $F_{V(\omega)}$ be the cumulative distribution function (continuous from the right) of random variable $V(\omega)$ which is such that $F_{V(\omega)}(w) = P(V(\omega) \leq w)$. For $0 < p < 1$, the p-th quantile or fractile of $F_{V(\omega)}$ is defined as

$$\zeta(p) = \inf\{w : F_{V(\omega)}(w) \geq p\} \quad . \quad (16)$$

Then the upper envelope $v^+(\omega)$ and the lower envelope $v^-(\omega)$ of the confidence region are defined by

$$v^+(\omega) = \zeta(1 - P_c) \quad , \quad v^-(\omega) = \zeta(P_c) \quad . \quad (17)$$

The estimation of $v^+(\omega)$ and $v^-(\omega)$ is performed by using the sample quantiles [Serfling R.J. (1980)]. Let $v_1(\omega) = V(\omega; \theta_1), \ldots, v_{n_r}(\omega) = V(\omega; \theta_{n_r})$ be the n_r independent realizations of random variable $V(\omega)$. Let $\widetilde{v}_1(\omega) < \ldots < \widetilde{v}_{n_r}(\omega)$ be the order statistics associated with $v_1(\omega), \ldots, v_{n_r}(\omega)$. Therefore, one has the following estimation

$$v^+(\omega) \simeq \widetilde{v}_{j^+}(\omega) \quad , \quad j^+ = \text{fix}(n_r(P_c)) \quad (18)$$

$$v^-(\omega) \simeq \widetilde{v}_{j^-}(\omega) \quad , \quad j^- = \text{fix}(n_r(1 - P_c)) \quad (19)$$

in which fix(z) is the integer part of the real number z.

4 APPLICATION TO THE VIBROACOUSTIC RESPONSE OF A TRIMMED BODY

4.1 Mean model of the trimmed body

The mean model is constituted of a trimmed body as currently built by PSA engineers [Sol A. and al. (2001)]. We are interested in studying acoustic energy in the low-frequency band $B = [20, 300]Hz$, for applied unitary forces (12 loaded structural DOF). The finite element model of the structure has 947298 DOF. The finite element model of the internal acoustic cavity has 6883 DOF corresponding to 34796 acoustic fluid finite elements. It should be noted that the two meshes are incompatible on the fluid-structure coupling interface which is a cause of uncertainty.

Fig. 1 displays two interface meshes corresponding to two computations of the acoustic energy. On Fig. 1, the upper left mesh is a rough interface mesh, constructed by a spatial sampling of coupled panels of the mean structural model. The upper right mesh is a detailed interface mesh corresponding, for most of the coupled panels, to the mean structural model.

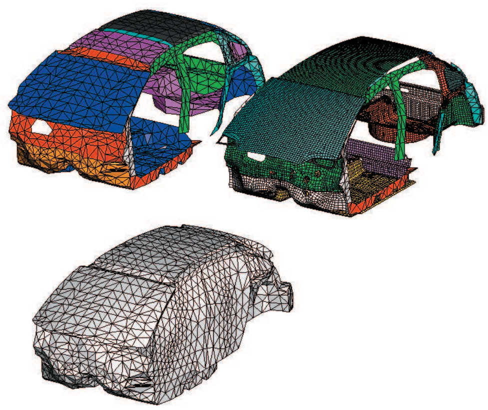

Figure 1. Fluid-structure interface meshes (upper right and upper left), Finite element model of the internal cavity (lower)

Fig. 2 displays the graph of acoustic energies with respect to the frequency corresponding to the previous interface meshes. The horizontal axis is the frequency axis in Hertz, the vertical axis is $10\log_{10}(E(\omega))$. The thick solid line is relative to the detailed fluid-structure interface mesh, the thick dashed line is relative to the rough fluid-structure interface mesh. From this point, we will associate the rough interface mesh with the mean model and the detailed one with a numerical realization -an experiment-. The "distance" between the two graphes may be considered as a model error. We use the Monte-Carlo simulation to

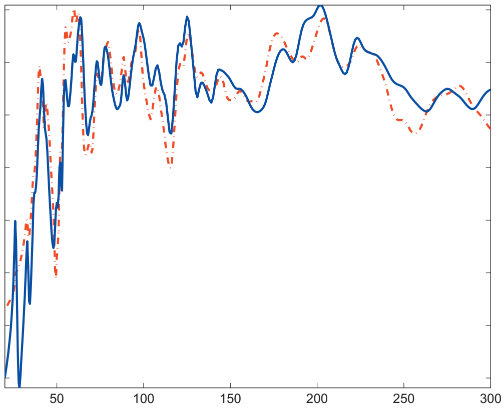

Figure 2. Graph of Acoustic Energy with respect to the frequency: mean model prediction (thick dashed line), numerical realization (thick solid line)) - 1 graduation = 5dB -

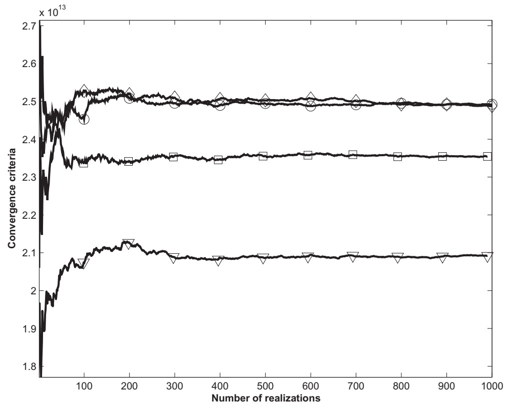

Figure 3. Graph of function $n_r \mapsto \text{conv}^F(n_r)$ related to the convergence of the random acoustic generalized coordinates. Horizontal axis: n_r, Vertical axis: $\text{conv}^F(n_r)$. (n, m) corresponding to a modal truncation of (250 Hz, 250 Hz) (▽), (300 Hz, 300 Hz) (□), (400 Hz, 350 Hz) (◇), (500 Hz, 400 Hz) (⊙)

compute the stochastic solution of Eq. (15). To ensure the convergence of the stochastic solution, we need to study the mean square convergence of the stochastic generalized coordinates.

4.2 Convergence study of the Monte-Carlo Simulation

The random Eq. (15) is solved by using the Monte Carlo numerical simulation performed with a number of realizations n_r and with a given value of $\delta_{\mathcal{C}}$. The random displacement field of the structure and the random pressure field inside the internal acoustic cavity are then computed by using the equations (see Eqs. (5) and (6)),

$$\underline{\mathbf{U}}^S(\omega) = [\underline{\Psi}]\,\mathbf{Q}^S(\omega) \quad , \quad \underline{\mathbf{P}}^F(\omega) = i\omega[\underline{\Phi}]\,\widetilde{\mathbf{Q}}^F(\omega) \quad .$$

The convergence of the random solution is analyzed in studying the graph of the function $n_r \mapsto \text{conv}^H(n_r)$ defined by

$$\text{conv}^H(n_r) = \frac{1}{n_r}\sum_{\ell=1}^{n_r} \int_B ||\mathbf{Q}^H(\omega;\theta_\ell)||^2 d\omega$$

in which exponent H is either S for the structure or F for the fluid and where $\mathbf{Q}^H(\omega;\theta_1), \ldots, \mathbf{Q}^H(\omega;\theta_{n_r})$ are the n_r realizations of the vector-valued random variable $\mathbf{Q}^H(\omega)$.This convergence indicator may be seen as the energy of either the structural part either the internal acoustic cavity.

Fig. 3 displays the graph of the function $n_r \mapsto \text{conv}^F(n_r)$ for the internal acoustic cavity. It can be deduced that a reasonable mean-square convergence of the random structural -acoustic system is reached for $n_r = 600$ realizations, n structural modes corresponding to a modal truncation of 400 Hz and m acoustic modes corresponding to a modal truncation of 350 Hz.

4.3 Random acoustic energy of the structural-acoustic system with vibroacoustic coupling uncertainties

In order to take coupling interface error into account, a Monte-Carlo simulation is carried out with a $\delta_{\mathcal{C}}$ value such that the confidence region computed from the mean model includes the numerical realization. The confidence region of the random acoustic energy was constructed with the following values of the parameters: number of realizations for the Monte Carlo numerical simulation $n_r = 600$, n structural modes corresponding to a modal truncation of 400 Hz and m acoustic modes corresponding to a modal truncation of 350 Hz. Quantiles are used to construct the confidence regions with an associated confidence level $P_c = 98\%$ [Serfling R.J. (1980)].

Fig. 4 is relative to the confidence region prediction of the random acoustic energy. The thick dashed line is the graph of the response of the mean model, the thin solid line is the mean value of the random frequency response function and the grey region is the confidence region for a probability level $P_c = 0.98$. The thick solid line is the graph of the response of the numerical realization.

Usually, effects of uncertainties increase with the frequency [Soize C. (2000, 2001, 2005), Durand and al. (2004, 2005)], this is not the case with this type of uncertainty, we can notice a wide confidence re-

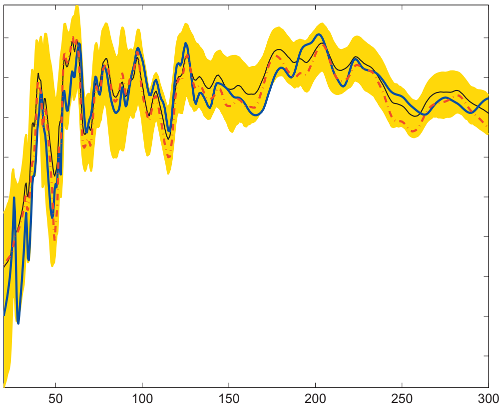

Figure 4. Confidence region associated to $P_c = 0.98$, of the random acoustic energy for **vibroacoustic coupling** uncertainties: mean model prediction (thick dashed line), mean response of the stochastic model (thin solid line), confidence region (grey region), numerical realization (thick solid line) - 1 graduation = 5dB -

gion at the low frequencies with a decreasing width at higher frequencies. This would mean that vibroacoustic coupling terms are the most sensitive at lower frequencies. A very interesting point concerns the frequency range around 26 Hz.

Concerning the numerical realization (thick solid line) :

- The acoustic peak at the frequency-value 26 Hz is mainly due to a global structural mode coupled with the acoustic constant pressure mode.

Concerning the mean model (thick dashed line) :

- At this frequency, the coupling interface deformation does not induce internal pressure variation for the mean model response.

On Fig. 4, we can see that the predicted confidence region accounts for the coupling effect at the frequency-value 26Hz, it is remarkable that non-parametric approach of vibroacoustic uncertainties can take into account effects that were, apparently, absent from the mean model prediction.

5 CONCLUSION

We have presented an approach that allows the random uncertainties to be modelled even for industrial car models, particularly model uncertainties which are the main cause of prediction failures. This paper is dedicated to vibroacoustic coupling uncertainties with an approach based on different fluid-structure interface mesh sizes, from a rough one to a refine one. The results highlight the effects of such model errors. The stochastic response of the proposed model presents a good behavior : the emergence of the acoustic resonance at the frequency-value 26Hz which was, apparently absent from the mean model, is well predicted by the confidence region. We can add that the non-parametric approach is easily practicable, and provides a realistic order of magnitude of the random responses. It should be noted that, with this method, all the realizations of the Monte-Carlo simulation model perfectly a mathematical-mechanical model.

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