Robust design optimization for uncertain complex dynamical systems

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Abstract
This paper deals with the design optimization problem of a structural-acoustic system in presence of uncertainties. The uncertain vibroacoustic numerical model is constructed by using a recent nonparametric probabilistic model which takes into account model uncertainties and data uncertainties. The formulation of the design optimization problem includes the effect of uncertainties and consists in minimizing a cost function with respect to an admissible set of design parameters. The numerical application consists in designing an uncertain master structure in order to minimize the acoustic pressure in a coupled internal cavity which is assumed to be deterministic and excited by an acoustic source. The results of the design optimization problem, solved with and without the uncertain numerical model show significant differences.

1 Introduction

The design of dynamical systems has become a challenge of interest in many industrial areas. The design optimization is performed with a numerical model. It is known that for complex dynamical systems such as structural-acoustic systems, the mathematical-mechanical modeling can induce important model uncertainties and data uncertainties. Consequently, uncertainties have to be taken into account in the numerical model which is used to perform design optimization. It should be noted that the quality of the design optimization strongly depends on the probabilistic model of uncertainties. Design optimization for structural-acoustic systems in a deterministic context (without uncertainties) can be found in [1, 2, 3, 4]. Nevertheless, there is *a priori* no reason for which the performance for such an optimal system yields an optimal performance for the real system manufactured from this optimal system because model and data uncertainties are not taken into account in the numerical model used. For this reason, the formulation of the design optimization has to contain the effect of uncertainties. There exists two classes of methodologies which allow the design optimization in a probabilistic context to be solved: the reliability-based design optimization formulations (see for example [5] in the context of aerostructural analysis and [6, 7] in the context of structural mechanics) and the robust design optimization formulation (see for example [8, 9, 10] in the context of linear or non-linear structural mechanics). This latter formulation allows the robustness of the design system with respect to uncertainties to be improved. The cost function used in this formulation is not defined for the objective performances of the design system but for the objective performances of the stochastic system modeling the real system. The optimal design is the solution of a stochastic non-linear constrained optimization problem solved by minimizing such a cost function with respect to an admissible set of design parameters. In structural dynamics, if several formulations for robust design optimization with respect to data uncertainties have been proposed [11, 12], the concept of robust design optimization with respect to model uncertainties is relatively recent [13]. Such an approach is based on the use of a recent nonparametric probabilistic approach of model uncertainties [14, 15, 16, 17]. In particular, it should be noted that this probabilistic approach has been experimentally and numerically validated for complex structural-acoustics systems [18, 19]. In the present paper, a robust design optimization formulation with respect to model and data uncertainties is proposed in the context of a structural-acoustic system in the low-frequency range. The paper is voluntary limited to the design of a master system with stiffness uncertainties coupled to a deterministic internal acoustic cavity which is excited by an acoustic source and whose noise level has to be reduced for the best. Clearly, the extension to mass and damping uncertainties is straightforward. The paper compares the design points and the performances of their corresponding real systems obtained with the deterministic design optimization and with the robust design optimization.

2 Mean structural-acoustic system

The structural-acoustic system under consideration is made up of an internal acoustic cavity coupled with a master structure which has to be designed (see Figure 1). Let $\mathbf{r} = (r_1, \ldots, r_s)$ be the $\mathbb{C}^s$-vector of the design parameters (geometry, elasticity properties, boundary conditions, etc.). The vector of the design parameters belongs to an admissible set $\mathcal{R}$ defined by the set of constraints prescribed by the design. For a given $\mathbf{r}$ in $\mathcal{R}$, the linear vibrations of the structural-acoustic system are studied around a static equilibrium state taken as a natural state at rest.

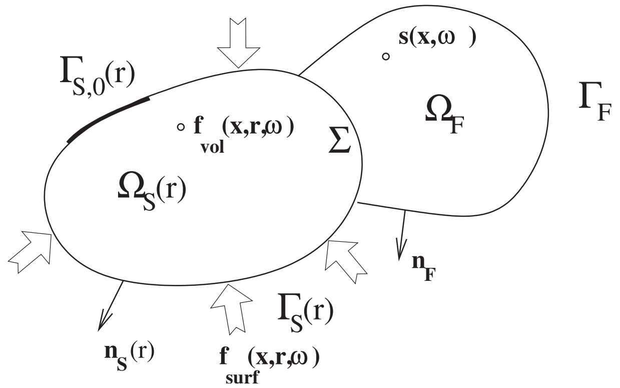

Figure 1: Structural-acoustic system

The master structure is constituted of a nonhomogeneous and anisotropic viscoelastic material without memory, occupying a three-dimensional bounded domain $\Omega_S(\mathbf{r})$ of the physical space $\mathbb{R}^3$ with boundary $\partial\Omega_S(\mathbf{r}) = \Gamma_{S,0}(\mathbf{r}) \cup \Gamma_S(\mathbf{r}) \cup \Sigma$. The master structure is fixed on $\Gamma_{S,0}(\mathbf{r})$. The internal acoustic cavity occupies a three-dimensional bounded domain Ω_F of $\mathbb{R}^3$ with boundary $\partial\Omega_F = \Gamma_F \cup \Sigma$ and is filled with a dissipative acoustic fluid. It is coupled to the master structure through boundary Σ and has rigid wall conditions on Γ_F. Les $\mathbf{n}_S(\mathbf{r})$ and $\mathbf{n}_F$ be the outward unit normals to $\partial\Omega_S(\mathbf{r})$ and $\partial\Omega_F$. Note that $\mathbf{n}_S = -\mathbf{n}_F$ on Σ. Let $\mathbf{x}$ be the generic point of $\mathbb{R}^3$. The equations are written in the frequency domain of analysis and the low-frequency band of analysis is denoted as $\mathbb{B}$. A formulation in terms of displacements field $\mathbf{u}(\mathbf{x}, \mathbf{r}, \omega)$ for the master structure and in terms of pressure field $p(\mathbf{x}, \mathbf{r}, \omega)$ for the internal acoustic cavity is chosen. For $\mathbf{r}$ fixed in $\mathcal{R}$ and for ω fixed in $\mathbb{B}$, the equations related to the mean structural-acoustic system [20] are written as

$$-\omega^2 \rho_S \mathbf{u} - \operatorname{div} \sigma_S = \mathbf{f}_{\text{vol}} \quad \text{in} \quad \Omega_S(\mathbf{r}) \quad , \tag{1}$$

$$\mathbf{u} = \mathbf{0} \quad \text{on}\, \Gamma_{S,0}(\mathbf{r}) \quad , \tag{2}$$

$$\sigma_S \cdot \mathbf{n}_S = \mathbf{f}_{\text{surf}} \quad \text{on} \quad \Gamma_S(\mathbf{r}) \quad , \tag{3}$$

$$\sigma_S \cdot \mathbf{n}_S = \mathbf{f}_{\text{surf}} - p\, \mathbf{n}_S \quad \text{on} \quad \Sigma \quad , \tag{4}$$

$$\sigma_S = \mathbb{a}_S : \varepsilon(\mathbf{u}) + i\,\omega\, \mathbb{b}_S : \varepsilon(\mathbf{u}) \quad , \tag{5}$$

$$-\frac{\omega^2}{\rho_F\, c_F^2}\, p - i\,\omega\, \frac{\tau}{\rho_F}\, \Delta p - \frac{1}{\rho_F} \Delta p = -\frac{\tau\, c_F^2}{\rho_F}\, \Delta\, s + \frac{i\,\omega}{\rho_F}\, s \quad \text{in} \quad \Omega_F \quad , \tag{6}$$

$$\frac{(1 + i\,\omega\,\tau)}{\rho_F} \frac{\partial p}{\partial \mathbf{n}_F} = \tau\, \frac{c_F^2}{\rho_F} \frac{\partial s}{\partial \mathbf{n}_F} \quad \text{on} \quad \Gamma_F \quad , \tag{7}$$

$$\frac{(1 + i\,\omega\,\tau)}{\rho_F} \frac{\partial p}{\partial \mathbf{n}_F} = \tau\, \frac{c_F^2}{\rho_F} \frac{\partial s}{\partial \mathbf{n}_F} + \omega^2\, \mathbf{u} \cdot \mathbf{n}_F \quad \text{on} \quad \Sigma \quad , \tag{8}$$

in which $\sigma_S(\mathbf{x}, \mathbf{r}, \omega)$ is the stress tensor, $\varepsilon(\mathbf{u})$ is the linearized strain tensor, $\mathbb{a}_S(\mathbf{x}, \mathbf{r})$ and $\mathbb{b}_S(\mathbf{x}, \mathbf{r})$ are fourth-order tensors, $\rho_S(\mathbf{x}, \mathbf{r})$ is the mass density of the master structure, $\mathbf{f}_{\text{vol}}(\mathbf{x}, \mathbf{r}, \omega)$ and $\mathbf{f}_{\text{surf}}(\mathbf{x}, \mathbf{r}, \omega)$ are the body force and the surface force fields for the master structure, $\rho_F(\mathbf{x})$ is the mass density of the fluid, c_F is the sound velocity, τ is a coefficient due to the viscosity of the fluid and $s(\mathbf{x}, \omega)$ is the acoustic source density assuming that $\lim_{\omega \mapsto 0} \dfrac{s(\mathbf{x}, \omega)}{\omega^2} = \lim_{\omega \mapsto 0} \dfrac{\nabla\, s(\mathbf{x}, \omega)}{\omega^2} = \lim_{\omega \mapsto 0} \dfrac{\Delta\, s(\mathbf{x}, \omega)}{\omega^2} = 0$.

The structural-acoustic system is then discretized with the finite element method assuming that the finite element meshes of the master structure and of the internal acoustic cavity are compatible on the coupling interface Σ.

A mean reduced matrix model of the structural-acoustic system is then constructed. Let $\underline{\mathbf{u}}(\mathbf{r}, \omega)$ be the $\mathbb{C}^{n_S}$-vector of the n_S DOF (independent of $\mathbf{r}$) of the master structure and let $\underline{\mathbf{p}}(\mathbf{r}, \omega)$ be the $\mathbb{C}^{n_F}$-vector corresponding to the finite element discretization of the pressure field of the internal acoustic cavity. For a given $\mathbf{r}$ in $\mathcal{R}$, let $[\underline{\Phi}_S(\mathbf{r})]$ be the $n_S \times N_S$ real matrix whose columns are the N_S structural modes related to the N_S first positive structural eigenfrequencies of the master structure *in vacuo*. The generalized eigenvalue problem of the internal acoustic cavity with fixed coupling interface yields one zero eigenvalue corresponding to the constant pressure mode and $n_F - 1$ acoustic eigenmodes [20]. Let $[\underline{\Phi}_F]$ be the $n_F \times N_F$ real matrix whose columns are (1) the constant pressure eigenmode and (2) the $N_F - 1$ acoustic eigenmodes related to the $N_F - 1$ first positive acoustic eigenfrequencies. Note that each eigenmode is normalized with respect to its corresponding mass matrix. The projection basis allowing the mean reduced matrix model to be constructed is given by

$$\begin{bmatrix} \underline{\mathbf{u}}(\mathbf{r}, \omega) \\ \underline{\mathbf{p}}(\mathbf{r}, \omega) \end{bmatrix} = \begin{bmatrix} [\underline{\Phi}_S(\mathbf{r})] & [\mathbb{0}] \\ [\mathbb{0}] & [\underline{\Phi}_F] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{q}}_S(\mathbf{r}, \omega) \\ \underline{\mathbf{q}}_F(\mathbf{r}, \omega) \end{bmatrix} \quad , \tag{9}$$

in which $\underline{\mathbf{q}}_S(\mathbf{r}, \omega)$ and $\underline{\mathbf{q}}_F(\mathbf{r}, \omega)$ are the $\mathbb{C}^{N_S}$-vector and the $\mathbb{C}^{N_F}$-vector of the generalized coordinates related to the master structure and to the internal acoustic cavity and are solution of the matrix equation

$$\begin{bmatrix} [\underline{\mathcal{A}}_S(\mathbf{r}, \omega)] & [\underline{\mathcal{C}}(\mathbf{r})] \\ -\omega^2\, [\underline{\mathcal{C}}(\mathbf{r})]^T & [\underline{\mathcal{A}}_F(\omega)] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{q}}_S(\mathbf{r}, \omega) \\ \underline{\mathbf{q}}_F(\mathbf{r}, \omega) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{F}}_S(\mathbf{r}, \omega) \\ \boldsymbol{\mathcal{F}}_F(\omega) \end{bmatrix} \quad , \tag{10}$$

in which the symmetric $N_S \times N_S$ complex matrix $[\underline{\mathcal{A}}_S(\mathbf{r}, \omega)]$ and the diagonal $N_F \times N_F$ complex matrix $[\underline{\mathcal{A}}_F(\omega)]$ are the generalized dynamical stiffness matrices of the master structure and of the internal acoustic cavity respectively. The rectangular $N_S \times N_F$ real matrix $[\underline{\mathcal{C}}(\mathbf{r})]$ is the generalized coupling matrix. In Eq. (10) the $\mathbb{C}^{N_S}$-vector $\boldsymbol{\mathcal{F}}_S(\mathbf{r}, \omega)$ and the $\mathbb{C}^{N_F}$-vector $\boldsymbol{\mathcal{F}}_F(\omega)$ are the generalized force vectors related to the master structure and to the internal acoustic cavity respectively.

3 Design optimization of the structural-acoustic system without uncertainties

In this Section, the design optimization problem is formulated assuming that there is no uncertainties in the structural-acoustic system. This formulation will be used to compare the solution of the deterministic design optimization problem with the solution obtained with the robust design optimization formulation which includes the effects of uncertainties and which will be described in Section 5. Let $\underline{\mathbf{w}}(\mathbf{r},\omega)$ be the vector in $\mathbb{C}^k$ of the observations of the mean model of the internal acoustic cavity, defined as a function of the acoustic pressure such that

$$\underline{\mathbf{w}}(\mathbf{r},\omega) \;=\; \mathbf{b}_\omega(\underline{\mathbf{p}}(\mathbf{r},\omega)) \quad , \tag{11}$$

where $\mathbf{b}_\omega$ is a given function from $\mathbb{C}^{n_F}$ into $\mathbb{C}^k$ depending on the frequency ω. Recalling that the objective of the paper is to design the master structure for minimizing the acoustic pressure in the internal acoustic cavity over given frequency band $\mathbb{B}$, the cost function $\underline{j}(\mathbf{r})$ is formulated as follows

$$\underline{j}(\mathbf{r}) \;=\; \frac{\max_{\omega\in\mathbb{B}}||\underline{\mathbf{w}}(\mathbf{r},\omega)||}{\max_{\omega\in\mathbb{B}}||\underline{\mathbf{w}}(\mathbf{r}_0,\omega)||} \quad , \tag{12}$$

in which $||\underline{\mathbf{w}}(\mathbf{r},\omega)||$ is the Hermitian norm of vector $\underline{\mathbf{w}}(\mathbf{r},\omega)$ and where $\mathbf{r}_0 \in \mathcal{R}$ is the $\mathbb{C}^s$-vector corresponding to the initial value of the design parameter. The design optimization problem is formulated as the minimization of the cost function $\underline{j}(\mathbf{r})$ with respect to the design parameter $\mathbf{r}$ in the admissible set $\mathcal{R}$ and is written as: find $\mathbf{r}^{\mathrm{D}}$ in $\mathcal{R}$ such that

$$\underline{j}(\mathbf{r}^{\mathrm{D}}) \;\le\; \underline{j}(\mathbf{r}),\ \text{for all } \mathbf{r} \text{ in } \mathcal{R} \quad . \tag{13}$$

4 Stochastic structural-acoustic system

As explained in the Introduction, the objective of this paper is to include the effects of data uncertainties and model uncertainties in the formulation of the design optimization problem. In this Section, the nonparametric probabilistic approach of uncertainties [14, 17] is briefly summarized. It is assumed that the structural stiffness of the mean master model only contains model uncertainties and data uncertainties. The dynamic stiffness reduced matrix of the mean master structure is written as $[\underline{\mathcal{A}}_S(\mathbf{r},\omega)] \;=\; -\omega^2\,[\underline{I}] + i\,\omega\,[\underline{\mathcal{D}}_S(\mathbf{r})] + [\underline{\mathcal{K}}_S(\mathbf{r})]$ in which $[\underline{I}]$ is the $N_S \times N_S$ identity matrix and where $[\underline{\mathcal{D}}_S(\mathbf{r})]$ and $[\underline{\mathcal{K}}_S(\mathbf{r})]$ are the $N_S \times N_S$ real symmetric and diagonal positive-definite generalized damping and stiffness matrices of the mean model of the master system. The methodology of the nonparametric probabilistic approach consists in replacing matrix $[\underline{\mathcal{K}}_S(\mathbf{r})]$ by a random matrix $[\boldsymbol{\mathcal{K}}_S(\mathbf{r})]$ such that $\mathcal{E}\{[\boldsymbol{\mathcal{K}}_S(\mathbf{r})]\} \;=\; [\underline{\mathcal{K}}_S(\mathbf{r})]$ in which $\mathcal{E}$ is the mathematical expectation and for which the probability distribution is known. The random matrix $[\boldsymbol{\mathcal{K}}_S(\mathbf{r})]$ is written as $[\boldsymbol{\mathcal{K}}_S(\mathbf{r})] \;=\; [\underline{L}_{K_S}(\mathbf{r})]^T\,[\mathbf{G}_{K_S}]\,[\underline{L}_{K_S}(\mathbf{r})]$ in which $[\underline{L}_{K_S}(\mathbf{r})]$ is a $N_S \times N_S$ real diagonal matrix such that $[\underline{\mathcal{K}}_S(\mathbf{r})] \;=\; [\underline{L}_{K_S}(\mathbf{r})]^T\,[\underline{L}_{K_S}(\mathbf{r})]$ and where $[\mathbf{G}_{K_S}]$ is a full random matrix with value in the set of all the positive-definite symmetric $N_S \times N_S$ matrices. The probability model of random matrix $[\mathbf{G}_{K_S}]$ is constructed by using the maximum entropy principle with the available information. All the details concerning the construction of this probability model can be found in [14, 15]. The dispersion of the random matrix $[\mathbf{G}_{K_S}]$ is controlled by one real positive parameter δ_{K_S} called the dispersion parameter. In addition, there exists an algebraic representation of this random matrix useful to the Monte Carlo numerical simulation.

In coherence with the notation of Section 2, let $\mathbf{U}(\mathbf{r},\omega)$ be the $\mathbb{C}^{n_S}$-valued random vector of the n_S DOF and let $\mathbf{P}(\mathbf{r},\omega)$ be the $\mathbb{C}^{n_F}$-valued random vector of the acoustic pressure. The equations of the stochastic reduced structural-acoustic system constructed with the nonparametric approach of uncertainties are given by

$$\begin{bmatrix}\mathbf{U}(\mathbf{r},\omega)\\ \mathbf{P}(\mathbf{r},\omega)\end{bmatrix} \;=\; \begin{bmatrix}[\underline{\Phi}_S(\mathbf{r})] & [\mathbb{0}]\\ [\mathbb{0}] & [\underline{\Phi}_F]\end{bmatrix}\begin{bmatrix}\mathbf{Q}_S(\mathbf{r},\omega)\\ \mathbf{Q}_F(\mathbf{r},\omega)\end{bmatrix} \quad , \tag{14}$$

where $\mathbf{Q}_S(\mathbf{r},\omega)$ and $\mathbf{Q}_F(\mathbf{r},\omega)$ are the $\mathbb{C}^{N_S}$-valued random vector and the $\mathbb{C}^{N_F}$-valued random vector of the generalized coordinates related to the master structure and to the internal acoustic cavity respectively, solution of the random matrix equation

$$\begin{bmatrix} [\boldsymbol{\mathcal{A}}_S(\mathbf{r},\omega)] & [\underline{\mathcal{C}}(\mathbf{r})] \\ -\omega^2\,[\underline{\mathcal{C}}(\mathbf{r})]^T & [\underline{\mathcal{A}}_F(\omega)] \end{bmatrix} \begin{bmatrix} \mathbf{Q}_S(\mathbf{r},\omega) \\ \mathbf{Q}_F(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{F}}_S(\mathbf{r},\omega) \\ \boldsymbol{\mathcal{F}}_F(\omega) \end{bmatrix} \quad , \tag{15}$$

in which the matrix $[\boldsymbol{\mathcal{A}}_S(\mathbf{r},\omega)]$ is such that $[\boldsymbol{\mathcal{A}}_S(\mathbf{r},\omega)] = -\omega^2\,[\underline{\mathcal{I}}] + i\,\omega\,[\underline{\mathcal{D}}_S(\mathbf{r})] + [\boldsymbol{\mathcal{K}}_S(\mathbf{r})]$.

5 Design optimization of the structural-acoustic system with uncertain stiffness in the master structure numerical model

In this Section, the model uncertainties and the data uncertainties are taken into account for the stiffness operator of the master structure in the formulation of the design problem, using the nonparametric probabilistic approach described in Section 4. This design optimization problem consists in minimizing a cost function with respect to the admissible set $\mathcal{R}$ of the design parameter. Contrary to the design optimization problem described in Section 3, the cost function is not defined for the performance of the mean model of the structural-acoustic system but is defined with respect to the performance of the stochastic model of the structural-acoustic system representing the real structural-acoustic system. The cost function is thus constructed with the uncertain numerical model introduced in Section 4. For $\mathbf{r}$ fixed in $\mathcal{R}$, the $\mathbb{C}^k$-valued random vector $\mathbf{W}(\mathbf{r},\omega)$ of the acoustic observation is introduced in coherence with the notation of Section 3. For $\mathbf{r}$ fixed in $\mathcal{R}$ and for ω fixed in $\mathbb{B}$, let $w^+(\mathbf{r},\omega)$ be the $99\,\%$ quantile of random variable $||\mathbf{W}(\mathbf{r},\omega)||$, such that $\mathcal{P}(||\mathbf{W}(\mathbf{r},\omega)|| \leq w^+(\mathbf{r},\omega)) = 0.99$ [21], in which $\mathcal{P}$ denotes the probability. The cost function is then written as

$$j(\mathbf{r}) = \frac{\max_{\omega\in\mathbb{B}} w^+(\mathbf{r},\omega)}{\max_{\omega\in\mathbb{B}} ||\underline{\mathbf{w}}(\mathbf{r}_0,\omega)||} \quad . \tag{16}$$

For given dispersion parameter δ_{K_S}, such a design optimization problem is formulated as: find $\mathbf{r}^{\mathrm{RD}}$ in $\mathcal{R}$ such that

$$j(\mathbf{r}^{\mathrm{RD}}) \leq j(\mathbf{r}),\ \text{for all } \mathbf{r} \text{ in } \mathcal{R} \quad . \tag{17}$$

It should be noted that the formulation of such a robust design optimization is coherent with respect to the deterministic design optimization problem given in Section 3, i.e. $\lim_{\delta_{K_S}\mapsto 0} \mathbf{r}^{\mathrm{RD}} = \mathbf{r}^D$. In addition, Eqs. (16) and (17) mean that the acoustic level corresponding to the upper envelope of the confidence region is minimized.

6 Application

6.1 Mean finite element model of the structural-acoustic system

The mean model of the structural-acoustic system is a heterogeneous system made up of a master structure coupled with an internal acoustic cavity. The master structure is located in the plane (OX, OY) of a cartesian coordinate system $(O\,X\,Y\,Z)$. The master structure is made up of a rectangular frame with four plates as shown in figure 2. The frame has length $L_1 = 1\,m$, width $L_2 = 0.9\,m$, is fixed at each of its corner and is constituted of tubes with square section $0.08\,m \times 0.08\,m$ and thickness $1\,10^{-4}\,m$. The plates have length $0.5\,m$, width $0.45\,m$ and constant thickness $0.0035\,m$ except for the plate coupled with the internal acoustic cavity whose constant thickness is the design parameter r. Each substructure is constituted of a homogeneous, isotropic elastic material with mass density $7800\,Kg.m^{-3}$, Poisson ratio 0.29 and Young modulus $2\,10^{11}\,N.m^{-2}$. The damping part of the constitutive equation is modeled by a hysteretic model with a mean loss factor 0.02. The internal acoustic cavity is a six-sided box with no parallel sides whose corners are located at points $(0, 0, 0)$, $(0.5, 0, 0)$, $(0, 0.45, 0)$, $(0.5, 0.45, 0)$, $(0.1, 0.45, 0.12)$, $(0.4, 0.45, 0.12)$, $(0.48, 0, 0.15)$ and $(0, 0, 0.15)$. All the walls are rigid except the wall made up of the elastic plate with constsnt thickness r. The bounded internal acoustic cavity is filled with an acoustic fluid with mass density $\rho_F = 1.16\,Kg.m^{-3}$, with sound velocity $c_F = 343\,m.s^{-1}$. Parameter τ in Eq. (6) is such that $\tau(\omega) = \dfrac{0.001}{\omega}$.

The mean finite element model of the master structure is constituted of 228 Euler beams elements with two nodes (the tubes), 1440 bending thin plate elements with four nodes and has $n_S = 10927$ DOF. The mean finite element model of the internal acoustic cavity is constituted of 2160 acoustic finite elements and has $n_F = 2793$ DOF. The finite element mesh of the structural-acoustic system is shown in Fig. 2. The internal acoustic cavity is excited by a localized deterministic acoustic source density, which is constant in the frequency band $\mathbb{B} = [1060\,,\,1300]\,Hz$. Let $\mathcal{J}$ be the set of indices corresponding to the nodes of the finite element mesh of the internal cavity located at points $(0.423, 0.450, 0.040)$, $(0.424, 0.425, 0.041)$, $(0.445, 0.450, 0.040)$, $(0.446, 0.425, 0.041)$, $(0.41, 0.45, 0.06)$, $(0.412, 0.425, 0.061)$, $(0.43, 0.45, 0.06)$, $(0.432, 0.425, 0.061)$. The spatial distribution of the acoustic source is such that the vector of the generalized acoustic forces is written as $\boldsymbol{\mathcal{F}}_F(\omega) = \mathbb{1}_{\mathbb{B}}(\omega) \sum_{j\in\mathcal{J}} [\underline{\Phi}_F]^T\,\mathbf{e}_j$ in which $\mathbf{e}_1, \ldots, \mathbf{e}_{n_F}$ are the canonical basis vectors of $\mathbb{R}^{n_F}$ and where $\mathbb{1}_{\mathbb{B}}(\omega) = 1$ if $\omega \in \mathbb{B}$ and $\mathbb{1}_{\mathbb{B}}(\omega) = 0$ if $\omega \notin \mathbb{B}$. The chosen observation is the spectral acoustic energy $\underline{w}(r, \omega) = \dfrac{V_F}{\rho_F\, c_F^2}\, \tilde{p}(r, \omega)^2$ with $\tilde{p}(r, \omega)^2 = \dfrac{1}{n_F} \displaystyle\sum_{j=1}^{n_F} |\underline{p}_j(r, \omega)|^2$, in which V_F is the volume of the internal acoustic cavity and where $\underline{p}_j(r, \omega)$ is the component number j of vector $\underline{\mathbf{p}}(r, \omega)$.

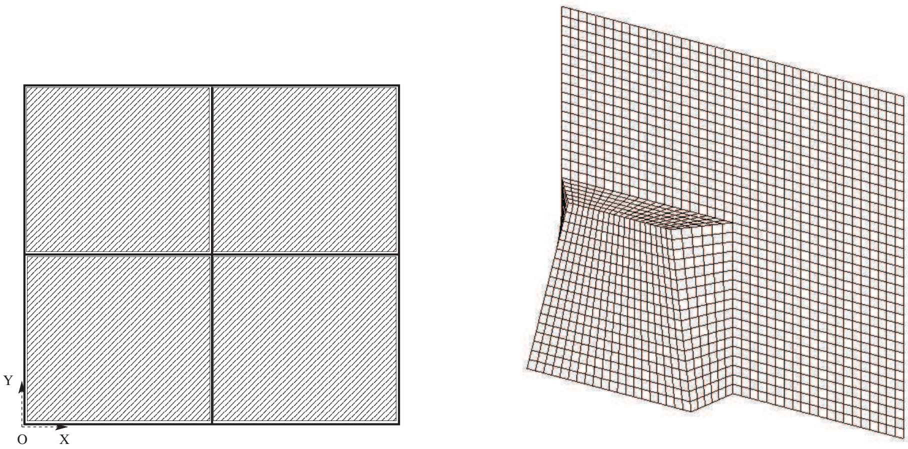

Figure 2: Mean model of the master structure : plates (filled domain), frame (thick black line) (left) - mean finite element model of the structural-acoustic system (right).

6.2 Estimation of the numerical parameters for the robust design optimization

In the present analysis, the initial structural-acoustic system corresponds to the value of the design parameter $r_0 = 0.005\,m$. The frequency band of analysis for which the acoustic level has to be reduced is $\mathbb{B} = [1060\,,\,1300]\,Hz$.

The Monte Carlo numerical simulation is chosen for solving the design optimization problem. The numerical parameters related to the stochastic reduced equation Eq. (15) have to be fixed first. These numerical parameters are the number N_S of structural modes, the number $N_F - 1$ of acoustic modes, which have to be kept in the modal reduction and the number n_r of realizations used in the Monte Carlo numerical simulation. Consequently, a convergence analysis has to be carried out with respect to n_r, N_F and N_S. The computation is performed for the initial structural-acoustic system with dispersion parameter $\delta_{K_S} = 0.25$. Let $W^0(\omega)$

be the initial random observation defined by $W^0(\omega) = W(r_0, \omega)$ and corresponding to the random spectral acoustic energy of the initial structural-acoustic system. The mean square convergence is analyzed by studying the function $(n_r, N_F, N_S) \mapsto \text{Conv}(n_r, N_F, N_S)$ defined by

$$\text{Conv}^2(n_r, N_F, N_S) = \frac{1}{n_r} \sum_{i=1}^{n_r} \left(W^0_{\mathbb{B},\infty}(\theta_i)\right)^2 \quad , \tag{18}$$

in which $W^0_{\mathbb{B},\infty}(\theta_i)$ is the realization number i of the random variable $W^0_{\mathbb{B},\infty}$ defined by $W^0_{\mathbb{B},\infty} = \max_{\omega \in \mathbb{B}} W^0(\omega)$. Note that random variable $W^0_{\mathbb{B},\infty}$ is computed with a reduced model of dimension $N_S + N_F$.

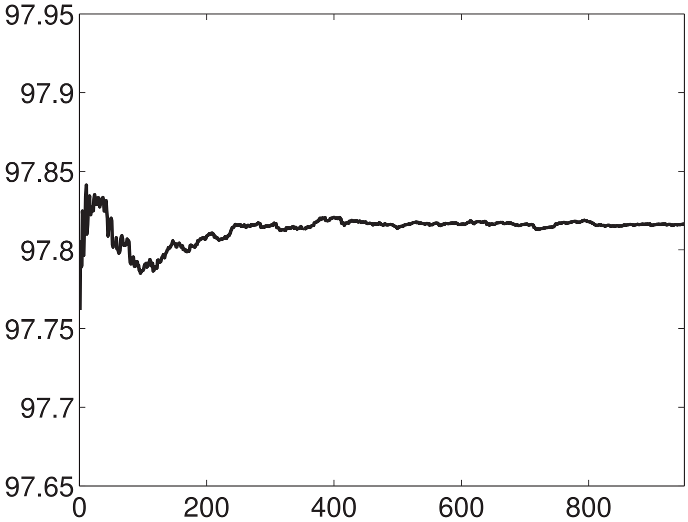

Figure 3: Convergence analysis : graph of function $n_r \mapsto \text{Conv}(n_r, 51, 100)$ for the structural-acoustic system with $r_0 = 0.005\,m$ and $\delta_{K_S} = 0.25$.

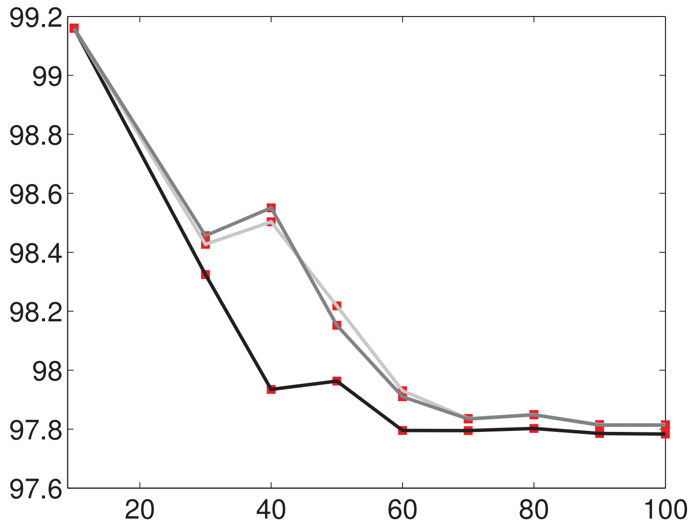

Figure 4: Convergence analysis : graph of function $N_S \mapsto \text{Conv}(500, N_F, N_S)$ for the structural-acoustic system with $r_0 = 0.005\,m$ and $\delta_{K_S} = 0.25$ and for $N_F = 11$ (black line), $N_F = 41$ (dark gray line) and $N_F = 51$ (light gray line).

Figure 3 displays the graph $n_r \mapsto \text{Conv}(n_r, 51, 100)$. It can be seen that a reasonable convergence is reached for $n_r = 500$. Figure 4 displays the graph $N_S \mapsto \text{Conv}(500, N_F, N_S)$ for several values of N_F. Convergence is reached for $N_F = 41$ and $N_S = 90$.

6.3 Specification of the design optimization

Below, the robust design optimization is carried out with $\delta_{K_S} = 0.25$, $N_F = 41$, $N_S = 90$ and $n_r = 500$. The admissible set $\mathcal{R}$ for the design parameter r is defined such that $r \in [0.005\,,\,0.007]\,m$. Note that the convergence of the results has been verified over admissible set $\mathcal{R}$ with these numerical parameters. Similarly to the stochastic case, let $\underline{w}^0(\omega) = \underline{w}(r_0, \omega)$ be the observation corresponding to the spectral acoustic energy of the mean initial structural-acoustic system. Figure 5 shows the observation $\omega \mapsto 10\,\log_{10}(\underline{w}^0(\omega))$ and the confidence region of random observation $W^0(\omega)$ obtained with a probability level $P_c = 0.98$. It can be seen that the confidence region is narrow over frequency band $\mathbb{B}$ except for the frequency band $[1130\,,\,1160]\,Hz$. Consequently the structural-acoustic system is robust with respect to model uncertainties and to data uncertainties in frequency band $\mathbb{B} \setminus [1130\,,\,1160]\,Hz$.

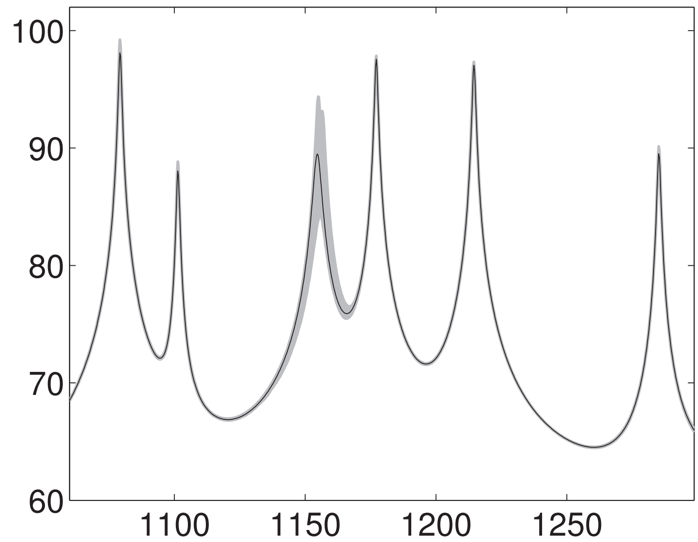

Figure 5: Observation of the initial structural-acoustic system. Graph of function $\nu \mapsto 10\,\log_{10}(\underline{w}^0(2\pi\nu))$ (thin black line). Confidence region (gray region) of random observation $W^0(2\pi\nu)$ obtained with a probability level $P_c = 0.98$. Horizontal axis is the frequency ν in Hz.

The design optimization problem consists in finding the design of the structural-acoustic system which allows the spectral acoustic energy over frequency band $\mathbb{B}$ to be reduced for the best. It is assumed that the precision of design parameter r is $50\,\mu m$. The robust optimization problem is then solved by computing the cost function with repect to admissible set $\mathcal{R}$ and by using Monte Carlo numerical simulation. For r in $\mathcal{R}$, let $g_{\mathbb{B}}(r)$ and $g_{\mathbb{B}}^{\text{real}}(r)$ be the acoustic gains defined with respect to the acoustic level corresponding to the upper envelope of the confidence region of the initial structural-acoustic system and defined by

$$g_{\mathbb{B}}(r) = 10\,\log_{10}\left(\frac{\underline{w}_{\mathbb{B},\infty}(r)}{w^+_{\mathbb{B},\infty}(r_0)}\right) \quad , \quad g_{\mathbb{B}}^{\text{real}}(r) = 10\,\log_{10}\left(\frac{w^+_{\mathbb{B},\infty}(r)}{w^+_{\mathbb{B},\infty}(r_0)}\right) \quad , \tag{19}$$

in which $\underline{w}_{\mathbb{B},\infty}(r) = \max_{\omega\in\mathbb{B}}\underline{w}(r,\omega)$ and $w^+_{\mathbb{B},\infty}(r) = \max_{\omega\in\mathbb{B}} w^+(r,\omega)$. For a given r in $\mathcal{R}$, the scalar $g_{\mathbb{B}}(r)$ represents the acoustic gain predicted with the mean model of the designed system and the scalar $g_{\mathbb{B}}^{\text{real}}(r)$ represents the acoustic gain predicted with the stochastic model constructed from this mean model. We are interested in comparing the acoustic gain obtained from the designed system solution of the design optimization presented in Section 3 and from the designed system solution of the robust design optimization presented in Section 5.

6.4 Robust design optimization over a narrow frequency band of analysis

The design analysis is limited to the narrow frequency band $\mathbb{B}_1 = [1190\,,\,1260]\,Hz$ for which the initial structural-acoustic system is robust with respect to model uncertainties and to data uncertainties. Figure 6 displays the graphs $r \mapsto 10\,\log_{10}(w^+_{\mathbb{B}_1,\infty}(r))$ and $r \mapsto 10\,\log_{10}(\underline{w}_{\mathbb{B}_1,\infty}(r))$. It can be seen that the deterministic design optimization and the robust design optimization yield optimal design parameters $r^{\mathrm{D}} = 5.9\,10^{-3}\,m$ and $r^{\mathrm{RD}} = 5.95\,10^{-3}\,m$. Let $W^{\mathrm{D}}(\omega)$ and $W^{\mathrm{RD}}(\omega)$ be the random observations defined by $W^{\mathrm{D}}(\omega) = W(r^{\mathrm{D}},\omega)$ and $W^{\mathrm{RD}}(\omega) = W(r^{\mathrm{RD}},\omega)$. Similarly to the stochastic case, let $\underline{w}^{\mathrm{D}}(\omega) = \underline{w}(r^{\mathrm{D}},\omega)$ and $\underline{w}^{\mathrm{RD}}(\omega) = \underline{w}(r^{\mathrm{RD}},\omega)$.

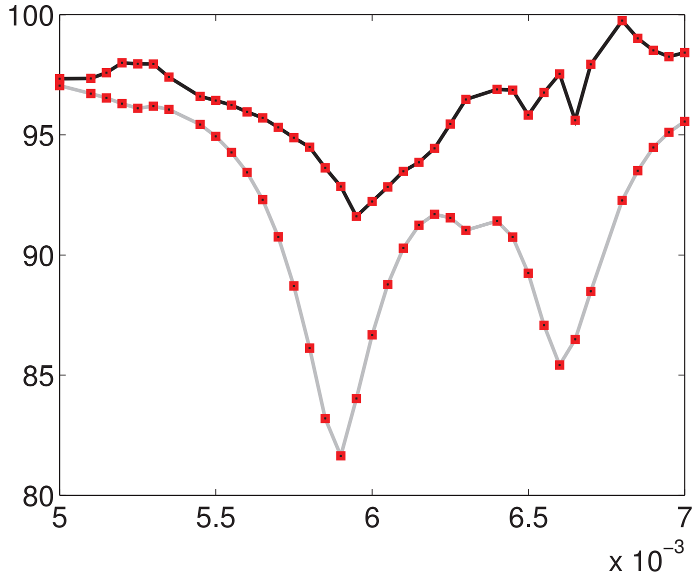

Figure 6: Comparison between the design optimization and the robust design optimization. Graph of functions $r \mapsto 10\,\log_{10}(w^+_{\mathbb{B}_1,\infty}(r))$ (black line) and $r \mapsto 10\,\log_{10}(\underline{w}_{\mathbb{B}_1,\infty}(r))$ (gray line). Horizontal axis is design parameter r.

Figure 7 shows the spectral acoustic energy $\omega \mapsto 10\,\log_{10}(\underline{w}^{\mathrm{D}}(\omega))$ and the confidence region of random observation $W^{\mathrm{D}}(\omega)$ corresponding to the design optimization. It can be seen that the resonance peaking of the spectral acoustic energy $\underline{w}^{\mathrm{D}}(\omega)$ has been considerably reduced. Indeed, the value r^{D} of the design parameter yields a mean master structure for which there exists a structural mode which couples with the acoustic mode of the internal acoustic cavity. The resonance peaking corresponds to an elasto-acoustic mode for this mean structural-acoustic system. At this resonance, the transfer of energy from the internal acoustic cavity to the master structure is optimal. From Fig. 6, it should be noted that this energy pumping phenomenon is very sensitive to the design parameter.

Figure 7 displays a broad confidence region for random observation $W^{\mathrm{D}}(\omega)$. By comparing Fig. 5 and 7, it can be seen that the robustness of the structural-acoustic system (corresponding to the design optimization point r^{D}) with respect to model and data uncertainties has drastically decreased in comparison to the robustness of the initial structural-acoustic system. This lack of robustness is due to the amount of uncertainty in the master structure. Indeed, the structural mode (related to the uncertain master structure with fixed coupling interface), which is likely to couple with the acoustic mode of the internal cavity *in vacuo* is uncertain. The width of the support corresponding to the probability distribution of its corresponding structural eigenvalue is an increasing function of dispersion parameter δ_{K_S}. In the present case, the value of the dispersion parameter is relatively important ($\delta_{K_S} = 0.25$), yielding realizations of the corresponding structural eigenmode which couple weakly with the acoustic mode of the internal acoustic cavity. Consequently, such realizations do not yield optimal elasto-acoustic coupling. The gain $g^{\mathrm{real}}_{\mathbb{B}_1}(r^{\mathrm{D}})$ predicted with the stochastic model of the structural-acoustic system is lower than the gain $g_{\mathbb{B}_1}(r^{\mathrm{D}})$ predicted with the mean model of the structural-acoustic system. We have $g^{\rm real}_{\mathbb{B}_1}(r^{\rm D}) = 4.2\, dB \leq g_{\mathbb{B}_1}(r^{\rm D}) = 15.7\, dB$. In addition, it should be noted that the deterministic design optimization yields a secondary optimum $r^{\rm D'}$ for which $g^{\rm real}_{\mathbb{B}_1}(r^{\rm D'}) < 0$.

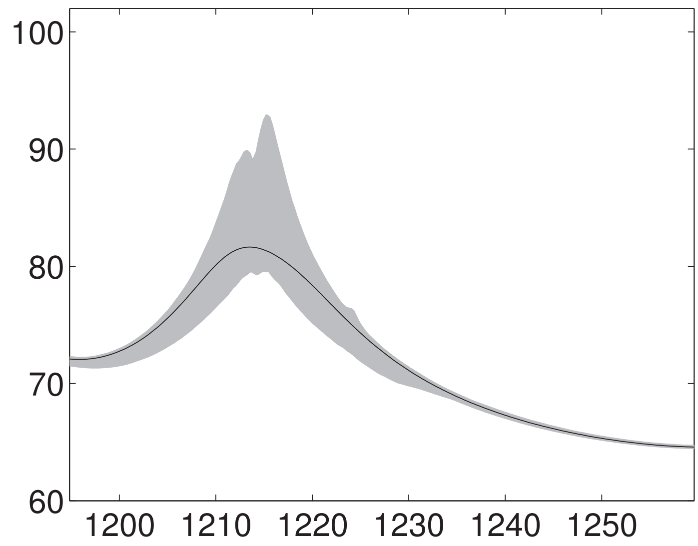

Figure 7: Graphs of function $\nu \mapsto 10\, \log_{10}(\underline{w}^{\rm D}(2\pi\nu))$ (thin black line) and of the confidence region (gray region) of random observation $10\, \log_{10}(W^{\rm D}(2\pi\nu))$ corresponding to the design optimization. Horizontal axis is the frequency ν in Hz.

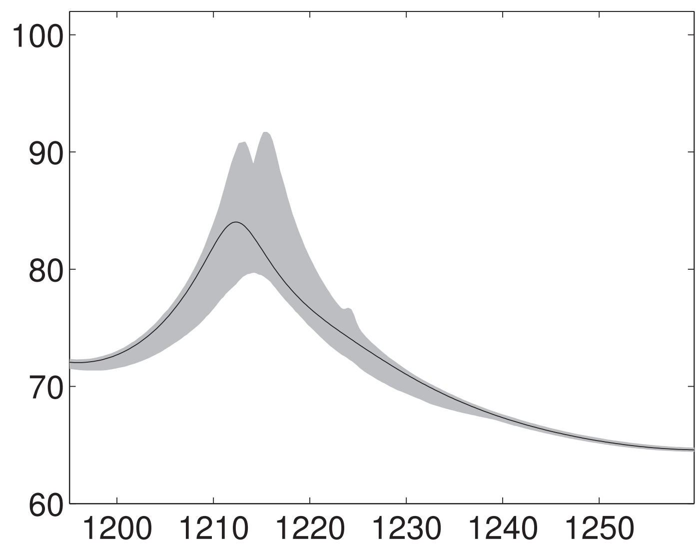

Figure 8: Graphs of function $\nu \mapsto 10\, \log_{10}(\underline{w}^{\rm RD}(2\pi\nu))$ (thin black line) and of the confidence region (gray region) of random observation $10\, \log_{10}(W^{\rm RD}(2\pi\nu))$ corresponding to the design optimization. Horizontal axis is the frequency ν in Hz.

Figure 8 shows the spectral acoustic energy $\omega \mapsto 10\, \log_{10}(\underline{w}^{\rm RD}(\omega))$ and the confidence region of random observation $W^{\rm RD}(\omega)$ corresponding to the robust design optimization. Figure 6 shows that the design optimization and the robust design optimization yields close design points. It can be seen that $g_{\mathbb{B}_1}(r^{\rm RD}) < g_{\mathbb{B}_1}(r^{\rm D})$ which means that the performance of the designed system solution of the robust design optimization is not as good as the performance of the designed system solution of the deterministic design optimization. Nevertheless, we have $g^{\text{real}}_{\mathbb{B}_1}(r^{\text{RD}}) = 5.4\,dB > g^{\text{real}}_{\mathbb{B}_1}(r^{\text{D}})$. Clearly, the real structural-acoustic system manufactured from the optimal designed system solution of the robust design optimization yields the most optimal performance.

6.5 Robust design optimization over a broad frequency band of analysis

The robust design analysis is carried out over the broad frequency band $\mathbb{B} = [1060\,,\,1300]\,Hz$. Figure 9 displays the graphs $r \mapsto 10\,\log_{10}(w^{+}_{\mathbb{B},\infty}(r))$ and $r \mapsto 10\,\log_{10}(\underline{w}_{\mathbb{B},\infty}(r))$. It can be seen that $r^{\text{D}} = 5.45\,10^{-3}\,m$ and $r^{\text{RD}} = 5.80\,10^{-3}\,m$. Figure 10 shows the spectral acoustic energy $\omega \mapsto 10\,\log_{10}(\underline{w}^{\text{D}}(\omega))$ and the confidence region of random observation $W^{\text{D}}(\omega)$ corresponding to the design optimization. Figure 11 shows the spectral acoustic energy $\omega \mapsto 10\,\log_{10}(\underline{w}^{\text{RD}}(\omega))$ and the confidence region of random observation $W^{\text{RD}}(\omega)$ corresponding to the robust design optimization.

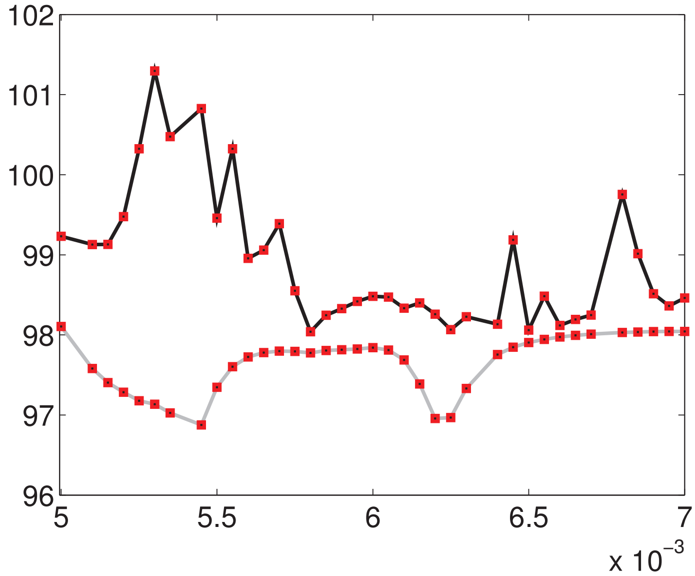

Figure 9: Comparison between the design optimization and the robust design optimization. Graph of functions $r \mapsto 10\,\log_{10}(w^{+}_{\mathbb{B},\infty}(r))$ (black line) and $r \mapsto 10\,\log_{10}(\underline{w}_{\mathbb{B},\infty}(r))$ (gray line). Horizontal axis is design parameter r.

From figure 9, it can be seen that the design optimization yields $g^{\text{real}}_{\mathbb{B}}(r^{\text{D}}) = -1.57\,dB$. The comparison between the confidence region of figure 4 and figure 10 shows that the resonance peaking number 2 is drastically softened at the expense of the resonance peaking number 1. In the present case, since $g^{\text{real}}_{\mathbb{B}}(r^{\text{D}}) < 0$, the deterministic optimization yields an erroneous optimal structural-acoustic system. The structural-acoustic system which is manufactured with this erroneous optimal design yields an acoustic pressure level which is contradictory to the prescribed objective. By comparing figure 4 and figure 11, it can be seen that the robust design optimization yields $g^{\text{real}}_{\mathbb{B}}(r^{\text{RD}}) = 1.20\,dB$. These results show that the model uncertainties and the data uncertainties have to be taken into account in the formulation of design optimization problems.

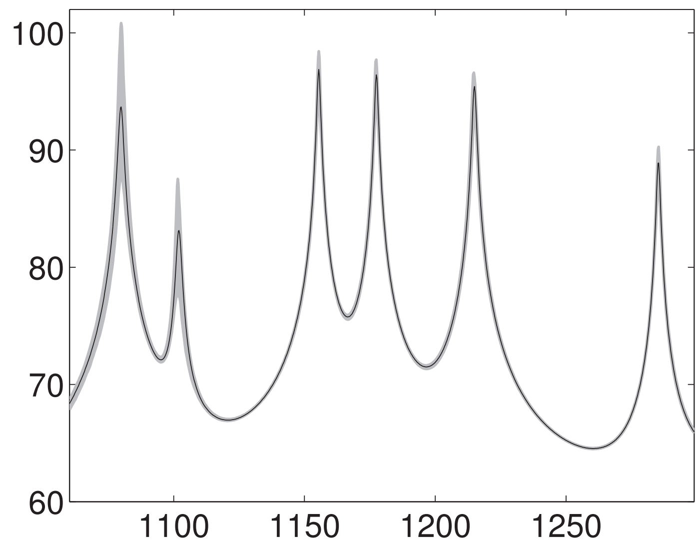

Figure 10: Graphs of function $\nu \mapsto 10\ \log_{10}(\underline{w}^{\mathrm{D}}(2\pi\nu))$ (thin black line) and of the confidence region (gray region) of random observation $10\ \log_{10}(W^{\mathrm{D}}(2\pi\nu))$ corresponding to the design optimization. Horizontal axis is the frequency ν in Hz.

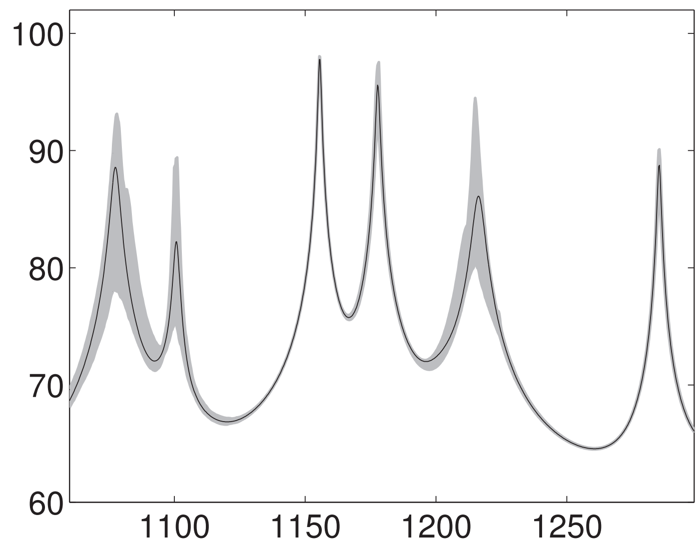

Figure 11: Graphs of function $\nu \mapsto 10\ \log_{10}(\underline{w}^{\mathrm{RD}}(2\pi\nu))$ (thin black line) and of the confidence region (gray region) of random observation $10\ \log_{10}(W^{\mathrm{RD}}(2\pi\nu))$ corresponding to the design optimization. Horizontal axis is the frequency ν in Hz.

7 Conclusion

An approach which allows the robust design optimization problem to be formulated and solved in presence of model uncertainties has been presented in the context of structural-acoustics. Model uncertainties are taken into account with a nonparametric probabilistic approach. The numerical application shows that the usual design optimization can produce a non optimal result. The proposed approach can be easily extended to any complex uncertain structural-acoustic system.

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