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Identification of a sound-insulation layer modelled by fuzzy structure theory - Experimental validation

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Abstract

One proposes a novel approach to model sound-insulation layers based on the use of the fuzzy structure theory which presents the advantage to account for variability as well as no addition of DOF in computational models. The keypoint of the method is the construction of a mean elastoacoustic sound-insulation layer model whose parameters are simply the modal density, the coefficient of participating mass and the damping coefficient. In this paper, the detailed construction of the mean model and an identification methodology of the mean parameters are presented. The modal density is identified from a stand-alone detailed finite element model of the sound-insulation layer. The coefficient of participating mass is then obtained by solving an inverse problem which is formulated as an optimization problem. Finally, the theory is validated with experiments for a vibroacoustic system made up of a steel plate connected to an elastic framework on its edges, covered with a sound-insulation layer and coupled with a bounded acoustic cavity.

1 Introduction

This paper deals with the construction of a simplified model of sound-insulation layers required for computational vibroacoustic simulation of complex systems in low- and medium-frequency ranges. The sound-insulation layer is assumed to behave as a resonant continuous dynamical system in the frequency band of analysis. In this paper, we will not consider the high-frequency range for which other phenomena appear. For such a modelling in the low- and medium-frequency ranges, a usual approach consists in modelling a sound-insulation layer as a poro-elastic medium using the Biot theory ; the finite element method is then classically used to solve the associated boundary value problem. In this case, both vibroacoustic system and sound-insulation layers are modelled by the finite element method (see for instance Refs. [1, 2, 3, 4, 5, 6, 7, 8]). When the first thickness eigenfrequencies belong to the frequency band of analysis, as assumed here, such a finite element model of sound-insulation layer requires a large number of physical degrees of freedom (DOF) in the computational model and introduces numerous elastic modes in the band. The size of the associated reduced computational model can then be very large. For instance, in a car booming noise analysis (frequency range [20, 200] Hz later referred as low-frequency range), the finite element model may involve up to two millions of DOF for the structure and the reduced model requires about one thousand elastic modes. If the sound-insulation layers were modelled by the finite element method, an additional number of about five millions of DOF would be necessary. Twenty thousand additional elastic modes then appear in the reduced computational model exceeding the limits of current computational resources. Consequently, there is a great interest to construct simplified sound-insulation layer models without adding neither physical DOF nor generalized DOF. Representing the sound-insulation layer by an adapted wall impedance can be a way to avoid the increase of DOF number (see for instance Refs. [9, 10, 11, 12, 13] for the notion of wall impedance in vibroacoustics). A great number of publications has been devoted to this subject in the last three decades.

It should be noted that the largest part of these works deals with the medium- and high-frequency ranges, where the sound-insulation layer behavior differs from the lower frequency range that is investigated here. Since the objective of this paper is not to give a state of the art on this particular topic, we simply refer to a few papers such as Refs. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

In this paper, an alternate construction to the finite element approach or to the usual wall impedances is presented. Firstly, this construction does not increase the number of physical and generalized DOF in the computational vibroacoustic model. Secondly, it does not involve the poroelastic equations because the sound-insulation layers considered here have a rather simple dynamic behavior which does not require advanced material modelling. As explained above, in the frequency band of analysis, the simplified model can be originated from a single DOF dynamical system. Due to the actual variability of thicknesses, curvature and materials properties, the sound-insulation layer is considered as complex and therefore, a statistical description of its internal dynamical DOF is proposed. We are naturally led to use the fuzzy structure theory [24] which fits this framework and has already been validated. A representation of the sound-insulation layer based on the fuzzy structure theory benefits from an understandable interpretation of complex dynamical systems behavior. It is simply characterized by a few physical parameters: the participating mass, the modal density and the internal damping rate. The fuzzy structure theory was introduced twenty years ago in order to model the effects on a master structure of complex subsystems imprecisely known (see Refs. [24, 25, 26, 27, 28]). This theory is developed in the context of the probability theory which is well adapted to this kind of problem that carries many uncertainties (geometry, material and boundary conditions). Many other works have been developed in this field, completing the initial construction (see Refs. [29, 30, 31, 32, 33, 34, 35, 36, 37]). Most of these developments are related to complex structural subsystems coupled to a master vibroacoustic system. No attempt has been performed to develop a specific sound-insulation layer model using the fuzzy structure theory. The known results have to be extended in order to build an elastoacoustic element. That is the aim of this paper.

Section 2 is devoted to the construction of a simplified mean model of the sound-insulation layer using the fuzzy structure theory. Since this model is a part of the complex vibroacoustic system, the complete vibroacoustic model is presented. Section 3 deals with the finite element approximation which allows a computational vibroacoustic mean model to be constructed. In Section 4, we present the first step of the methodology performing the experimental identification of the mean parameters of the sound-insulation layer simplified model. Finally, Section 5 deals with the prediction of the system's vibroacoustic response using the previously identified computational mean model.

2 CONSTRUCTION OF A SIMPLIFIED MEAN MODEL OF THE SOUND-INSULATION LAYER USING THE FUZZY STRUCTURE THEORY IN A VIBROACOUSTIC SYSTEM

2.1 Definition of the vibroacoustic system

The physical space \mathbb{R}^3 is referred to a cartesian system for which the generic point is denoted by $\mathbf{x} = (x_1, x_2, x_3)$. The Fourier transform with respect to time t is denoted by $u(\omega) = \int_{\mathbb{R}} e^{-i\omega t} u(t) dt$. The vibroacoustic system is analyzed in the frequency band $\mathbb{B} = [\omega_{\min}, \omega_{\max}]$. The structure occupies a three-dimensional bounded domain Ω_s and is modelled by a nonhomogeneous anisotropic viscoelastic material. The boundary of Ω_s is written as $\partial\Omega_s = \Gamma_s \cup \Gamma_0 \cup \Gamma_1 \cup \Gamma_2$ (see Fig.1.a) and the outward unit normal to $\partial\Omega_s$ is $\mathbf{n}^s(\mathbf{x})$. The structure is fixed on Γ_0 , a surface force field $\mathbf{g}^{\text{surf}}(\mathbf{x}, \omega)$ is given on Γ_1 and a body force field $\mathbf{g}^{\text{vol}}(\mathbf{x}, \omega)$ is given in Ω_s . The acoustic cavity Ω_a is filled with a dissipative acoustic fluid, its boundary is written as $\partial\Omega_a = \Gamma \cup \Gamma_2$ and the inward unit normal to $\partial\Omega_a$ is $\mathbf{n}(\mathbf{x})$ (note that $\mathbf{n}(\mathbf{x}) = \mathbf{n}^s(\mathbf{x})$ on Γ_2 and that without sound-insulation layer, the coupling surface $\partial\Omega_a$ between the structure and the cavity would be $\Gamma_s \cup \Gamma_2$). A sound-insulation layer which occupies a bounded domain Ω_h with boundary $\partial\Omega_h = \Gamma \cup \Gamma_s$ is attached to the part Γ_s of the boundary of the structure (see Fig.1.a). Let $\mathbf{x} \mapsto \mathbf{u}^s(\mathbf{x}, \omega) =$

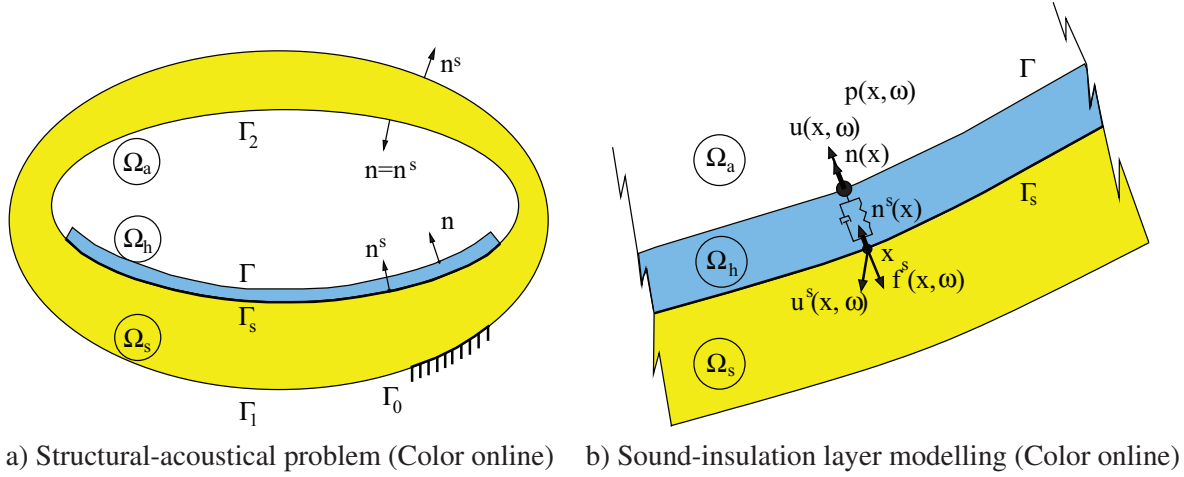


FIG. 1 – Sketches

$(u_1^s(\mathbf{x}, \omega), u_2^s(\mathbf{x}, \omega), u_3^s(\mathbf{x}, \omega))$ be the structural displacement field defined on Ω_s with values in \mathbb{C}^3 and which is equal to zero on Γ_0 . Let $\mathbf{x} \mapsto p(\mathbf{x}, \omega)$ be the pressure field inside Ω_a with value in \mathbb{C} for which the value on $\partial\Omega_a = \Gamma \cup \Gamma_2$ is still denoted by $p(\mathbf{x}, \omega)$. Let $\mathbf{x} \mapsto \mathbf{u}^h(\mathbf{x}, \omega) = (u_1^h(\mathbf{x}, \omega), u_2^h(\mathbf{x}, \omega), u_3^h(\mathbf{x}, \omega))$ be the sound-insulation layer displacement field defined on Ω_h with values in \mathbb{C}^3 whose value on interface Γ is still denoted by $\mathbf{x} \mapsto \mathbf{u}^h(\mathbf{x}, \omega)$. Finally, we need to introduce the admissible spaces for the three fields of the problem. Let \mathcal{C}_0^s be the space of the admissible displacement fields of the structure such that $\mathbf{u}^s = 0$ on Γ_0 , let \mathcal{C}^a be the space of the admissible pressure fields in the acoustic cavity and \mathcal{C}^h be the space of the admissible displacement fields of the sound-insulation layer.

2.2 Coupling force fields

The coupling force field on boundary Γ_s that the structure exerts on the sound-insulation layer is denoted by $\mathbf{x} \mapsto \mathbf{f}^s(\mathbf{x}, \omega) = (f_1^s(\mathbf{x}, \omega), f_2^s(\mathbf{x}, \omega), f_3^s(\mathbf{x}, \omega))$ and can be written for all \mathbf{x} fixed in Γ_s as $\mathbf{f}^s(\mathbf{x}, \omega) = f^s(\mathbf{x}, \omega)\mathbf{n}^s(\mathbf{x}) + \mathbf{f}_{\text{tang}}^s(\mathbf{x}, \omega)$. It is assumed that the tangential force field $\mathbf{f}_{\text{tang}}^s(\mathbf{x}, \omega)$ exerted by the structure on the sound-insulation layer is equal to zero. This hypothesis is reasonable in vibroacoustics for the majority of the cases met in the technologies such as the ones used in the automotive industry. Consequently, we have

$$\mathbf{f}^s(\mathbf{x}, \omega) = f^s(\mathbf{x}, \omega)\mathbf{n}^s(\mathbf{x}) \quad . \quad (1)$$

Note that dimension of $f_i^s(\mathbf{x}, t)$ is $[M][L]^{-1}[T]^{-2}$. The coupling force field $\mathbf{x} \mapsto \mathbf{f}^p(\mathbf{x}, \omega) = (f_1^p(\mathbf{x}, \omega), f_2^p(\mathbf{x}, \omega), f_3^p(\mathbf{x}, \omega))$ on boundary $\partial\Omega_a = \Gamma \cup \Gamma_2$ that the acoustic fluid exerts on the structure (interface Γ_2) and the sound-insulation layer (interface Γ) is written as,

$$\mathbf{f}^p(\mathbf{x}, \omega)ds(\mathbf{x}) = -p(\mathbf{x}, \omega)\mathbf{n}(\mathbf{x})ds(\mathbf{x}) \quad (2)$$

in which ds is the surface element on $\partial\Omega_a$. The equations of the boundary value problem for the vibroacoustic system made up of the structure coupled with an internal acoustic cavity and with the sound-insulation layer are given in Appendix A. Eqs. (41), (42) and (43) are the equations for the structure, the acoustic cavity and the sound-insulation layer. There are coupling terms in these three equations. In particular, the coupling term between the sound-insulation layer and the structure in the Eq. (41) of the structure is represented by the term $c_{\Gamma_s}(\delta\mathbf{u}^s; \omega)$; the coupling term between the sound-insulation layer and the acoustic cavity in Eq. (42) of the acoustic cavity is represented by the term $c_{\Gamma}(\mathbf{u}^h, \delta p)$. The principle of construction of the simplified model for the sound-insulation layer consists in replacing Eq. (43) by a simplified model obtained in using the fuzzy structure theory[24, 26, 27] for which a synthesis is given in Ref [13]. This means that the two coupling terms $c_{\Gamma_s}(\delta\mathbf{u}^s; \omega)$ and $c_{\Gamma}(\mathbf{u}^h, \delta p)$ have to be expressed as a function of \mathbf{u}^s and p using the fuzzy structure theory. This theory consists (1) in introducing an underlying deterministic dynamical model (see

Section 2.3), (2) in introducing a probabilistic model of the eigenfrequencies of this dynamical model (see Section 2.4) and (3) in performing a statistical averaging (see Section 2.5).

2.3 Definition of the underlying deterministic model for the fuzzy structure

We introduce the following hypothesis for the sound-insulation layer (see Fig.1.b): the surfaces Γ and Γ_s are assumed to be geometrically equivalent and consequently, for all \mathbf{x} in $\Gamma_s \simeq \Gamma$, $\mathbf{n}^s(\mathbf{x}) \simeq \mathbf{n}(\mathbf{x})$. The normal component to Γ_s of the structural displacement field is

$$\mathbf{n}^s(\mathbf{x}) \cdot \mathbf{u}^s(\mathbf{x}, \omega) = w^s(\mathbf{x}, \omega) \quad , \quad \mathbf{x} \in \Gamma_s \quad . \quad (3)$$

The normal component to Γ of the displacement field of the sound-insulation layer is

$$\mathbf{n}(\mathbf{x}) \cdot \mathbf{u}^h(\mathbf{x}, \omega) = w(\mathbf{x}, \omega) \quad (4)$$

in which $\mathbf{n}(\mathbf{x}) \simeq \mathbf{n}^s(\mathbf{x})$, for $\mathbf{x} \in \Gamma_s \simeq \Gamma$. Using the first step of the fuzzy structure theory and taking into account the hypothesis introduced in Section 2.2, the underlying deterministic model is made up of a density of damped linear oscillators acting in the normal direction to Γ . For a fixed frequency ω and for a fixed \mathbf{x} in Γ_s , the displacement of the base of an oscillator is $w^s(\mathbf{x}, \omega)$ and the displacement of its mass $\mu(\mathbf{x}, \omega) > 0$ is $w(\mathbf{x}, \omega)$. The mass density $\mu(\mathbf{x}, \omega)$ ($[M][L]^{-2}$) is distributed on Γ . The corresponding stiffness density $k(\mathbf{x}, \omega)$ associated with this oscillator is $k(\mathbf{x}, \omega) = \mu(\mathbf{x}, \omega)\omega_p^2(\mathbf{x}, \omega)$ where $\omega_p(\mathbf{x}, \omega) > 0$ is the eigenfrequency (rad.s^{-1}) of the undamped linear oscillator with fixed base. The damping rate of this oscillator is denoted by $\xi(\mathbf{x}, \omega)$. Let $\mathbf{f}^s(\mathbf{x}, \omega)$ be the force applied to the base of this oscillator and corresponding to the force density induced by the structure on the sound-insulation layer (see Eq. (1)). Let $\mathbf{f}^p(\mathbf{x}, \omega)$ be the force applied to the mass of the oscillator and corresponding to the force density induced by the acoustic pressure $p(\mathbf{x}, \omega)$ on the sound-insulation layer (see Eq. (2)). Removing \mathbf{x} and ω for brevity, the equation of this oscillator is written as

$$\mu \begin{bmatrix} -\omega^2 + 2i\omega\xi\omega_p + \omega_p^2 & -2i\omega\xi\omega_p - \omega_p^2 \\ -2i\omega\xi\omega_p - \omega_p^2 & 2i\omega\xi\omega_p + \omega_p^2 \end{bmatrix} \begin{bmatrix} w(\mathbf{x}, \omega) \\ w^s(\mathbf{x}, \omega) \end{bmatrix} = \begin{bmatrix} -p(\mathbf{x}, \omega) \\ \mathbf{f}^s(\mathbf{x}, \omega) \end{bmatrix} . \quad (5)$$

For all ω in \mathbb{B} , from Eq. (5), it can be deduced that

$$w(\mathbf{x}, \omega) = a^c(\mathbf{x}, \omega)w^s(\mathbf{x}, \omega) + \frac{1}{\omega^2}a^a(\mathbf{x}, \omega)p(\mathbf{x}, \omega) , \quad (6)$$

$$\mathbf{f}^s(\mathbf{x}, \omega) = a^s(\mathbf{x}, \omega)w^s(\mathbf{x}, \omega) + a^c(\mathbf{x}, \omega)p(\mathbf{x}, \omega) \quad (7)$$

in which

$$a^s(\mathbf{x}, \omega) = \frac{-\omega^2 \mu(\mathbf{x}, \omega) (2i\omega \xi(\mathbf{x}, \omega)\omega_p(\mathbf{x}, \omega) + \omega_p(\mathbf{x}, \omega)^2)}{-\omega^2 + 2i\omega \xi(\mathbf{x}, \omega)\omega_p(\mathbf{x}, \omega) + \omega_p(\mathbf{x}, \omega)^2} , \quad (8)$$

$$a^a(\mathbf{x}, \omega) = \frac{-\omega^2/\mu(\mathbf{x}, \omega)}{-\omega^2 + 2i\omega \xi(\mathbf{x}, \omega)\omega_p(\mathbf{x}, \omega) + \omega_p(\mathbf{x}, \omega)^2} , \quad (9)$$

$$a^c(\mathbf{x}, \omega) = \frac{2i\omega \xi(\mathbf{x}, \omega)\omega_p(\mathbf{x}, \omega) + \omega_p(\mathbf{x}, \omega)^2}{-\omega^2 + 2i\omega \xi(\mathbf{x}, \omega)\omega_p(\mathbf{x}, \omega) + \omega_p(\mathbf{x}, \omega)^2} . \quad (10)$$

As explained in Section 2.2, we have to express the two terms $c_\Gamma(\mathbf{u}^h, \delta p)$ and $c_{\Gamma_s}(\delta \mathbf{u}; \omega)$. Using $\Gamma \simeq \Gamma_s$, substituting Eq. (4) into $c_\Gamma(\mathbf{u}^h, \delta p)$ defined by Eq. (39), substituting Eq. (6) again into Eq. (39) and using Eq. (3) yield

$$\omega^2 c_\Gamma(\mathbf{u}^h, \delta p) = \omega^2 \int_{\Gamma_s} a^c(\mathbf{x}, \omega) \mathbf{n}^s(\mathbf{x}) \cdot \mathbf{u}^s(\mathbf{x}, \omega) \delta p(\mathbf{x}) ds(\mathbf{x}) + \int_{\Gamma_s} a^a(\mathbf{x}, \omega) p(\mathbf{x}, \omega) \delta p(\mathbf{x}) ds(\mathbf{x}) . \quad (11)$$

Substituting Eq. (1) into $c_{\Gamma_s}(\delta \mathbf{u}; \omega; \omega)$ defined by Eq. (40), substituting Eq. (7) again into Eq. (40) and using Eq. (3) yield

$$\begin{aligned} c_{\Gamma_s}(\delta \mathbf{u}^s; \omega) &= \int_{\Gamma_s} a^s(\mathbf{x}, \omega) (\mathbf{n}^s(\mathbf{x}) \cdot \mathbf{u}^s(\mathbf{x}, \omega)) (\mathbf{n}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x})) ds(\mathbf{x}) \\ &+ \int_{\Gamma_s} a^c(\mathbf{x}, \omega) p(\mathbf{x}, \omega) (\mathbf{n}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x})) ds(\mathbf{x}). \end{aligned} \quad (12)$$

2.4 Probabilistic model of the eigenfrequency of the oscillators

The second step of the fuzzy structure consists in modelling $\omega_p(\mathbf{x}, \omega)$ by a random variable $\Omega_p(\mathbf{x}, \omega)$. In this section, we then introduce the random bilinear form associated with $c_{\Gamma}(\mathbf{u}^h, \delta p)$ and the random linear form associated with $c_{\Gamma_s}(\delta \mathbf{u}^s; \omega)$ defined by Eqs. (11) and (12). For all ω in \mathbb{B} , we choose to represent $\mu(\mathbf{x}, \omega)$ and $\xi(\mathbf{x}, \omega)$ by their mean values $\mu(\mathbf{x}, \omega) = \underline{\mu}(\omega) > 0$ and $\xi(\mathbf{x}, \omega) = \underline{\xi}(\omega)$ where $\omega \mapsto \underline{\mu}(\omega)$ and $\omega \mapsto \underline{\xi}(\omega)$ are two deterministic functions independent of \mathbf{x} with $0 < \underline{\xi}(\omega) < 1$. The mean participating mass can be written [24, 26, 27] as $\underline{\mu}(\omega) = \underline{\nu}(\omega) m_{tot} / |\Gamma_s|$ where $0 \leq \underline{\nu}(\omega) \leq 1$ is the mean coefficient of participating mass, m_{tot} is the total mass of the density of oscillators and $|\Gamma_s|$ is the measure of surface Γ_s . It should be noted that if there are several sound-insulation layers with different values of parameters $\underline{\mu}$ and $\underline{\xi}$, domain Ω_h is subdivided into several subdomains and thus, their parameters have to be constant with respect to \mathbf{x} in every subdomain. For all \mathbf{x} fixed in Γ_s and ω fixed in \mathbb{B} , the eigenfrequency $\omega_p(\mathbf{x}, \omega)$ is modelled by a positive random variable $\Omega_p(\mathbf{x}, \omega)$ whose probability distribution $P_{\Omega_p(\mathbf{x}; \omega)}(d\omega_p, \omega)$ is assumed to be independent of \mathbf{x} and is defined by the probability density function $p_{\Omega_p(\mathbf{x}; \omega)}(\omega_p, \omega)$ with respect to $d\omega_p$, such that [24, 26]

$$p_{\Omega_p(\mathbf{x}; \omega)}(\omega_p, \omega) = \ell(\omega) \mathbb{1}_{[a(\omega), b(\omega)]}(\omega_p) \quad , \quad (13)$$

with $\mathbb{1}_B(\mathbf{x}) = 1$ if $\mathbf{x} \in B$ and $= 0$ if $\mathbf{x} \notin B$ and where

$$a(\omega) = \sup \left\{ 0, \omega - \frac{1}{2\underline{n}(\omega)} \right\} \quad , \quad b(\omega) = \omega + \frac{1}{2\underline{n}(\omega)} \quad , \quad \ell(\omega) = \frac{1}{b(\omega) - a(\omega)} \quad (14)$$

in which $\underline{n}(\omega)$ is the mean modal density of the sound-insulation layer. In order to better explain the meaning of parameters $\underline{n}(\omega)$ and $\underline{\mu}(\omega)$, we define them in the simplest case for which the fuzzy structure Ω_h would be made up of N_{osc} oscillators uniformly distributed in the frequency band \mathbb{B} and uniformly distributed on surface Γ_s . In this case, the mass of each oscillator would be m_{osc} . Consequently, the total mass of the fuzzy structure would be $m_{tot} = N_{osc} m_{osc}$ and for all \mathbf{x} in Γ_s , we would have $\mu(\mathbf{x}, \omega) = \underline{\mu}(\omega) = \sqrt{N_{osc} m_{osc} / |\Gamma_s|}$, $\int_{\mathbb{B}} \underline{n}(\omega) d\omega = \sqrt{N_{osc}}$ and $\int_{\mathbb{B}} \underline{\mu}(\omega) \underline{n}(\omega) d\omega = m_{tot} / |\Gamma_s|$. Coming back to the general case, for all \mathbf{x} fixed in Γ_s and ω fixed in \mathbb{B} , the coefficients $a^s(\mathbf{x}, \omega)$, $a^a(\mathbf{x}, \omega)$ and $a^c(\mathbf{x}, \omega)$ defined by Eqs. (8), (9) and (10) become random variables denoted by $A^s(\mathbf{x}, \omega)$, $A^a(\mathbf{x}, \omega)$ and $A^c(\mathbf{x}, \omega)$. For all \mathbf{u}^s and $\delta \mathbf{u}^s$ in C_0^s and for all p and δp in C^a , the forms $c_{\Gamma}(\mathbf{u}^h, \delta p)$ and $c_{\Gamma_s}(\delta \mathbf{u}^s; \omega)$ defined by Eqs. (11) and (12) become random variables which are rewritten in terms of $\mathbf{u}^s, p, \delta \mathbf{u}^s$ and δp as $C_{\Gamma}(\mathbf{u}^s, p, \delta p; \omega)$ and $C_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega)$ and which are such that

$$\omega^2 C_{\Gamma}(\mathbf{u}^s, p, \delta p; \omega) = \omega^2 \int_{\Gamma_s} A^c(\mathbf{x}, \omega) \mathbf{n}^s(\mathbf{x}) \cdot \mathbf{u}^s(\mathbf{x}, \omega) \delta p(\mathbf{x}) ds(\mathbf{x}) + \int_{\Gamma_s} A^a(\mathbf{x}, \omega) p(\mathbf{x}, \omega) \delta p(\mathbf{x}) ds(\mathbf{x}) \quad (15)$$

and

$$\begin{aligned} C_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega) &= \int_{\Gamma_s} A^s(\mathbf{x}, \omega) (\mathbf{n}^s(\mathbf{x}) \cdot \mathbf{u}^s(\mathbf{x}, \omega)) (\mathbf{n}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x})) ds(\mathbf{x}) \\ &+ \int_{\Gamma_s} A^c(\mathbf{x}, \omega) p(\mathbf{x}, \omega) (\mathbf{n}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x})) ds(\mathbf{x}) \quad . \end{aligned} \quad (16)$$

2.5 Statistical averaging and simplified mean model of the sound-insulation layer

The last step of the fuzzy structure theory consists in defining the simplified mean model taking the statistical averaging of random variables $C_\Gamma(\mathbf{u}^s, p, \delta p; \omega)$ and $C_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega)$ defined by Eqs. (15) and (16). As we have explained in section 2.2, the simplified mean model thus consists in replacing the two coupling terms $c_\Gamma(\mathbf{u}^h, \delta p)$ and $c_{\Gamma_s}(\delta \mathbf{u}^s; \omega)$ by $\underline{c}_\Gamma(\mathbf{u}^s, p, \delta p; \omega)$ and $\underline{c}_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega)$ such that,

$$\underline{c}_\Gamma(\mathbf{u}^s, p, \delta p; \omega) = \mathcal{E}\{C_\Gamma(\mathbf{u}^s, p, \delta p; \omega)\} \quad , \quad (17)$$

$$\underline{c}_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega) = \mathcal{E}\{C_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega)\} \quad (18)$$

in which \mathcal{E} is the mathematical expectation. Analyzing Eqs. (15) and (16) leads us to introduce the following deterministic bilinear forms $b^s(\mathbf{u}^s, \delta \mathbf{u}^s)$ on $\mathcal{C}_0^s \times \mathcal{C}_0^s$, $c^s(p, \delta \mathbf{u}^s)$ on $\mathcal{C}^a \times \mathcal{C}_0^s$ and $b^a(p, \delta p)$ on $\mathcal{C}^a \times \mathcal{C}^a$,

$$b^s(\mathbf{u}^s, \delta \mathbf{u}^s) = \int_{\Gamma_s} (\mathbf{n}^s(\mathbf{x}) \cdot \mathbf{u}^s(\mathbf{x})) (\mathbf{n}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x})) ds(\mathbf{x}) \quad , \quad (19)$$

$$c^s(p, \delta \mathbf{u}^s) = \int_{\Gamma_s} p(\mathbf{x}) (\mathbf{n}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x})) ds(\mathbf{x}) \quad , \quad (20)$$

$$b^a(p, \delta p) = \int_{\Gamma_s} p(\mathbf{x}) \delta p(\mathbf{x}) ds(\mathbf{x}) \quad . \quad (21)$$

From Eqs. (15) and (16) and using Eqs. (17) and (18) with Eqs. (13) to (14) yield

$$\omega^2 \underline{c}_\Gamma(\mathbf{u}^s, p, \delta p; \omega) = \omega^2 \underline{a}^c(\omega) c^s(\delta p, \mathbf{u}^s) + \underline{a}^a(\omega) b^a(p, \delta p) \quad , \quad (22)$$

$$\underline{c}_{\Gamma_s}(\mathbf{u}^s, p, \delta \mathbf{u}^s; \omega) = \underline{a}^s(\omega) b^s(\mathbf{u}^s, \delta \mathbf{u}^s) + \underline{a}^c(\omega) c^s(p, \delta \mathbf{u}^s) \quad (23)$$

in which

$$\underline{a}^s(\omega) = -\omega^2 \underline{a}_R^s(\omega) + i\omega \underline{a}_I^s(\omega) \quad , \quad (24)$$

$$\underline{a}^a(\omega) = \underline{a}_R^a(\omega) + i\omega \underline{a}_I^a(\omega) \quad , \quad (25)$$

$$\underline{a}^c(\omega) = \underline{a}_R^c(\omega) + i\omega \underline{a}_I^c(\omega) \quad , \quad (26)$$

with

$$\underline{a}_R^s(\omega) = \underline{\mu}(\omega) \underline{n}(\omega) \left[\frac{1}{\underline{n}(\omega)} - \omega \underline{\lambda}(\omega) \Theta_R(\omega) \right] \quad , \quad \underline{a}_I^s(\omega) = \underline{\mu}(\omega) \underline{n}(\omega) \omega^2 \underline{\lambda}(\omega) \Theta_I(\omega) \quad , \quad (27)$$

$$\underline{a}_R^a(\omega) = \omega \underline{n}(\omega) \frac{\underline{\lambda}(\omega)}{\underline{\mu}(\omega)} \Theta_R(\omega) \quad , \quad \underline{a}_I^a(\omega) = \underline{n}(\omega) \frac{\underline{\lambda}(\omega)}{\underline{\mu}(\omega)} \Theta_I(\omega) \quad , \quad (28)$$

$$\underline{a}_R^c(\omega) = 1 - \omega \underline{n}(\omega) \underline{\lambda}(\omega) \Theta_R(\omega) \quad , \quad \underline{a}_I^c(\omega) = -\omega \underline{n}(\omega) \underline{\lambda}(\omega) \Theta_I(\omega) \quad . \quad (29)$$

In these equations, the functions $\underline{\lambda}$, Θ_R and Θ_I are defined in Appendix B. We then deduce the simplified mean model of the sound-insulation layer: Find \mathbf{u}^s in \mathcal{C}_0^s and p in \mathcal{C}^a such that, for all $\delta \mathbf{u}^s$ in \mathcal{C}_0^s and δp in \mathcal{C}^a , we have

$$\underbrace{\left[\begin{array}{l} -\omega^2 m^s(\mathbf{u}^s, \delta \mathbf{u}^s) + i\omega d^s(\mathbf{u}^s, \delta \mathbf{u}^s; \omega) \\ + k^s(\mathbf{u}^s, \delta \mathbf{u}^s; \omega) + c_{\Gamma_2}(\delta \mathbf{u}^s, p) \\ + \underline{a}^s(\omega) b^s(\mathbf{u}^s, \delta \mathbf{u}^s) + \underline{a}^c(\omega) c^s(p, \delta \mathbf{u}^s) = l^s(\delta \mathbf{u}^s; \omega) \end{array} \right]}_{\text{STRUCTURE}} \quad \underbrace{\left[\begin{array}{l} -\omega^2 m^a(p, \delta p) + i\omega d^a(p, \delta p; \omega) + k^a(p, \delta p) \\ + \omega^2 c_{\Gamma_2}(\mathbf{u}^s, \delta p) + \omega^2 \underline{a}^c(\omega) c^s(\delta p, \mathbf{u}^s) \\ + \underline{a}^a(\omega) b^a(p, \delta p) = l^a(\delta p; \omega) \end{array} \right]}_{\text{ACOUSTIC CAVITY}} \quad (30)$$

in which the bilinear forms b^s , c^s and b^a are defined by Eqs. (19), (20) and (21) and where m^s , d^s , k^s , m^a , d^a , k^a and c_{Γ_2} are defined by Eqs. (44) to (47).

3 COMPUTATIONAL VIBROACOUSTIC MEAN MODEL

The finite element discretization [38, 13] of the system of equations (30) yields the following matrix equation on $\mathbb{C}^{m_s} \times \mathbb{C}^{m_a}$ defined by Eqs. (15) and (16),

$$\begin{bmatrix} [\underline{\mathbb{A}}^s(\omega)] + \underline{a}^s(\omega)[\underline{\mathbb{B}}^s] & [\underline{\mathbb{C}}] + \underline{a}^c(\omega)[\underline{\mathbb{C}}^s] \\ \omega^2 \{ [\underline{\mathbb{C}}]^T + \underline{a}^c(\omega)[\underline{\mathbb{C}}^s]^T \} & [\underline{\mathbb{A}}^a(\omega)] + \underline{a}^a(\omega)[\underline{\mathbb{B}}^a] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{u}}^s(\omega) \\ \underline{\mathbf{p}}(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{f}}^s(\omega) \\ \underline{\mathbf{f}}^a(\omega) \end{bmatrix}, \quad (31)$$

where $[\underline{\mathbb{A}}^s(\omega)]$ is a complex $(m_s \times m_s)$ matrix such that

$$[\underline{\mathbb{A}}^s(\omega)] = -\omega^2 [\underline{\mathbb{M}}^s] + i\omega [\underline{\mathbb{D}}^s(\omega)] + [\underline{\mathbb{K}}^s(\omega)] \quad (32)$$

in which $[\underline{\mathbb{M}}^s]$, $[\underline{\mathbb{D}}^s(\omega)]$ and $[\underline{\mathbb{K}}^s(\omega)]$ are the mass, damping and stiffness matrices of the structure *in vacuo*. In Eq. (31), $[\underline{\mathbb{A}}^a(\omega)]$ is a complex $(m_a \times m_a)$ matrix,

$$[\underline{\mathbb{A}}^a(\omega)] = -\omega^2 [\underline{\mathbb{M}}^a] + i\omega [\underline{\mathbb{D}}^a(\omega)] + [\underline{\mathbb{K}}^a] \quad (33)$$

in which $[\underline{\mathbb{M}}^a]$, $[\underline{\mathbb{D}}^a(\omega)]$ and $[\underline{\mathbb{K}}^a]$ are the “mass”, “damping” and “stiffness” matrices of the acoustic cavity with rigid walls. The real $(m_s \times m_a)$ matrix $[\underline{\mathbb{C}}]$ is the usual vibroacoustic coupling matrix relative to boundary Γ_2 (which is without sound-insulation layer). The matrices $[\underline{\mathbb{B}}^s]$, $[\underline{\mathbb{C}}^s]$ and $[\underline{\mathbb{B}}^a]$ correspond to the finite element approximation of the bilinear forms defined by Eqs. (19), (20) and (21) respectively. Using n_s structural elastic modes *in vacuo* and n_a acoustic modes of the cavity with rigid walls including the constant pressure mode, the mean reduced matrix model of the vibroacoustic system is written as

$$\underline{\mathbf{u}}^s(\omega) = [\underline{\Phi}^s] \underline{\mathbf{q}}^s(\omega) \quad , \quad \underline{\mathbf{p}}(\omega) = [\underline{\Phi}^a] \underline{\mathbf{q}}^a(\omega) \quad , \quad (34)$$

$$\begin{bmatrix} [\underline{\mathbb{A}}^s(\omega)] + \underline{a}^s(\omega)[\underline{\mathbb{B}}^s] & [\underline{\mathbb{C}}] + \underline{a}^c(\omega)[\underline{\mathbb{C}}^s] \\ \omega^2 \{ [\underline{\mathbb{C}}]^T + \underline{a}^c(\omega)[\underline{\mathbb{C}}^s]^T \} & [\underline{\mathbb{A}}^a(\omega)] + \underline{a}^a(\omega)[\underline{\mathbb{B}}^a] \end{bmatrix} \begin{bmatrix} \underline{\mathbf{q}}^s(\omega) \\ \underline{\mathbf{q}}^a(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{f}}^s(\omega) \\ \underline{\mathbf{f}}^a(\omega) \end{bmatrix} \quad (35)$$

in which $[\underline{\Phi}^s]$ is the $(m_s \times n_s)$ real matrix of the structural modes, $[\underline{\Phi}^a]$ is the $(m_a \times n_a)$ real matrix of the acoustic modes, $[\underline{\mathbb{C}}]$ is the $(n_s \times n_a)$ generalized vibroacoustic coupling matrix, $\underline{a}^s(\omega)[\underline{\mathbb{B}}^s]$ is the $(n_s \times n_s)$ generalized matrix, $\underline{a}^c(\omega)[\underline{\mathbb{C}}^s]$ is the $(n_s \times n_a)$ generalized matrix and $\underline{a}^a(\omega)[\underline{\mathbb{B}}^a]$ is the $(n_a \times n_a)$ generalized matrix corresponding to the vibroacoustic coupling induced by the sound-insulation layer. The $(n_s \times n_s)$ matrix $[\underline{\mathbb{A}}^s(\omega)]$ and the $(n_a \times n_a)$ matrix $[\underline{\mathbb{A}}^a(\omega)]$ are written as

$$[\underline{\mathbb{A}}^s(\omega)] = -\omega^2 [\underline{\mathbb{M}}^s] + i\omega [\underline{\mathbb{D}}^s(\omega)] + [\underline{\mathbb{K}}^s(\omega)] \quad , \quad (36)$$

$$[\underline{\mathbb{A}}^a(\omega)] = -\omega^2 [\underline{\mathbb{M}}^a] + i\omega [\underline{\mathbb{D}}^a(\omega)] + [\underline{\mathbb{K}}^a] \quad (37)$$

in which $[\underline{\mathbb{M}}^s]$, $[\underline{\mathbb{D}}^s(\omega)]$ and $[\underline{\mathbb{K}}^s(\omega)]$ are the generalized mass, damping and stiffness matrices of the structure and $[\underline{\mathbb{M}}^a]$, $[\underline{\mathbb{D}}^a(\omega)]$ and $[\underline{\mathbb{K}}^a]$ are the generalized “mass”, “damping” and “stiffness” matrices of the acoustic cavity.

4 EXPERIMENTAL IDENTIFICATION OF THE MEAN PARAMETERS OF THE FUZZY STRUCTURE MODEL FOR THE SOUND-INSULATION LAYER - DESIGN METHODOLOGY PART 1

We propose to validate the simplified mean model of the sound-insulation layer by using experiments. The methodology used is the following:

- (1) We consider a structure for which the experimental frequency response functions (FRF) are measured on frequency band \mathbb{B} .

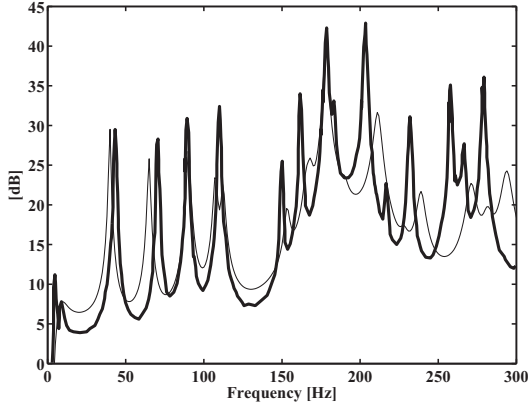
- (2) A sound-insulation layer is laid on this structure and the experimental FRF are measured again for the structure coupled with the sound-insulation layer.
- (3) A mean computational model of the structure is developed and the model is updated using the experimental FRF measured in point (1) above.
- (4) A mean computational model of the structure coupled with the sound-insulation layer is developed using the updated mean computational model of the structure and the simplified mean model of the sound-insulation layer. This simplified model provided by the fuzzy structure theory depends on unknown parameters $\underline{\xi}(\omega)$, $\underline{n}(\omega)$ and $\underline{\nu}(\omega)$ that we propose to identify using the experimental FRF measured in point (2) above. The following methodology is carried out:
- (4.i) Over all the frequency band \mathbb{B} , the mean damping rate $\underline{\xi}$ of the fuzzy part is assumed to be independent of ω and is fixed to its estimated value corresponding to the damping rate at the first experimental eigenfrequency.
- (4.ii) The mean modal density $\underline{n}(\omega)$ is obtained by performing a modal analysis with a very fine mesh finite element model of the sound-insulation layer embedded on its base. We then calculate the mean number $\underline{N}(\omega)$ of eigenfrequencies in the frequency band $[0, \omega]$ and then, by a numerical derivative, we deduce the mean modal density $\underline{n}(\omega)$ which is such that $\underline{N}(\omega) = \int_0^\omega \underline{n}(\alpha) d\alpha$. The first eigenfrequency for which the mean modal density is different from zero is defined as the cut-off frequency Ω_C . This cut-off frequency can be viewed as the frequency for which the sound-insulation layer begins to act as a power flow transmitter due to its own thickness resonances (the internal dynamical resonances taken into account by the fuzzy structure theory).
- (4.iii) We introduce the quantity $\Pi(\omega) = \underline{n}(\omega)\underline{\nu}(\omega)$. Over all the frequency band of analysis \mathbb{B} , the parameter $\Pi(\omega)$ is experimentally identified by solving an inverse problem which is formulated as an optimization problem.
- (4.iv) For $\omega \geq \Omega_C$, the coefficient of participating mass $\underline{\nu}(\omega)$ is deduced from the equation $\underline{\nu}(\omega) = \Pi(\omega)/\underline{n}(\omega)$ in which $\underline{n}(\omega)$ is calculated from point(4.ii) above.
- (4.v) For $\omega \leq \Omega_C$, we consider that the sound-insulation layer is equivalent to an added mass for which the coefficient of participating mass is then taken equal to 1 and with a dissipation due to the Joule effect. Theoretically, the modal density $\underline{n}(\omega)$ is zero for $\omega \leq \Omega_C$. Nevertheless, in order to obtain continuous functions in ω on \mathbb{B} for \underline{a}^s , \underline{a}^a and \underline{a}^c (defined by Eqs (24), (25), (26)) and since the coefficient of participating mass $\underline{\nu}(\omega) = 1$, we deduce the value of the mean modal density from the equation $\underline{n}(\omega) = \Pi(\omega)/\underline{\nu}(\omega) = \Pi(\omega)$.

4.1 Experimental configuration and measurements

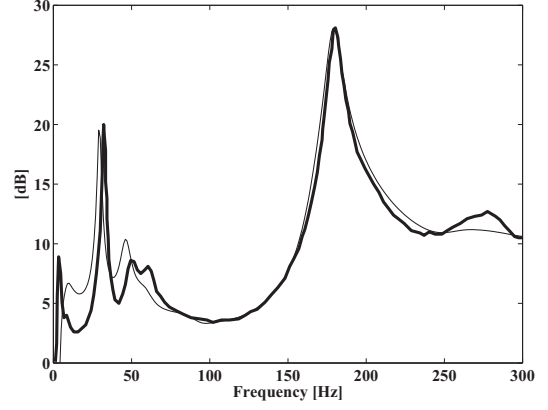
The experimental configuration is made up of a homogeneous, isotropic and slightly damped thin plate (steel plate with a constant thickness) connected to an elastic framework on its edges. This structure is set horizontally and is hung up by four soft springs in order to avoid rigid body modes. The highest eigenfrequency of suspension is 9 Hz while the lowest elastic mode of the structure is 43 Hz. The excitation is a point force applied to the framework and excites the dynamical system mainly in bending mode in the frequency band of analysis $\mathbb{B} =]0, 300]$ Hz. The number of sampling frequencies is $n_{\text{freq}} = 300$. The frequency resolution is $\Delta f = 1$ Hz. Only one experiment is performed for this structure. The frequency response functions $\omega \mapsto \gamma_i^{\text{exp}}(\omega)$ are identified on frequency band \mathbb{B} for $n_{\text{obs}} = 60$ normal accelerations in the plate measured with a laser velocimeter. We then construct the following experimental frequency response function, $\omega \mapsto r^{\text{exp}}(\omega) = 10 \log_{10} \left(\sum_{i=1}^{n_{\text{obs}}} |\gamma_i^{\text{exp}}(\omega)|^2 \right)$.

4.2 Experimental updating of the mean model of the structure without sound-insulation layer

The mean computational model of the structure is made up of a finite element model having $m_s = 57,768$ structural DOF. The reduced mean computational model is constructed with $n_s = 240$ structural modes. The



a) System without the sound-insulation layer



b) System with sound-insulation layer

FIG. 2 – Graphs of $\omega \mapsto r(\omega)$ (thin solid line) and $\omega \mapsto r^{\text{exp}}(\omega)$ (thick solid line)

mean computational model has been updated with respect to the Young modulus, the mass density and the damping rate of the plate and of the elastic framework using the experimental eigenvalues corresponding to the two first elastic modes and the ninth elastic mode (first elastic torsion mode) of the structure. The updated mean computational model will simply be called below the mean computational model. We introduce the function $\omega \mapsto r(\omega) = 10 \log_{10} \left(\sum_{i=1}^{n_{\text{obs}}} |\gamma_i(\omega)|^2 \right)$. Figure 2.a shows the comparison between the experimental measurements and the updated FRF of the mean computational model for the structure without the sound-insulation layer.

4.3 Experimental Identification of the parameters of the simplified model of the sound-insulation layer using a design methodology

A similar experimentation to the experiments described in section 4.2 has been carried out when the structure is coupled with the sound-insulation layer which is made up of a heterogeneous, anisotropic, poroelastic foam and of a heavily damped septum (EPDM). The sound-insulation layer is laid on the plate and is not connected to the elastic framework. The reduced mean computational model is written (see Eq. (35)) as $[\underline{A}^s(\omega) + \underline{a}^s(\omega)[\underline{B}^s]] \underline{q}^s(\omega) = \underline{f}^s(\omega)$. We use this reduced mean computational model to identify the three parameters $\underline{\xi}$, $\underline{n}(\omega)$ and $\underline{\nu}(\omega)$ of the simplified model of the sound-insulation layer for ω in \mathbb{B} . The methodology used is the following:

(4.3.1) As previously explained, the mean modal density is calculated using a refined finite element model of the sound-insulation layer (33, 210 DOF for the foam and 13, 284 DOF for the septum; there are $N = 1900$ elastic modes in the frequency band $[0, 450]$ Hz). The cut-off frequency Ω_C and the mean modal density $\underline{n}(\omega)$ are then deduced in the frequency band \mathbb{B} . We obtain $\Omega_C = 67 \times 2\pi \text{ rad.s}^{-1}$ and the graph of the smoothed function $\omega \mapsto \underline{n}(\omega)$ is given in Fig.3.a for frequency band \mathbb{B} . It should be noted that the modal density increases in the frequency band $[250, 300]$ Hz (it can also be seen in Fig.3.b this increase in the frequency band $[300, 450]$ Hz).

(4.3.2) In the frequency band $]0, \Omega_C[$, we remind that the sound-insulation layer is equivalent to an added mass and to a dashpot. The expression for the fuzzy coefficient $\underline{a}^s(\omega)$ is then $\underline{a}^s(\omega) = -\underline{\mu}\omega^2 + 2i\omega\underline{\mu}\underline{\xi}\Omega_C$, for $\omega \leq \Omega_C$. As explained in point (4.i) above, $\underline{\xi}$ is experimentally identified as the damping rate of the first elastic eigenmode of the structure with the sound-insulation layer. The identified value is $\underline{\xi} = 0.01$ and $\underline{\mu} = 5.9 \text{ kg.m}^{-2}$.

(4.3.3) We have chosen to directly identify the function $\omega \mapsto \Pi(\omega)$ as explained in point (4.iii) above. The optimization problem consists in minimizing the distance between the model and the experiments for the FRF $\omega \mapsto r(\omega; \Pi(\omega)) = 10 \log_{10} \left(\sum_{i=1}^{n_{\text{obs}}} |\gamma_i(\omega; \Pi(\omega))|^2 \right)$. Figure 3.b displays the graph of the identified function $\omega \mapsto \Pi(\omega)$ on the frequency band $[0, 450]$ Hz. It should be noted that we have not used an explicit

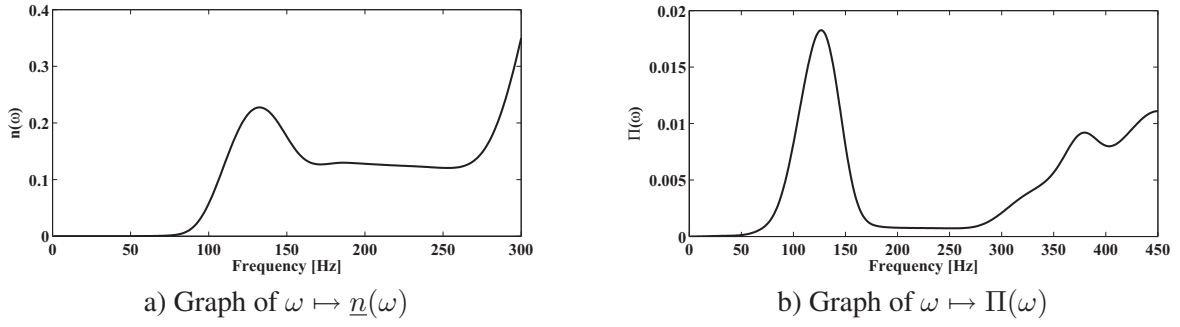


FIG. 3 – Mean modal density and Π function

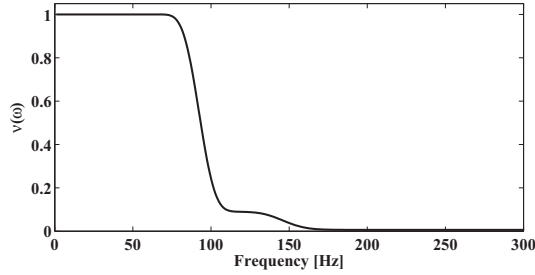


FIG. 4 – Graph of $\omega \mapsto \underline{\nu}(\omega)$

expression for $\omega \mapsto \Pi(\omega)$ such as the prototype mass-natural-frequency-distribution proposed in Ref [29]. In fact, Fig.3.b shows that the graph of $\omega \mapsto \Pi(\omega)$ which has experimentally been identified cannot be fitted by a Rayleigh distribution.

(4.3.4) In the frequency band $[\omega_{\min}; \Omega_C]$, we have $\underline{\nu}(\omega) = 1$ and the modal density $\underline{n}(\omega)$ which is theoretically equal to zero is in fact estimated as explained in point (4.v) above and yields a modal density which is very small but not exactly zero. In the frequency band $[\Omega_C, \omega_{\max}]$, we use the calculated mean modal density $\underline{n}(\omega)$ and the mean damping rate $\underline{\xi}$ in order to identify the mean coefficient of the participating mass $\underline{\nu}(\omega)$ as explained in point (4.iv) above. Figure 4 displays the updated mean coefficient $\underline{\nu}(\omega)$ of the participating mass and Fig.2.b shows the comparison between the experimental FRF and the FRF of the mean computational model for the structure with the sound-insulation layer calculated with the parameters experimentally identified.

(4.3.5) The use of the structural part of the vibroacoustic model allows the identification of the mean parameters $\underline{\nu}(\omega)$, $\underline{\xi}$ and $\underline{n}(\omega)$. It should be noted that this identification allows not only the coefficient $\underline{a}^s(\omega)$ to be identified but also the coefficients $\underline{a}^c(\omega)$ and $\underline{a}^a(\omega)$. Therefore, we only need the structural part of the vibroacoustic model to identify all of the mean computational simplified model of the sound-insulation layer (*i.e.* the structural, the coupling and the acoustic parts).

5 Prediction of the vibroacoustic responses with the identified computational stochastic model

In this section, we use the identified computational stochastic model to predict a vibroacoustic response. The response of the identified model is compared to a reference solution calculated with a commercial software. The structure and the sound-insulation layer models are defined in Section 4. The sound-insulation layer is coupled with a parallelepipedic acoustic cavity ($m_a = 23,354$ DOF and $n_a = 67$ modes) which is assumed to be without uncertainties and which is filled with air. We observe the pressure at $m_{\text{obs}} = 120$ points in the acoustic cavity while the excitation force is applied to the elastic framework of the structure as in Section 4.

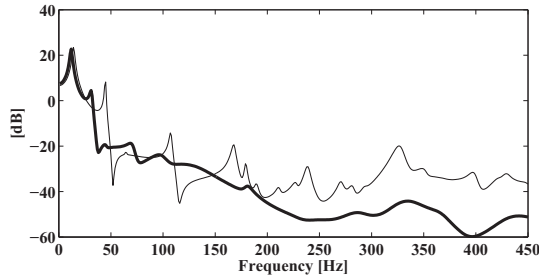


FIG. 5 – Graphs of $\omega \mapsto r^a(\omega)$ for the reference solution without sound-insulation layer (thin solid line) and with sound-insulation layer (thick solid line)

We then compute the FRF $\omega \mapsto r^a(\omega)$ relative to the acoustic cavity and Fig.5 displays its graph for the mean acoustic system with and without the sound-insulation layer. It can be seen that the two graphs match pretty well in the frequency band $[0, 200]$ Hz. This is a mean model obtained with the use of simplifications which introduce model uncertainties. In order to improve the prediction, it should be implemented a probabilistic model of uncertainties.

6 Conclusion

In this paper, a new extension of the fuzzy structure theory to elastoacoustic element is presented in order to construct a simplified model of sound-insulation layers. Such a simplified model, based on an extension of the fuzzy structure theory, (1) allows the dynamics of the sound-insulation layer to be taken into account without increasing the number of DOF in the computational vibroacoustic model and (2) allows a representation of the sound-insulation in terms of physical parameters such as its participating mass, its modal density and its internal damping rate to be obtained. This approach allows several kinds of sound-insulation layers to be simultaneously taken into account in the computational vibroacoustic model of a complex system such as a car with a very small increase of the computational cost. The complete related developments are given and an experimental validation is presented. Finally, an efficient design methodology is proposed to identify the parameters of the simplified model of the sound-insulation layer. The mean parameters are identified by solving an inverse problem formulated as an optimization problem using an experimental database. This methodology can be generalized to any kind of sound-insulation layer presenting internal resonances. In order to make the predictions more robust, it can be implemented a non-parametric probabilistic model of uncertainties. Such a modelling can address not only data uncertainties but also model uncertainties. That work has been made in [40] only for a structure coupled with a sound-insulation layer without acoustic cavity.

Acknowledgements

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A Formulation of the vibroacoustic problem with a sound-insulation layer

In this section, we give additional explanations relative to Section 2 useful for the construction of the simplified model of the sound-insulation layer. Let \mathcal{C}_0^s be the space of the admissible displacement fields of the structure, \mathcal{C}^a be the space of the admissible pressure fields in the acoustic cavity and \mathcal{C}^h be the space of the admissible displacement fields of the sound-insulation layer. For all ω in \mathbb{B} , we introduce the following bilinear form defined on $\mathcal{C}_0^s \times \mathcal{C}^a$,

$$c_{\Gamma_2}(\mathbf{u}^s, p) = \int_{\Gamma_2} \mathbf{u}^s(\mathbf{x}) \cdot \mathbf{n}^s(\mathbf{x}) p(\mathbf{x}) ds(\mathbf{x}) \quad , \quad (38)$$

the bilinear form defined on $\mathcal{C}^h \times \mathcal{C}^a$,

$$c_{\Gamma}(\mathbf{u}^h, p) = \int_{\Gamma} \mathbf{u}^h(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) p(\mathbf{x}) ds(\mathbf{x}) \quad , \quad (39)$$

and the linear form defined on \mathcal{C}_0^s or \mathcal{C}^h ,

$$c_{\Gamma_s}(\mathbf{u}; \omega) = \int_{\Gamma_s} \mathbf{f}^s(\mathbf{x}, \omega) \cdot \mathbf{u}(\mathbf{x}) ds(\mathbf{x}) \quad . \quad (40)$$

The weak formulation of the vibroacoustic boundary value problem is formulated as follows (see Ref. [13]). For all ω in \mathbb{B} , find $(\mathbf{u}^s, p, \mathbf{u}^h)$ in $\mathcal{C}_0^s \times \mathcal{C}^a \times \mathcal{C}^h$ such that for all $(\delta \mathbf{u}^s, p, \delta \mathbf{u}^h)$ in $\mathcal{C}_0^s \times \mathcal{C}^a \times \mathcal{C}^h$, we have, for the structure,

$$-\omega^2 m^s(\mathbf{u}^s, \delta \mathbf{u}^s) + i\omega d^s(\mathbf{u}^s, \delta \mathbf{u}^s; \omega) + k^s(\mathbf{u}^s, \delta \mathbf{u}^s; \omega) + c_{\Gamma_2}(\delta \mathbf{u}^s, p) = -c_{\Gamma_s}(\delta \mathbf{u}^s; \omega) + l^s(\delta \mathbf{u}^s; \omega) \quad , \quad (41)$$

for the acoustic cavity,

$$-\omega^2 m^a(p, \delta p) + i\omega d^a(p, \delta p; \omega) + k^a(p, \delta p) + \omega^2 \left\{ c_{\Gamma_2}(\mathbf{u}^s, \delta p) + c_{\Gamma}(\mathbf{u}^h, \delta p) \right\} = l^a(\delta p; \omega) \quad , \quad (42)$$

and for the sound-insulation layer,

$$-\omega^2 m^h(\mathbf{u}^h, \delta \mathbf{u}^h) + i\omega d^h(\mathbf{u}^h, \delta \mathbf{u}^h; \omega) + k^h(\mathbf{u}^h, \delta \mathbf{u}^h; \omega) + c_{\Gamma}(\delta \mathbf{u}^h, p) = c_{\Gamma_s}(\delta \mathbf{u}^h; \omega) \quad . \quad (43)$$

The bilinear forms m^s, d^s, k^s , (respectively m^a, d^a, k^a and respectively m^h, d^h, k^h) relative to the mass, damping and stiffness of the structure (respectively of the acoustic cavity and respectively of the sound-insulation layer) and the linear forms l^s and l^a related to the structural and acoustical excitations are defined in Ref. [13]. For instance, we have

$$m^s(\mathbf{u}^s, \delta \mathbf{u}^s) = \int_{\Omega_s} \rho^s(\mathbf{x}) \mathbf{u}^s(\mathbf{x}) \cdot \delta \mathbf{u}^s(\mathbf{x}) d\mathbf{x} \quad , \quad m^a(p, \delta p) = \frac{1}{\rho_0 c_0^2} \int_{\Omega_a} p(\mathbf{x}) \delta p(\mathbf{x}) d\mathbf{x}, \quad (44)$$

$$d^s(\mathbf{u}^s, \delta \mathbf{u}^s; \omega) = \int_{\Omega_s} b_{ijkl}(\mathbf{x}, \omega) \varepsilon_{kh}(\mathbf{u}^s) \varepsilon_{ij}(\delta \mathbf{u}^s) d\mathbf{x} \quad , \quad d^a(p, \delta p; \omega) = \tau(\omega) k^a(p, \delta p) \quad , \quad (45)$$

$$k^s(\mathbf{u}^s, \delta \mathbf{u}^s; \omega) = \int_{\Omega_s} a_{ijkl}(\mathbf{x}, \omega) \varepsilon_{kh}(\mathbf{u}^s) \varepsilon_{ij}(\delta \mathbf{u}^s) d\mathbf{x} \quad , \quad k^a(p, \delta p) = \frac{1}{\rho_0} \int_{\Omega_a} \nabla p \cdot \nabla \delta p d\mathbf{x}, \quad (46)$$

$$c_{\Gamma_2}(\mathbf{u}^s, \delta p) = \int_{\Gamma_2} \mathbf{u}^s(\mathbf{x}) \cdot \mathbf{n}^s(\mathbf{x}) \delta p(\mathbf{x}) ds(\mathbf{x}) \quad . \quad (47)$$

B Functions of the fuzzy coefficients introduced in section 2.5

For all $\omega \in \mathbb{B}$,

$$\Theta_R(\omega) = \frac{1}{4\sqrt{1-\underline{\xi}(\omega)^2}} \ln \left\{ \frac{N^+(\tilde{b}(\omega), \underline{\xi}(\omega)) N^-(\tilde{a}(\omega), \underline{\xi}(\omega))}{N^-(\tilde{b}(\omega), \underline{\xi}(\omega)) N^+(\tilde{a}(\omega), \underline{\xi}(\omega))} \right\}, \quad (48)$$

$$\Theta_I(\omega) = \frac{1}{2\sqrt{1-\underline{\xi}(\omega)^2}} \left[\Lambda(\tilde{b}(\omega), \underline{\xi}(\omega)) - \Lambda(\tilde{a}(\omega), \underline{\xi}(\omega)) \right], \quad (49)$$

$$N^\pm(u, \xi) = u^2 \pm 2u\sqrt{1-\xi^2} + 1, \quad \Lambda(u, \xi) = \arctan \left\{ \frac{u^2 + 2\xi^2 - 1}{2\xi\sqrt{1-\xi^2}} \right\}, \quad (50)$$

$$\lambda(\omega) = \frac{\tilde{\ell}(\omega)}{\omega \underline{n}(\omega)}, \quad \tilde{\ell}(\omega) = \frac{1}{\tilde{b}(\omega) - \tilde{a}(\omega)}, \quad \tilde{a}(\omega) = \frac{1}{\omega} a(\omega), \quad \tilde{b}(\omega) = \frac{1}{\omega} b(\omega). \quad (51)$$

C Graphs of the fuzzy coefficients

Figs.6.a-f display the graphs of the fuzzy coefficients defined by Eqs. (27) to (29) with Eqs. (48) to (51) for $\xi = 0.01$, $\underline{n}(\omega)$ given by Fig.3.a and $\underline{\nu}(\omega)$ given by Fig.4.

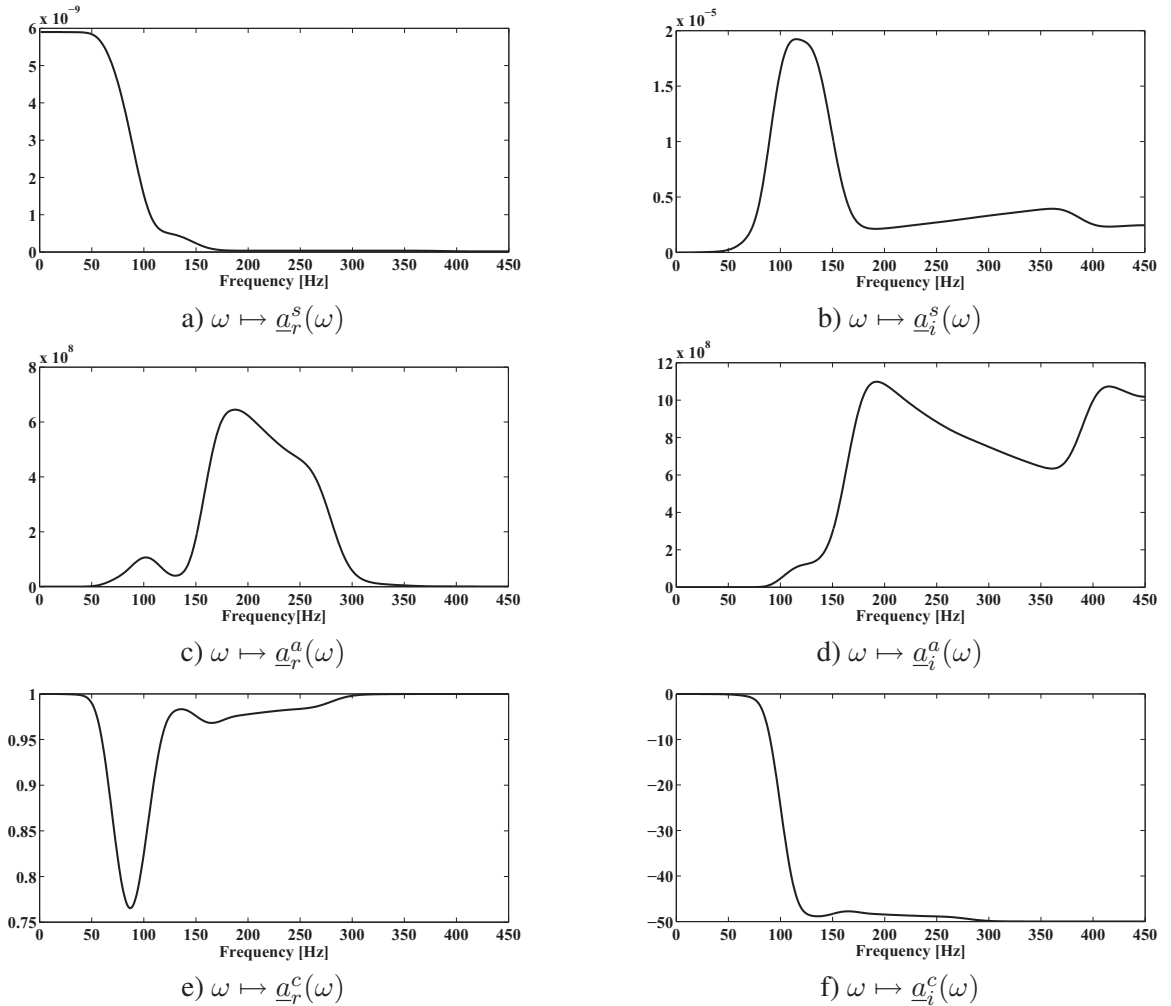


FIG. 6 – Graphs of the real and imaginary parts of the identified fuzzy coefficients