Probabilistic model and experimental identification of screw-attachment in plasterboard

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Abstract

In this paper, one provides a robust modelling for the screw-attachment of large light partition wall between plasterboard (CPC) plate and metallic frame. The analysis of shear behaviour of this attachment under mechanic loads has been carried out by using an experimental approach taking into account the complexity of the mechanical systems. A deterministic model is then proposed to fit the experimental results. A mean model is identified using the experiments. Since there is variability in the experimental results and since the mean model corresponds to a rough approximation, there are uncertainties in the mean model which are taken into account with a parameter probabilistic approach. The probabilistic approach of uncertain parameters is constructed using the Maximum Entropy Principle under the constraints defined by the available information. The identification of unknown parameters of the probability model is performed using the experimental data which lead us to an optimization problem which has to be solved. Finally, the numerical results are presented and validated with experiments.

Keywords: Screw attachment; Plasterboard; Probabilistic model; Experimental identification.

1 Introduction

Nowadays, lightweight metal frames are widely used in construction. This type of frames has many advantages such as, rapidity construction and building flexibility, the facility of assemblage and of dismantiling… They are used either as load-bearing elements such as in residential, office or industrials buildings, or as non load-bearing elements in partition walls and suspended ceilings. In this research, we are concerned by the behaviour of a non load-bearing element. The chosen element is a large light partition wall with plasterboard using metallic frame. The plasterboard [cardboard-plaster-cardboard (CPC) multilayer] screwed with a metal frame on both sides, and are made of a body of plaster stickled with two sheets of cardboard on both sides. They are linked with the metal frame using screws. The dimension of a large light partition currently reaches more than 10 meters. Its mechanical and thermomechanical behaviour can be analyzed with computational models such as finite element models. Validation can be obtained using experimental tests . However, experimental tests cannot be carried out when the structural dimensions exceed those of the testing furnaces (generally up to three meters). Given the complexity of such a mechanical system, uncertainties exist in the system parameters. One very efficient way to take into account uncertainties in the computational model is using the probability theory. Some previous works have been carried out in this field and a deterministic and a probabilistic model for thermomechanical analysis of plasterboard plate submitted to fire load was proposed [8][9][10] .The present work is a extension to large light partitions. The work is focused on the screwed attachment between the plasterboard plate and the metallic frame. A full computational model of the structure with the attachments would require to introduce a multiscale nonlinear micro-macro model to describe the behaviour of the screw between the plasterboard plates and the metallic frame. Such a model would be very difficult to develop and a lot of data would be missing to perform efficient caculations.This is why we didn't try to develop such an approach and we have preferred to analyze a shear behaviour of the screw in the plasterboard plate using an experimental analysis and then fitting an equivalent constitutive equation with the experimental databases. The first section deals with a shear analysis of such an attachment under mechanical loads which is carried out by using an experimental approach. The experimental results were performed by the load-displacement curves. In the second section, a deterministic model is then proposed to fit the average experimental results. The parameters of this mean model are identified experimentally. Since there are variability in the experimental results due to materials and manufacturing processes, and since the mean model corresponds to a rough approximation, uncertainties in the mean model are taken into account using a probabilistic approach. The next section consists in developing the probabilistic model which is constructed using the Maximum Entropy Principle [4][5] under the constraints defined by the available information. The identification of the unknown parameters of the probability model is performed again using the experimental data which leads us to the solution of the optimization problem to be solved. Finally, the numerical results are presented and validated with experiments.

Concerning the methodology used, the identification of the probabilistic model is performed in 2 steps. The first one is devoted to the first identification of the mean parameters of the shear behaviour for the screw attachment in minimizing a distance between the experimental average value and the average mean prediction. The second one consists in identifying both the mean parameter and the dispersion parameter of the probabilistic model starting from the mean value identified in step one. This means that step one must be viewed as the primary computation step to perform the global identification in step 2.

In this paper the number of experiments is limited to a small number which is 10. It should be noted that such a number is always small due to the cost of experiments. In this condition, the variability observed with this small number of experiments is not representative of real statistical fluctuations which could be observed if a large number of experiments was available. A simply average deterministic function with known limits for variability can not be used. Such a deterministic approach would not allow the probability to reach the bounds to be known. This is the reason why a probabilistic approach is used and the probability model is constructed with the powerful Information Theory. Finally, the great interest of such an approach is to propose a practical design solution based on a probabilistic approach and not in an usual deterministic approach. With such an approach, a nonlinear structural statistical probabilistic analysis of large light partition walls with plasterboards screwed with metallic frames on both sides can be carried out to take into account large statistical fluctuations in due to the shear behaviour for the screw attachment.

2 Experimental analysis of the shear behaviour for the screw-attachment

2.1 Description of the experimental data

In order to analyze the shear behaviour of the screw-attachment, experiments have been carried out using the experimental setup shown in Fig. 1 consisting in imposing a relative displacement between the plasterboard plate and the metallic frame. A sensor directly measures the vertical relative displacement between the plasterboard plate at the screw level and the metallic frame while another load sensor measures the load applied to the sample.

The experiments have been carried out with 10 samples. The relative displacement at the screw level has been limited to $x_{\max} = 5.17mm$. This limit corresponds to the upper value for practical application (see Figure 2 left). Figure 2 displays the measurements obtained.

2.2 Analysis of the experimental results

The experimental results for the 10 samples are presented by 10 load-displacement curves (see Figure 2 left). Figure 2 right displays the averaging of the 10 experimental curves. It can be seen that the experimental averaging curve is monotone

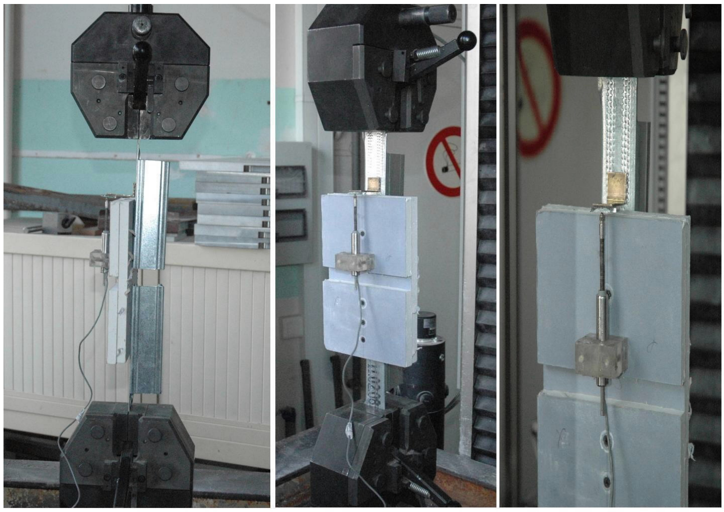

Figure 1: Photo of the experimental setup.

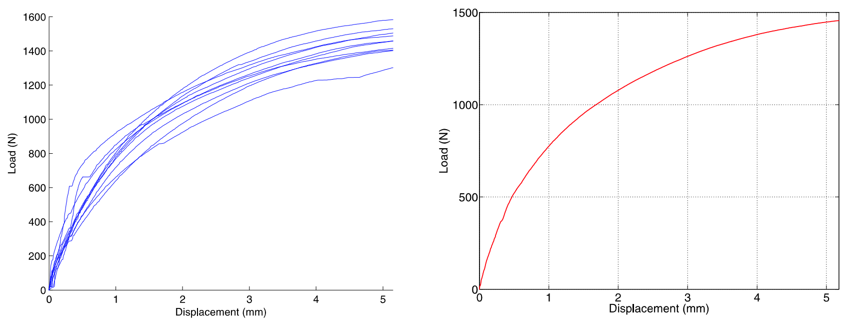

Figure 2: Experimental results of shear behavior of the screw-attachment. Load applied (vertical axis in N) as a function of displacement (horizontal axis in mm). 10 measures curves (left figure), averaging of the 10 curves (right figure).

increase, and then a strictly concave function on interval $[0, x_{\max}]$. The mean model of the shear behaviour which is constructed in the next Section will satisfy this fundamental property. It can also be seen that for the same value of the displacement, corresponding loads is uncertain, and conversely. Hence, a stochastic modelling is used to take into account these uncertainties

3 Mean model of shear behaviour of the screwed attachment and experimental identification of the mean model parameters.

The mean model of shear behaviour is constructed as an algebraic function which fits the experimental averaging curve. Denoting x the relative displacement and y the applied load, the mean model is written as

$$y\left(x\right) = a\left[\left(x+b\right)^{\alpha} - b^{\alpha}\right] \tag{1}$$

In Eq (1) a, b and α are three positive real parameters. parameter. We introduce the vector parameter $\mathbf{r}$ such that $\mathbf{r} = (a, b, \alpha)$ which belongs to an admissible subset $\Re$. Parameter $\mathbf{r}$ is a parameter which has to be identified using the experimental averaging curve and which will be called the identification parameter of the mean model.

Since function $x \mapsto y\left(x\right)$ must be strictly concave in $[0, x_{\max}]$ with positive values and such that the relative displacement is zero if load applied is zero, it can be deduced that for all $\mathbf{r}$ in $\Re$ and for all $x \in [0, x_{\max}]$,

$$\begin{cases} y\left(x\right) \geq 0 \\ y\left(0\right) = 0 \\ y'\left(x\right) = \alpha a\left(x+b\right)^{\alpha-1} > 0 \\ y''\left(x\right) = \alpha\left(\alpha-1\right)a\left(x+b\right)^{\alpha-2} < 0 \end{cases} \tag{2}$$

From Eq. (2), it can easily be deduced that parameters a, b and α have to be such that

$$a > 0, \quad b > 0, \quad 0 < \alpha < 1 \tag{3}$$

which shows that $\Re =]0,\ +\infty[\ \times\ ]0,\ +\infty[\ \times\ ]0,\ 1[\,.$

The mean model is fitted with the experimental average curve using the mean-square method solving the following optimization problem

$$\underline{\mathbf{r}} = \arg \min_{\mathbf{r}\,\in\,\Re} \int_0^{x_{\max}} \left(y\left(x\right) - \underline{y}^{\exp}\left(x\right)\right)^2 dx \tag{4}$$

where $\underline{y}^{\exp}$ is the experimental averaging curve.

4 Construction of the probability model to take into account uncertainties

As explained in Section 2, the variability of the experimental result are taken into account in modelling parameters a and b by two independent random variables A and B for which the mean values are $E\left\{A\right\} = \underline{a}$ and $E\left\{B\right\} = \underline{b}$ where E is the mathematical expectation. It should be noted that the independence hypothesis of random variables A and B is justified by the fact that no information variable concerning the statistical dependence of A and B. In addition, α is not modelled by a random variable and $\underline{\mathbf{r}}= (\underline{a}, \underline{b}, \underline{\alpha})$ will be considered as an updating deterministic parameter. Consequently, deterministic Eq. (1) is replaced by the random equation

$$Y\left(x\right)=A\left[\left(x+B\right)^{\underline{\alpha}}-B^{\underline{\alpha}}\right] \tag{5}$$

For physical reason, Y must be a second-order random variable which means that $E\left\{Y^2\right\}<+\infty$. It can be verified that this condition is satisfied if $E\left\{A^2\right\}<+\infty$ and $E\left\{B^2\right\}<+\infty$. From Eq. (5), it can be deduced that, if the applied load y is given, then the relative displacement x becomes a random variable X such that

$$X=\left(\frac{y}{A}+B^{\underline{\alpha}}\right)^{\frac{1}{\underline{\alpha}}}-B \tag{6}$$

Identically, for physical reason, $X^{\underline{\alpha}}$ must be a second-order random variable for all $\underline{\alpha}$ in $]0,\ 1[$ which means that $E\left\{X^{2\underline{\alpha}}\right\}<+\infty$. Such a condition is satisfied if $E\left\{A^{-2}\right\}<+\infty$. In addition this last condition implies that $E\left\{A^2\right\}<+\infty$. The available information of random variable A are then: (i) its support is $]0;\ +\infty[$, (ii) its mean value $E\left\{A\right\}=\underline{a}$, (iii) $E\left\{A^{-2}\right\}<+\infty$. The maximum entropy principle with this available informations yields for the probability density function $p_A(a)$ of A,

$$p_A(a)=\mathbf{1}_{]0,\ +\infty[}\left(a\right)\frac{1}{\underline{a}}\left(\frac{1}{\delta_A^2}\right)^{\left(\frac{1}{\delta_A^2}\right)}\frac{1}{\Gamma\left(\frac{1}{\delta_A^2}\right)}\left(\frac{a}{\underline{a}}\right)^{\frac{1}{\delta_A^2}-1}\exp\left(-\frac{a}{\underline{a}\delta_A^2}\right) \tag{7}$$

where $\delta_A=\sigma_A/\underline{a}$ is the coefficient of variation of A, satisfying $\delta_A<\sqrt{\alpha/2}$, σ_A is the standard deviation of A, Γ is the Gamma function and where $\mathbf{1}_K\left(a\right)=1$ if $a\in K$ and $=0$ if $a\notin K$. For the random variable B, the available information are (i) its support is $]0;\ +\infty[$, (ii) its mean value $E\left\{B\right\}=\underline{b}$, (iii) $E\left\{B^2\right\}=\underline{b}^2\left(1+\delta_B^2\right)<+\infty$. The probability density function is a truncated Gaussian function written as

$$p_B(b)=\mathbf{1}_{]0,\ +\infty[}\left(b\right)C_0\exp\left(-\lambda_1 b-\lambda_2 b^2\right) \tag{8}$$

where $(C_0,\lambda_1,\lambda_2)$ are the value calculated by solving the system of equations

$$\left\{\begin{array}{l} C_0\displaystyle\int_0^{+\infty} b.\exp\left(-\lambda_1 b-\lambda_2 b^2\right)db=\underline{b}\\ C_0\displaystyle\int_0^{+\infty} b^2.\exp\left(-\lambda_1 b-\lambda_2 b^2\right)db=\underline{b}^2\left(1+\delta_B^2\right)\\ C_0\displaystyle\int_0^{+\infty}\exp\left(-\lambda_1 b-\lambda_2 b^2\right)db=1\end{array}\right. \tag{9}$$

Consequently, probability density functions p_A and p_B depend only on vector $\underline{\mathbf{r}}$ and on dispersion vector parameter $\delta=(\delta_A,\delta_B)$ belonging to an admissible set Δ. Parameter δ allows the dispersion induced by uncertainties to be controlled.

5 Experimental identification of parameter

As explained in Section 4 there are two types of parameters which can be identified: the updating parameter $\underline{\mathbf{r}}$ and the dispersion parameter δ. Below these two parameters are identified by using the 10 experimental curves $y^{\exp,\,1}(x), ..., y^{\exp,\,10}(x)$ for $x \in [0, x_{\max}]$. The identification is performed in two steps. The first one consists in calculating $\underline{\mathbf{r}}_0$ and δ_0 as the solution of the optimization problem based on mean-squared method. The second step consists in improving this first identification using the maximum likelihood method. This non convex optimization problem is solved around the optimal points $(\underline{\mathbf{r}}_0, \delta_0)$ using the trial method.

6 Application and experimental validation

In this section, one presents the numerical application for the parameter identification and the validation with experimental data. The parameter of the mean model for shear behaviour screw attachment between plasterboard plate and the metallic frame defined in Section 3 is identified by minimizing the cost function defined in Eq. (4). The optimal parameter obtained is $\underline{\mathbf{r}}_0^{opt} = (\underline{a}, \underline{b}, \alpha) = (16598.73; 0.215; 0.028)$ .The comparison between mean model and average experimental result is presented in the figure 3.

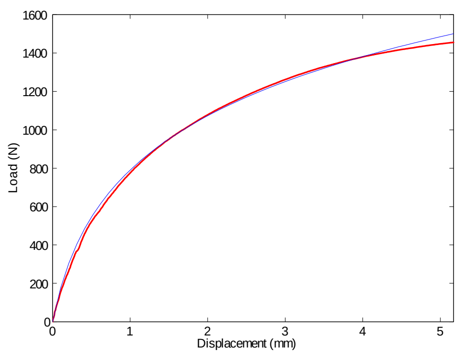

Figure 3: Comparison of the average experimental curve (thick solid line) with the mean model (thin solid line).

The stochastic model is then constructed by using Section 4. The vector-valued parameter $(\underline{\mathbf{r}}, \delta) = (\underline{a}, \underline{b}, \alpha, \delta_A, \delta_B)$ is identified as explained in Section 5 and yields $\underline{\mathbf{r}}^{opt} = (16210; 0.172; 0.0255)$ and $\delta^{opt} = (0.012, 0.2389)$. Figure 4 displays the confidence region for a probability level $P_C = 0.95$.

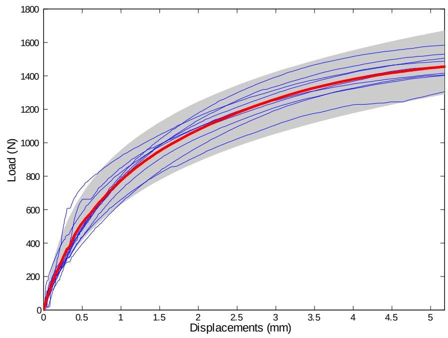

Figure 4: Maximum likelihood method. Comparison of the 10 experimental curves (ten thin solid lines) with (1) Average experimental data (thick solid line); (2) Confidence region of the optimal stochastic model (grey region)

7 Conclusion

In this paper, one has presented the construction and the experimental validation of a stochastic constitutive equation for screw-attachment. An experimental approach has been carried out to identify the shear behaviour of the attachment. A mean model then has been proposed to fit with the average experimental data. Due to data uncertainties and due to the variability of experimental data, a probabilistic model has been introduced to increase the robustness of the constitutive equation.

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