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UNCERTAINTIES IN DRILL DYNAMICS: THE ROLE OF A RANDOM
WEIGHT-ON-HOOK

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ABSTRACT

A drill-string is a slender structure that turns and drills into the rock in search of oil. This paper aims to investigate uncertainties on the weight-on-hook, which is the supporting force exerted by the hook at the top. In a drilling operation there are three parameters that can be continuously controlled: (1) the weight-on-hook, (2) the drilling fluid inlet velocity and (3) the rotational speed of the rotary table. The idea is to understand how the performance of the system (which is measured by the rate-of-penetration) is affected when the weight-on-hook is considered uncertain. A numerical model is developed using the Timoshenko beam theory and discretized by means of the Finite Element Method. The nonlinear dynamics presented includes bit-rock interaction, fluid-structure interaction and impacts. The probability theory is used to model the uncertainties. To construct the probability density function of the random variable, the Maximum Entropy Principle is employed, so that the probability distribution is derived from the mechanical properties of the weight-on-hook.

NOMENCLATURE

Π potential energy.
 T kinetic energy.
 U strain energy.
 W work done by nonconservative forces.
 δ variation symbol or dispersion parameter, depending on the context
 σ standard deviation.
 u displacement in x -direction.
 v displacement in y -direction.
 w displacement in z -direction.
 θ_a rotation about a -axis.
 N shape function.
 \mathbf{v} velocity vector.
 $\boldsymbol{\omega}$ angular velocity vector.
 $[I]$ cross section inertia matrix.
 I cross section moment of inertia.
 I_p cross section polar moment of inertia.
 ρ density.
 A area of the transversal section.
 D diameter.
 V volume.
 $\boldsymbol{\varepsilon}$ strain tensor (in voigt notation).

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- $[E]$ complete strain tensor.
- \mathbf{S} stress tensor (in voigt notation).
- E elasticity modulus.
- G shear modulus.
- \mathbf{X} position in the non-deformed configuration.
- \mathbf{p} displacement in the non-deformed configuration.
- F scalar force.
- p pressure.
- $[M]$ mass matrix.
- $[C]$ damping matrix.
- $[K]$ stiffness matrix.
- \mathbf{u} displacement vector.
- \mathbf{f} force vector.
- g acceleration of the gravity.
- S Shannon entropy measure.
- w_0 weight-on-hook.
- W_0 random weight-on-hook.
- $\mathbb{1}_B(x)$ assumes value 1 if x belong to B and 0 otherwise.

Subscripts

- br bit-rock.
- e element.
- f fluid.
- g geometric (for $[K]$) and gravity (for \mathbf{f}).
- ip impact.
- ke kinetic energy.
- se strain energy.
- NL nonlinear.
- S static response.
- ch channel (or borehole).
- i inside.
- o outside.

INTRODUCTION

In a drilling operation there are many sources of uncertainties as, for instance: the material properties of the column and the drilling fluid; the dimensions of the system, especially the borehole; the fluid-structure interaction; the bit-rock interaction, among others. This paper is concerned with the stochastic model of the weight-on-hook because it is one of the three parameters that are continuously controlled in a drilling operation (the other two are the drilling fluid inlet velocity and the rotational speed of the rotary table).

Figure 1 shows the general scheme of the system analyzed. The forces taking into account are: the motor torque (as a constant rotational speed at the top Ω_x); a constant hanging force w_0 ; the torque t_{bit} and force f_{bit} at the bit; the weight of the column; the fluid forces; the impact and rubbing between the column and the borehole; the forces due to the stabilizer; plus the elastic and kinetic forces due to the deformation and to the motion of the structure.

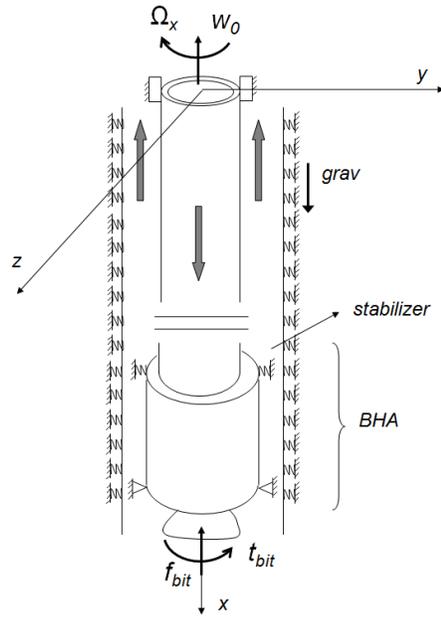


Figure 1. General scheme.

The sources of nonlinearity of the problem are the impact forces between the column and the borehole, and the bit-rock interaction forces (which are usually modeled with a dry friction model [1–4]). The nonlinear forces due to the finite strain are simplified because the system is linearized about a prestressed configuration. It is not obvious how uncertainties propagate in the nonlinear dynamics of a drill-string. To investigate that, we start by analyzing a random weight-on-hook which is a parameter that can be continuously controlled during a drilling operation. If we understand how uncertainties on the weight-on-hook affect the rate-of-penetration of the drill-string we may try to optimize the system performance.

There are few articles treating the stochastic problem of the drill-string dynamics, in special we might cite [5, 6]. In [5] the weight-on-bit is modeled as random in a simple two degrees of freedom drill-string model and in [6] lateral forces at the bit are modeled as random. In this paper, the probability density function of the weight-on-hook is constructed by means of the Maximum Entropy Principle [7–9].

The paper is organized as follows. First, the mean model is presented and the probabilistic model of the weight-on-hook is developed. Then the numerical results are shown and the concluding remarks are made.

MODEL

In this work the Timoshenko beam theory (which considers shear) is used because (1) it includes the Euler-Bernoulli theory and (2) it allows us to inquire about the importance of shear in the dynamics of this nonlinear system. A 3D beam model is used with six degrees of freedom in each interpolation point: the three translational displacements of the neutral line, axial u , lateral in y -axis v , and lateral in z -axis w , and the three rotations of the cross sections, about x -axis θ_x , about y -axis θ_y and about z -axis θ_z . It is assumed small angles (θ_y and θ_z) which is justified because the vibration of the column is constrained inside the borehole.

The equations of motion are derived using the extended Hamilton Principle, *i.e.*, the first variation of the potential Π must vanish:

$$\delta\Pi = \int_{t_1}^{t_2} (\delta U - \delta T - \delta W) dt = 0, \quad (1)$$

where U is the potential strain energy, T is the kinetic energy and W is the work done by the nonconservative forces and any force not accounted in the potential energy.

The system is discretized using the Finite Element Method: $u_e(x, t) = \mathbf{N}_u(x)\mathbf{u}_e(t)$, $v_e(x, t) = \mathbf{N}_v(x)\mathbf{u}_e(t)$, $w_e(x, t) = \mathbf{N}_w(x)\mathbf{u}_e(t)$, $\theta_{x_e}(x, t) = \mathbf{N}_{\theta_x}(x)\mathbf{u}_e(t)$, $\theta_{y_e}(x, t) = \mathbf{N}_{\theta_y}(x)\mathbf{u}_e(t)$, $\theta_{z_e}(x, t) = \mathbf{N}_{\theta_z}(x)\mathbf{u}_e(t)$. Where \mathbf{N} are the shape functions (see [10]) and $\mathbf{u}_e = (u_1 \ v_1 \ \theta_{z1} \ w_1 \ \theta_{y1} \ \theta_{x1} \ u_2 \ v_2 \ \theta_{z2} \ w_2 \ \theta_{y2} \ \theta_{x2})^T$.

Kinetic energy

The kinetic energy is written as

$$T = \frac{1}{2} \int_0^L (\rho A \mathbf{v}^T \mathbf{v} + \rho \omega^T [I] \omega) dx, \quad (2)$$

where ρ is the mass density, A is the cross sectional area, L is the length of the column, \mathbf{v} is the velocity vector, $[I]$ is the cross sectional inertia matrix and ω is the section angular velocity vector:

$$\mathbf{v} = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix}, [I] = \begin{pmatrix} I_p & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}, \omega = \begin{pmatrix} \dot{\theta}_x + \theta_y \dot{\theta}_z \\ \cos(\theta_x) \dot{\theta}_y - \sin(\theta_x) \dot{\theta}_z \\ \sin(\theta_x) \dot{\theta}_y + \cos(\theta_x) \dot{\theta}_z \end{pmatrix}.$$

The time derivative (d/dt) is denoted by a superposed dot. In the above expression I is the cross sectional moment of inertia and I_p is the polar moment of inertia. The angular velocity vector

ω is written in the inertial frame and it is derived by first rotating the inertial frame about the x -axis, θ_x , then rotating the resulting frame about the y -axis, θ_y and, finally, rotating the resulting frame about the z -axis, θ_z . It was done under the hypothesis of small angles θ_y and θ_z .

Note that $\dot{\theta}_x$ is not constant, therefore the discretization of the kinetic energy yields a constant mass matrix matrix $[M]$ and a nonlinear force \mathbf{f}_{ke} that couples axial, torsional and lateral vibrations.

Strain energy

The strain energy is written as:

$$U = \frac{1}{2} \int_V \boldsymbol{\varepsilon}^T \mathbf{S} dV, \quad (3)$$

where V is the volume, $\boldsymbol{\varepsilon}$ is the Green-Lagrange strain tensor and \mathbf{S} is the second Piola-Kirchhoff stress tensor. The stress-strain relationship is given by $\mathbf{S} = [D]\boldsymbol{\varepsilon}$ and may be written using the components

$$\begin{pmatrix} \sigma_{xx} \\ \tau_{xy} \\ \tau_{xz} \end{pmatrix} = \begin{bmatrix} E & 0 & 0 \\ 0 & k_s G & 0 \\ 0 & 0 & k_s G \end{bmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \gamma_{xy} \\ \gamma_{xz} \end{pmatrix}. \quad (4)$$

where E is the elasticity modulus, k_s is the shearing factor and G is the shear modulus. The complete strain tensor is obtained through

$$[E] = \frac{1}{2} \left[\left(\frac{d\mathbf{p}}{d\mathbf{X}} \right) + \left(\frac{d\mathbf{p}}{d\mathbf{X}} \right)^T + \left(\frac{d\mathbf{p}}{d\mathbf{X}} \right)^T \left(\frac{d\mathbf{p}}{d\mathbf{X}} \right) \right], \quad (5)$$

where $\varepsilon_{xx} = [E]_{11}$, $\gamma_{xy} = [E]_{21}$, $\gamma_{xz} = [E]_{31}$. The position in the non-deformed configuration is $\mathbf{X} = (x \ y \ z)^T$ and the displacements field $\mathbf{p} = (u_x \ u_y \ u_z)^T$, in the non-deformed configuration, is such that

$$\begin{aligned} u_x &= u - y\theta_z + z\theta_y, \\ u_y &= v + y(\cos(\theta_x) - 1) - z\sin(\theta_x), \\ u_z &= w + z(\cos(\theta_x) - 1) + y\sin(\theta_x), \end{aligned} \quad (6)$$

The discretization of the linear terms yields the stiffness matrix $[K]$ and the discretization of the nonlinear terms yields the

nonlinear force vector \mathbf{f}_{se} that couples axial, torsional and lateral vibrations.

In the numerical analysis, the dynamics is linearized about a prestress configuration, so the nonlinear forces are zero $\mathbf{f}_{se} = 0$ and the geometric stiffness matrix $[K_g(\mathbf{u}_S)]$ is used, where \mathbf{u}_S is the prestressed configuration.

Impact, rubbing and stabilizers

The drill-string may impact the borehole. If the radial displacement $r = \sqrt{v^2 + w^2}$ is large enough to hit the borehole there is a reaction force, *i.e.*, there is impact and rubbing if $(r - R_{ch} - R_o) > 0$, where R_{ch} is the radius of the borehole and R_o is the outer radius of the column. The impact is modeled as an elastic force

$$\begin{aligned} F_{yip} &= -k_{ip}(r - R_{ch} - R_o)\frac{v}{r}, \\ F_{zip} &= -k_{ip}(r - R_{ch} - R_o)\frac{w}{r}, \end{aligned} \quad (7)$$

where k_{ip} is the stiffness parameter of the impact. Rubbing between the column and the wall is modeled as a friction torque

$$T_{xip} = -\mu_{ip}F_n(R_o) \text{sign}(\dot{\theta}_x), \quad (8)$$

where $F_n = \sqrt{F_{yip}^2 + F_{zip}^2}$ and μ_{ip} is the friction coefficient.

Stabilizers are used in the BHA region to make the system stiffer, thus diminishing the amplitude of the lateral vibrations. Stabilizers are modeled as an elastic element and are located between the drill-pipes and the drill-collars:

$$F_y|_{x=stab} = k_{stab} v|_{x=stab} \quad \text{and} \quad F_z|_{x=stab} = k_{stab} w|_{x=stab}, \quad (9)$$

where k_{stab} is the stiffness parameter of the stabilizer and $x = stab$ is the location of the stabilizer.

Fluid-structure interaction

For short, the element equations are presented. These equations are an extension and an adaptation of the model developed

in [11].

$$\begin{aligned} [M_f]^{(e)} &= \int_0^1 (M_f + \chi \rho_f A_o) (\mathbf{N}_w^T \mathbf{N}_w + \mathbf{N}_v^T \mathbf{N}_v) l_e d\xi, \\ [K_f]^{(e)} &= \int_0^1 (-M_f U_i^2 - A_i p_i + A_o p_o - \chi \rho_f A_o U_o^2) (\mathbf{N}'_w^T \mathbf{N}'_w + \\ &+ \mathbf{N}'_v^T \mathbf{N}'_v) \frac{1}{l_e} d\xi + \int_0^1 \left(-A_i \frac{\partial p_i}{\partial x} + A_o \frac{\partial p_o}{\partial x} \right) (\mathbf{N}_{\theta_y}^T \mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T \mathbf{N}_{\theta_z}) l_e d\xi, \\ [C_f]^{(e)} &= \int_0^1 (-2M_f U_i + 2\chi \rho_f A_o U_o) (\mathbf{N}_{\theta_y}^T \mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T \mathbf{N}_{\theta_z}) l_e d\xi + \\ &+ \int_0^1 \left(\frac{1}{2} C_f \rho_f D_o U_o + k \right) (\mathbf{N}_w^T \mathbf{N}_w + \mathbf{N}_v^T \mathbf{N}_v) l_e d\xi, \\ \mathbf{f}_f^{(e)} &= \int_0^1 \left(M_f g - A_i \frac{\partial p_i}{\partial x} - \frac{1}{2} C_f \rho_f D_o U_o^2 \right) \mathbf{N}_u^T l_e d\xi. \end{aligned} \quad (10)$$

in which,

M_f is the fluid mass per unit length,

ρ_f is the density of the fluid,

$$\chi = \frac{(D_{ch}/D_o)^2 + 1}{(D_{ch}/D_o)^2 - 1} \quad (> 1),$$

D_{ch} is the borehole (channel) diameter,

D_i, D_o are the inside and outside diameters of the column,

U_i, U_o are the inlet and outlet flow velocities,

p_i, p_o are the pressures inside and outside the drill-string,

A_i, A_o are the inside and outside cross sectional area of the column,

C_f, k are the fluid viscous damping coefficients.

It is assumed that the inner and the outer pressures (p_i and p_o) vary linearly with x

$$p_i = (\rho_f g) x + p_{cte}, \quad (11)$$

$$p_o = \left(\rho_f g + \frac{F_{fo}}{A_o} \right) x, \quad (12)$$

where p_{cte} is a constant pressure and F_{fo} is the friction force due to the external flow given by

$$F_{fo} = \frac{1}{2} C_f \rho_f \frac{D_o^2 U_o^2}{D_h}. \quad (13)$$

In the above equation, D_h is the hydraulic diameter ($=4A_{ch}/S_{tot}$) and S_{tot} is the total wetted area per unit length ($\pi D_{ch} + \pi D_o$). Note that the reference pressure is $p_o|_{x=0} = 0$. Another assumption is that there is no head loss when the fluid passes from the drill-pipe to the drill-collar (and vice-versa). The head loss due to the change in velocity of the fluid at the bottom (it was going down, then it goes up) is given by

$$h = \frac{1}{2g}(U_i - U_o)^2. \quad (14)$$

Note that if the geometry and the fluid characteristics are given, only the inlet flow at $x = 0$ can be controlled because the fluid speed is calculated using the continuity equation and the pressures are calculated using the Bernoulli equation.

Bit-rock interaction model

The bit-rock interaction model chosen was the one developed in [2] basically for two reasons: (1) it is able to reproduce the main phenomena (as stick-slip oscillations); (2) it describes well the penetration of the bit into the rock (so we can analyze the rate-of-penetration-ROP). Usually the bit is considered fixed, [12–14], or an average rate of penetration is assumed, [1, 15]. The axial force and torque about the x -axis are written as

$$\begin{aligned} f_{\text{bit}} &= -\frac{\dot{u}_{\text{bit}}}{a_2 Z(\dot{\theta}_{\text{bit}})^2} + \frac{a_3 \dot{\theta}_{\text{bit}}}{a_2 Z(\dot{\theta}_{\text{bit}})} - \frac{a_1}{a_2}, \\ t_{\text{bit}} &= -\frac{\dot{u}_{\text{bit}} a_4 Z(\dot{\theta}_{\text{bit}})^2}{\dot{\theta}_{\text{bit}}} - a_5 Z(\dot{\theta}_{\text{bit}}), \end{aligned} \quad (15)$$

in which f_{bit} is the axial force, t_{bit} is the torque about the x -axis and $Z(\dot{\theta}_{\text{bit}})$ is the regularizing function.

In the above equation, the forces at the bit are written in function of the axial speed and the rotational speed of the bit: $f_{\text{bit}} = g_1(\dot{u}_{\text{bit}}, \dot{\theta}_{\text{bit}})$ and $t_{\text{bit}} = g_2(\dot{u}_{\text{bit}}, \dot{\theta}_{\text{bit}})$, where a_1, \dots, a_5 are positive constants which are related to the rock strength, the orientation of the cutting force, the geometry of the bit and the coefficient of friction. In [4] we can find a similar relationship with a detailed explanation about the constants of bit-rock interaction model.

In this model, the bit exerts only an axial force (f_{bit}) and a torque (t_{bit}) about the x -axis. The force and torque exerted by the rock at the bit depend on the axial speed (\dot{u}_{bit}) and the rotational speed ($\dot{\theta}_{\text{bit}}$) of the bit and they couple axial and torsional vibrations.

Discretized system

The final discretized system (considering the prestressed state) is written as [10]:

$$\begin{aligned} ([M] + [M_f])\ddot{\mathbf{u}} + ([C] + [C_f])\dot{\mathbf{u}} + ([K] + [K_f] + [K_g(\mathbf{u}_S)])\mathbf{u} = \\ = \mathbf{f}_{NL}(t, \mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}), \end{aligned} \quad (16)$$

where $\mathbf{u} (= \mathbf{u} - \mathbf{u}_S)$ is the response around the displacement vector $\mathbf{u}_S (= [K]^{-1}(\mathbf{f}_g + \mathbf{f}_c + \mathbf{f}_f))$ induced by the prestressed state. \mathbf{u} is the \mathbb{R}^m valued response in which m equals the number of degrees of freedom of the system. \mathbf{f}_g is the gravity force; \mathbf{f}_c is a concentrated reaction force at the bit; \mathbf{f}_f is the buoyancy force; $[M]$, $[C]$ and $[K]$ are the standard mass, damping and stiffness matrices; $[M_f]$, $[C_f]$, $[K_f]$ are the added fluid mass, damping and stiffness matrices; $[K_g(\mathbf{u})]$ is the geometric stiffness matrix; $\mathbf{f}_{NL}(t, \mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$ is the nonlinear force vector that is decomposed in:

$$\mathbf{f}_{NL}(t, \mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}) = \mathbf{f}_{ke}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}) + \mathbf{f}_{se}(\mathbf{u}) + \mathbf{f}_{ip}(\mathbf{u}) + \mathbf{f}_{br}(\dot{\mathbf{u}}) + \mathbf{g}(t). \quad (17)$$

where $\mathbf{f}_{ke}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$ are the quadratic forces due to the kinetic energy; $\mathbf{f}_{se}(\mathbf{u})$ are the quadratic and higher order forces due to the strain energy; $\mathbf{f}_{ip}(\mathbf{u})$ are the forces due to the impact and rubbing between the column and the borehole; $\mathbf{f}_{br}(\dot{\mathbf{u}})$ are the forces due to the bit-rock interactions; and $\mathbf{g}(t)$ is the force due to the Dirichlet boundary condition (imposed rotational speed at the top).

A reduced-order model is constructed from Eq. (16) using the normal modes obtained in the prestressed configuration.

Reduced model

Usually the final discretized FE system have big matrices (dimension $m \times m$) and the dynamic analysis may be time consuming, which is the case of the present analysis. One way to reduce the system is to project the nonlinear dynamical equation on a subspace $V_n \in \mathbb{R}^m$, with $n \ll m$, in which V_n is spanned by an algebraic basis of \mathbb{R}^n . In this paper, the basis used for the reduction corresponds to the normal modes projection, but, as it will be pointed out later, these normal modes have to properly be chosen (they can not be taken simply in the order that they appear). The normal modes are obtained from the following generalized eigenvalue problem,

$$([K] + [K_f] + [K_g(\mathbf{u}_S)])\phi = \omega^2([M] + [M_f])\phi, \quad (18)$$

where ϕ_i is the i -th normal mode and ω_i is the i -th natural frequency. Using the representation

$$\mathbf{u} = [\Phi] \mathbf{q}, \quad (19)$$

and substituting it in the equation of motion yields

$$([M] + [M_f])[\Phi]\ddot{\mathbf{q}} + ([C] + [C_f])[\Phi]\dot{\mathbf{q}} + ([K] + [K_f] + [K_g(\mathbf{u}_S)])[\Phi]\mathbf{q} = \mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}), \quad (20)$$

where $[\Phi]$ is a $(m \times n)$ real matrix composed by n normal modes obtained using the prestressed configuration. Projecting the equation on the subspace spanned by these normal modes yields

$$[\Phi]^T([M] + [M_f])[\Phi]\ddot{\mathbf{q}} + [\Phi]^T([C] + [C_f])[\Phi]\dot{\mathbf{q}} + [\Phi]^T([K] + [K_f] + [K_g(\mathbf{u}_S)])[\Phi]\mathbf{q} = [\Phi]^T \mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}), \quad (21)$$

which can be rewritten as

$$[M_r]\ddot{\mathbf{q}}(t) + [C_r]\dot{\mathbf{q}}(t) + [K_r]\mathbf{q}(t) = [\Phi]^T \mathbf{f}_{NL}(t, \bar{\mathbf{u}}, \dot{\bar{\mathbf{u}}}, \ddot{\bar{\mathbf{u}}}), \quad (22)$$

in which

$$[M_r] = [\Phi]^T([M] + [M_f])[\Phi], \quad [C_r] = [\Phi]^T([C] + [C_f])[\Phi]$$

$$[K_r] = [\Phi]^T([K] + [K_f] + [K_g(\mathbf{u}_S)])[\Phi] \quad (23)$$

are the reduced matrices.

PROBABILISTIC MODEL OF THE WEIGHT-ON-HOOK

The weight-on-hook w_0 is modeled as a random variable W_0 . The Maximum Entropy Principle [7, 8] which was introduced in the framework of the Information Theory by [9] is used to construct the probability density function. This principle consists in finding the probability density function that maximizes the entropy under the constraints defined by the available information.

The available information is derived from the mechanical properties of the weight-on-hook. These properties are: (1) the column must penetrate the soil, *i.e.*, W_0 must be lower than the weight w_2 of the column; (2) bucking must not occur, *i.e.*, W_0 must be greater than the bucking limit w_1 ; (3) the probability must go to zero when W_0 approaches w_1 ; (4) the probability must go to zero when W_0 approaches w_2 .

Conditions (1) and (2) are expressed by setting the support of the probability density function as $]w_1, w_2[$. Conditions (3) and (4) are expressed by $E\{\ln(W_0 - w_1)\} = \tilde{c}_1$ and $E\{\ln(w_2 - W_0)\} = \tilde{c}_2$, with $|\tilde{c}_1| < +\infty$ and $|\tilde{c}_2| < +\infty$. The reason why (\ln) is used is because it imposes a weak decreasing of the probability density function in w_1^+ and w_2^- . To facilitate

the calculus, we introduce a normalized random variable X with values in $]0, 1[$, such that:

$$W_0 = w_1(1 - X) + w_2X. \quad (24)$$

The expected value of W_0 is written as

$$E\{W_0\} = w_1(1 - E\{X\}) + w_2E\{X\}. \quad (25)$$

We introduce the notation $\underline{w}_0 = E\{W_0\}$ and $\underline{x} = E\{X\}$. The second moment of W_0 may be written as

$$E\{W_0^2\} = E\{X^2\}(w_1^2 - 2w_1w_2 + w_2^2) + \underline{x}(-2w_1^2 + 2w_1w_2) + w_1^2. \quad (26)$$

The available information is re-expressed in term of the new random variable:

1. $X \in]0, 1[$.
2. $E\{\ln(X)\} = c_1$.
3. $E\{\ln(1 - X)\} = c_2$.

(27)

with $|c_1| < +\infty$ and $|c_2| < +\infty$. The optimization problem is finally written as:

$$\max_{p_X \in \mathcal{C}} S_X(p_X), \quad (28)$$

where \mathcal{C} is the space of admissible probability density functions satisfying the constraints given by Eq. (27) and the entropy measure is given by [9]:

$$S_X(p_X) = - \int_{\mathbb{R}} p_X(x) \ln(p_X(x)) dx. \quad (29)$$

The probability density function, solution of the optimization problem defined by Eq. (28), is the Beta probability density function which may be written as:

$$p_X(x) = \mathbb{1}_{]0, 1[}(x) \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad (30)$$

where the Gamma function $\Gamma(y) = \int_0^{+\infty} t^{y-1} \exp(-t) dt$ for $y > 0$. Also, $\alpha > 2$ and $\beta > 2$ so that Eq. (27) holds. The random generator of independent realizations of the random variable X is already implemented in many computer codes. The mean value of X is given by

$$\bar{x} = \frac{\alpha}{\alpha + \beta}, \quad (31)$$

and coefficient of variation is given by

$$\delta_X = \sqrt{\frac{\beta}{\alpha(\alpha + \beta + 1)}}. \quad (32)$$

where $\delta_X = \sigma_X / \bar{x}$, in which σ_X is the standard deviation.

The probability density function p_{W_0} defined by Eq. (24) and (30) depends on the four parameters (w_1, w_2, α, β). For applications, w_1 and w_2 will be fixed. Parameters α and β have no physical meaning, consequently, we express them as function of the physical meaningful parameters \underline{w}_0 and δ . After some manipulations we obtain:

$$\bar{x} = \frac{w_0 - w_1}{w_2 - w_1}, \quad (33)$$

$$\delta_X = \sqrt{\frac{w_0^2(\delta^2 + 1) - (\underline{w}_0 - w_1)^2 - 2w_1(\underline{w}_0 - w_1) - w_1^2}{(\underline{w}_0 - w_1)^2}}, \quad (34)$$

and

$$\alpha = \frac{\bar{x}}{\delta^2} \left(\frac{1}{\bar{x}} - \delta_X^2 - 1 \right), \quad (35)$$

$$\beta = \frac{\bar{x}}{\delta_X^2} \left(\frac{1}{\bar{x}} - \delta_X^2 - 1 \right) \left(\frac{1}{\bar{x}} - 1 \right). \quad (36)$$

Using the probabilistic model of the weight-on-hook, the deterministic reduced model defined by Eqs. (19) and (22) is replaced by the following stochastic equations:

$$\begin{aligned} \ddot{\mathbf{U}} &= [\Phi] \mathbf{Q}, \\ ([M] + [M_f]) \ddot{\mathbf{Q}} + ([C] + [C_f]) \dot{\mathbf{Q}} + ([K] + [K_f] + [K_g(\mathbf{u}_S)]) \mathbf{Q} &= \\ &= [\Phi]^T (\mathbf{f}_{NL}(t, \bar{\mathbf{U}}, \dot{\bar{\mathbf{U}}}, \ddot{\bar{\mathbf{U}}}) + \mathbf{F}_{W_0}), \end{aligned} \quad (37)$$

where $\bar{\mathbf{U}}$ is the random response and \mathbf{F}_{W_0} is a vector for which the only nonzero component is related to the axial d.o.f. of the first node $\mathbf{F}_{W_0}(1) = (W_0 - \underline{w}_0)$. Note that \underline{w}_0 was subtracted because the response is calculated in the prestressed configuration.

NUMERICAL RESULTS

The drill-string was discretized using 56 finite elements. For the dynamics analysis it was used 10 lateral modes, 10 torsional modes, 10 axial modes and also the two rigid body modes of the structure (axial and torsional), so matrix $[\Phi]$ is composed by 32 modes. For the time integration procedure, a routine based on the Newmark integration scheme was implemented with a predictor and a fix point procedure to equilibrate the system response at each time step. The data used in the simulations is found in the Appendix.

The nonlinear forces \mathbf{f}_{ke} and \mathbf{f}_{se} are important to the dynamic response of the system (especially to the lateral vibrations), but the torsional and axial displacements are not very affected when $\mathbf{f}_{ke} = \mathbf{f}_{se} = 0$ in the first twenty seconds of simulations. The torsional and axial displacements are mainly dictated by the bit-rock interaction model. Moreover, the time to perform the numerical simulation is around: 60 minutes when \mathbf{f}_{ke} and \mathbf{f}_{se} are considered; and 80 seconds when $\mathbf{f}_{ke} = \mathbf{f}_{se} = 0$. As we want to investigate the influence of the probabilistic model of the weight-on-hook (axial force) on the rate-of-penetration (axial speed) we will set $\mathbf{f}_{ke} = \mathbf{f}_{se} = 0$ for the next simulations, knowing that it is an approximation of the problem analyzed.

Response of the stochastic system

For the system considered $w_1 = 408$ kN, $w_2 = 1078$ kN and $\underline{w}_0 = 978$ kN. Fig. 2 shows the 95% envelope (that is to say the confidence region constructed with a probability level of 0.95) for the rate-of-penetration and the rotational speed of the bit for a standard deviation $\sigma = 1000$ N, which means $\delta = \sigma / \underline{w}_0 \sim 0.001$. The envelopes (the upper and lower envelopes of the confidence region) are calculated using the method of quantiles [16].

We are plotting two important variables: the rate-of-penetration (ROP) and the rotational speed at the bit (ω_{bit}). So, we analyze the influence of the random weight-on-hook in the system performance. Fig. 3 shows the stochastic response of the torque and force on the bit.

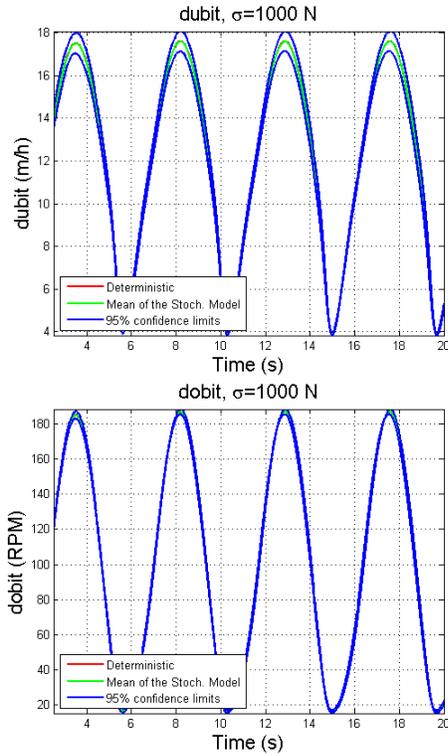


Figure 2. 95% envelope for $\sigma = 1000$ N. Upper: rate-of-penetration, ROP. Bottom: rotational speed of the bit.

It is noted that for $\sigma = 1000$ N, the response changes just a little, therefore σ will be increased in the next analysis. In our analysis we can not increase σ too much because the model used for the bit-rock interaction assumes a weight-on-bit $f_{bit} \sim -100$ kN, so the standard deviation σ of the W_0 is increased in a way that the f_{bit} has a maximum variation around 5%, that is to say that $\sigma_{max} = 5000$ N and therefore $\delta_{max} \sim 0.005$ (0.5% variation), which is a constraint to our analysis. But, as it will be seen, a small variation on the W_0 may cause a big variation in the system response.

Fig. 4 shows the system response for $\sigma = 5000$ N ($\delta \sim 0.005$). As expected, as σ increases the envelope of the response gets wider. We want to investigate how uncertainties on the weight-on-hook affects the performance of the system, so, Fig. 5 shows the evolution of the dispersion of the response for the: ROP, rotational speed of the bit, torque-on-bit and force-on-bit. The dispersion of the response is calculated by taken the square root of the variance divided by the value of the mean response for each time instant.

It is noted that, even for a small variation of W_0 ($\delta \sim 0.5\%$), there is a big dispersion in the response. See for instance the rate-of-penetration: the mean dispersion is 4.3%, which is more than eight times greater than the dispersion of the W_0 . It gets worse if

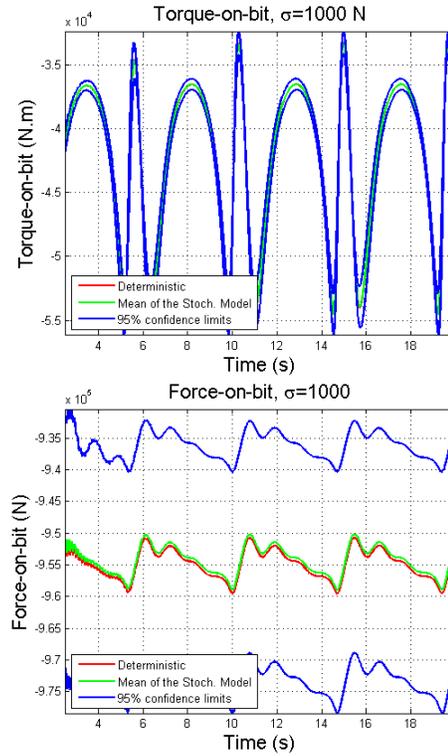


Figure 3. 95% envelope for $\sigma = 1000$ N. Upper: torque-on-bit. Bottom: force-on-bit.

we take the maximum dispersion, which is 16%. It means that if the W_0 has a dispersion of half percent, the variation in the ROP may achieve sixteen percent and the variation of the rotational speed of the bit may achieve twenty six percent!

CONCLUDING REMARKS

In this paper a stochastic model of the drill-string dynamics was analyzed. The main efforts that the column is subjected to were considered: rotation at the top; hanging force at the top; bit-rock interaction; fluid-structure interaction that takes into account the drilling fluid that flows downwards the column then goes upwards in the annulus; impact and rubbing between the column and the borehole; and the own weight of the column.

The weight-on-hook was modeled as a random variables with probability density function constructed using the Maximum Entropy Principle. This probability model is well suited for a random variable that assumes values in a finite interval $[a, b]$ and the only further information given is that the probability must go to zero when the random variable approaches each limit (a and b). It was noted that a small variation of the weight-on-bit (half percent) induces a considerable variation on the system response, for example: 6% on the force-on-bit, 12% on the torque-on-bit,

16% on the ROP and 26% on the ω_{bit} (these are the maximum dispersions observed). The system response is very sensitive to a dispersion of the weight-on-hook.

There are many sources of uncertainties in this problem, so more stochastic analysis should be done to identify the uncertainties that affect the most the performance of the system.

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DATA USED IN THE SIMULATION

$\Omega_x = 100$ [RPM] (imposed rotational speed about x -axis at $x = 0$ m), $f_c = 100$ [kN] (initial reaction force at the bit), $L_{dp} = 1400$ [m] (length of the drill pipe), $L_{dc} = 200$ [m] (length of the drill collar), $D_{odp} = .127$ [m] (outside diameter of the drill pipe), $D_{odc} = .2286$ [m] (outside diameter of the drill collar), $D_{idp} = .095$ [m] (inside diameter of the drill pipe), $D_{idc} = 0.0762$ [m] (inside diameter of the drill collar), $D_{ch} = 0.3$ [m] (diameter of the borehole (channel)), $x_{stab} = 1400$ [m] (location of the stabilizer), $k_{stab} = 17.5$ [MN/m] (stiffness of the stabilizer per meter), $E = 210$ [GPa] (elasticity modulus of the drill string material), $\rho = 7850$ [kg/m³] (density of the drill string material), $\nu = .29$ [-] (poisson coefficient of the drill string material), $k_s = 6/7$ [-] (shearing correcting factor), $c_1 = 0.05$ [N.s/m] (friction coefficient for the axial rigid body motion), $c_2 = 0.05$ [N.s/m] (friction coefficient for the rotation rigid body motion), $k_{ip} = 1e8$ [N/m] (stiffness per meter used for the impact), $\mu_{ip} = 0.0005$ [-] (friction coefficient between the string and the borehole), $U_i = 1.5$ [m/s] (flow velocity in the inlet), $\rho_f = 1200$ [kg/m³] (density of the fluid), $C_f = .0125$ [-] (fluid viscous damping coefficient), $k = 0$ [-] (fluid viscous damping coefficient), $g = 9.81$ [m/s²] (gravity acceleration), $a_1 = 3.429e - 3$ [m/s] (constant of the bit-rock interaction model), $a_2 = 5.672e - 8$ [m/(N.s)] (constant of the bit-rock interaction model), $a_3 = 1.374e - 4$ [m/rd] (constant of the bit-rock interaction model), $a_4 = 9.537e6$ [N.rd] (constant of the bit-rock interaction model), $a_5 = 1.475e3$ [N.m] (constant of the bit-rock interaction model), $e = 2$ [rd/s] (regularization parameter). The damping matrix is constructed using the relationship $[C] = \alpha([M] + [M_f]) + \beta([K] + [K_f] + [K_g(\mathbf{u}_S)])$ with $\alpha = .01$ and $\beta = .0003$.

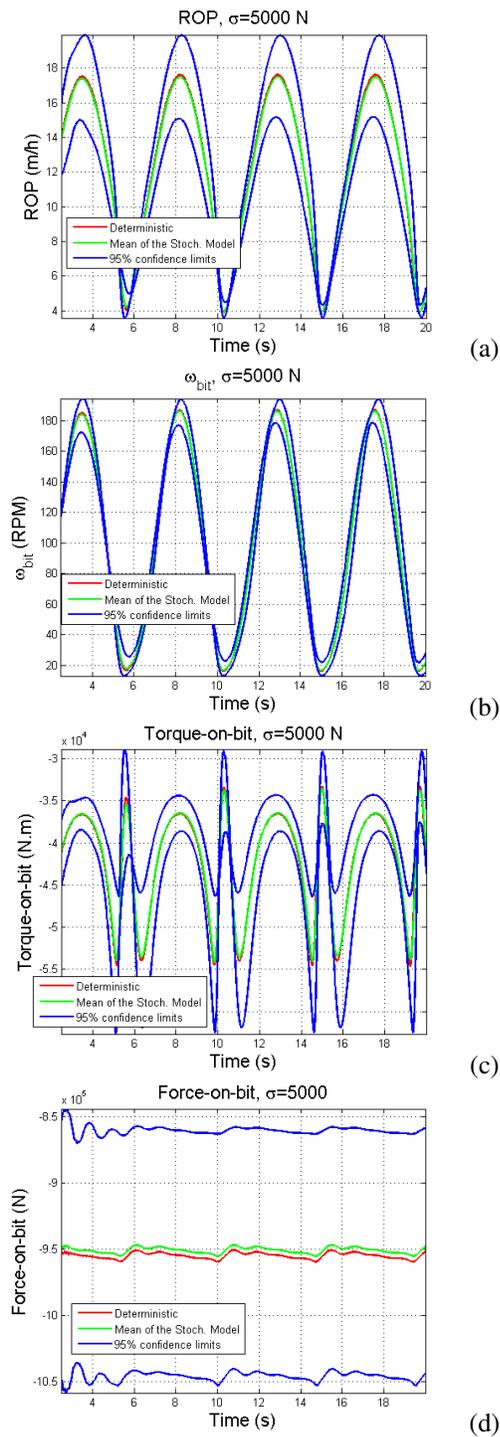


Figure 4. 95% envelope for $\sigma = 5000$ N. (a) rate-of-penetration, ROP; (b) rotational speed of the bit; (c) torque-on-bit; and (d) force-on-bit.

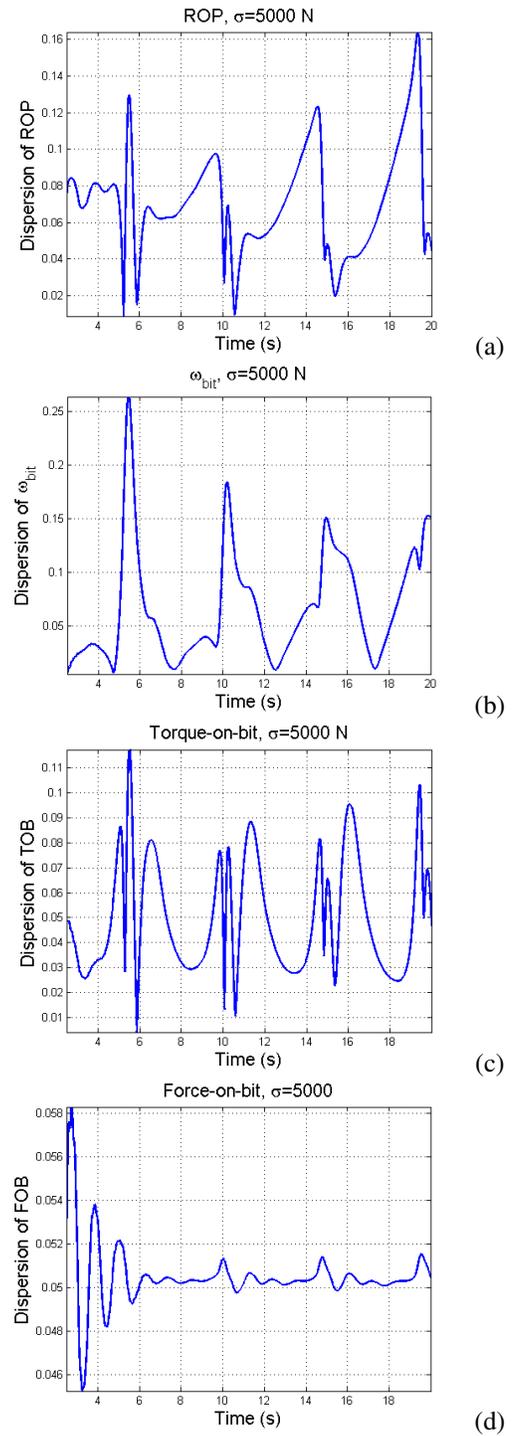


Figure 5. Dispersion of the response for $\sigma = 5000$ N. (a) rate-of-penetration, ROP; (b) rotational speed of the bit; (c) torque-on-bit; and (d) force-on-bit.